Stabilizing Receding Horizon Control for Three Dimensional Visual Feedback System with Fixed Camera Configuration

Toshiyuki Murao and Masayuki Fujita

***Abstract*— This paper investigates vision based robot control based on a receding horizon control strategy, as a first step for a predictive visual feedback control. Firstly, the brief summary of the 3D dynamic visual feedback system with fixed camera configuration is given. Next, a stabilizing receding horizon control for the 3D dynamic visual feedback system, a highly nonlinear and relatively fast system, is proposed. The stability of the receding horizon control scheme is guaranteed by using the terminal cost derived from an energy function of the visual feedback system.**

***Keywords*— Visual Feedback Control, Lyapunov Stability, Passivity, Receding Horizon Control**

I. Introduction

Robotics and intelligent machines need sensory information to behave autonomously in dynamical environments. Visual information is particularly suited to recognize unknown surroundings. In this sense, vision is one of the highest sensing modalities that currently exist. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, is important when we consider a mechanical system working in dynamical environments [1], [2].

In previous works, for the problem of three dimensional (3D) visual servo control, Kelly $et\ al.$ [3] considered an image based controller under the assumption that the objects' depths are known. Cowan $et\ al.$ [4] addressed the field-of-view problem for 3D dynamic visual feedback system using navigation functions. Although good solutions to the set-point problem are reported in those papers, few results have been obtained for the tracking problem of moving target objects in the full 3D dynamic visual feedback system that include not only the position and the orientation but also the manipulator dynamics. The authors discussed passivity based control for a moving target object in 3D workspace with eye-in-hand configuration [5], [6] and fixed camera configuration [7]. However, the control law proposed in [5]–[7] is not based on optimization, the desired control performance cannot be guaranteed explicitly.

Receding horizon control, also recognized as model predictive control is a well-known control strategy in which the current control action is computed by solving, a finite horizon optimal control problem on-line [8]. A large number of industrial applications using model predictive control can be found in chemical industries where the processes have relatively slow dynamics. On the contrary, for nonlinear and relatively fast systems such as in robotics, few implementations of the receding horizon control have been reported. For the receding horizon control, many researchers have tackled the problem of stability guarantees. An approach proposed by Parisini $et\ al.$ [9] is based on using a quadratic endpoint penalty of the form $ax^T(t+T)Px(t+T)$ for some $a > 0$, some positive definite matrix P and a terminal state $x(t+T)$. Jadbabaie $et\ al.$ [10] showed that closed-loop stability is ensured through the use of a terminal cost consisting of a control Lyapunov function. Moreover, these results were applied to the Caltech Ducted Fan to perform aggressive maneuvers [11], [12]. Visual feedback, however, is not considered here. Predictive control could be of significant benefit when used in conjunction with visual servoing. With the incorporation of visual information, the system could anticipate the target's future position and be waiting there to intercept it [13]. In [14], the authors proposed stabilizing receding horizon control for the planar visual feedback system. Moreover, for the 3D dynamic visual feedback system with eye-in-hand configuration, a stabilizing receding horizon control is derived in [15]. But the 3D dynamic visual feedback system has only three coordinate frames, while visual feedback systems typically use four coordinate frames as in Fig. 1. Because the camera is attached to the end-effector of robots, the camera frame represents the hand one in eye-in-hand configuration. This may occur that the visual feedback systems cannot be represented in eye-in-hand configuration, such as the autonomous injection of

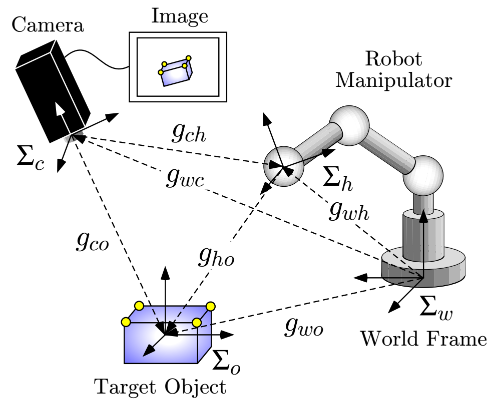

Fig. 1. Visual feedback system with fixed camera configuration.

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biological cells [16] and so on.

In this paper, as a first step for a predictive visual feedback control, stabilizing receding horizon control is applied to the 3D visual feedback system with fixed camera configuration in [7], a highly nonlinear and relatively fast system. Compared with the previous work [15], because the passivity-based visual feedback system with eye-in-hand configuration [5] can be regarded as the special case of that with fixed camera configuration [7], then the possible application area should be undoubtedly increasing. This represents a first step towards high performance visual servoing targeting more aggressive maneuvers. The main idea is the use of the terminal cost derived from an energy function of the visual feedback system.

Firstly, the brief summary of our prior work [7], which is the 3D dynamic visual feedback system with fixed camera configuration, is given. Next, a stabilizing receding horizon control for the 3D visual feedback system using a control Lyapunov function is proposed.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times 3}$ to represent the change of the principle axes of a frame Σ_b relative to a frame Σ_a. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. For simplicity we use $\hat{\xi}\theta_{ab}$ to denote $\hat{\xi}_{ab}\theta_{ab}$. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$', i.e., $so(3) \rightarrow \mathcal{R}^3$. Recall that a skew-symmetric matrix corresponds to an axis of rotation (via the mapping $a \mapsto \hat{a}$). We use the 4×4 matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \tag{1}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$ describing the configuration of a frame Σ_b relative to a frame Σ_a. The adjoint transformation associated with g_{ab} is denoted by $\mathrm{Ad}_{(g_{ab})}$ [17].

II. Passivity-based Visual Feedback System with Fixed Camera Configuration

In this section, the brief summary of our prior work in [7] is given. An energy function and a stabilizing control law, which play an important role for a predictive visual feedback control, are derived.

A. Basic Representation for Visual Feedback System

Visual feedback systems typically use four coordinate frames which consist of a world frame Σ_w, a target object frame Σ_o, a camera frame Σ_c and a hand (end-effector) frame Σ_h as in Fig. 1. Then, g_{wh}, g_{wc} and g_{wo} denote the rigid body motions from Σ_w to Σ_h, from Σ_w to Σ_c and from Σ_w to Σ_o, respectively. Similarly, the relative rigid body motions from Σ_c to Σ_h, from Σ_c to Σ_o and from Σ_h to Σ_o can be represented by g_{ch}, g_{co} and g_{ho}, respectively, as shown in Fig. 1. The objective of visual feedback control is to bring the actual relative rigid body motion g_{ho} to a given reference g_d. The reference g_d is assumed to be constant throughout this paper, because the end-effector of manipulator (we call hand) can track the moving target object in this case. Our goal is to determine the hand's motion using the visual information for this purpose.

The relative rigid body motion from Σ_c to Σ_o can be led by using the composition rule for rigid body transformations ([17], Chap. 2, pp. 37, eq. (2.24)) as follows:

$$g_{co} = g_{wc}^{-1} g_{wo}. \tag{2}$$

The relative rigid body motion involves the velocity of each rigid body. To this aim, let us consider the velocity of a rigid body as described in [17]. We define the body velocity of the camera relative to the world frame Σ_w as $V_{wc}^b = [v_{wc}^T \ \omega_{wc}^T]^T$, where v_{wc} and ω_{wc} represent the velocity of the origin and the angular velocity from Σ_w to Σ_c, respectively ([17] Chap. 2, eq. (2.55)).

Differentiating (2) with respect to time, the body velocity of the relative rigid body motion g_{co} can be written as follows [7].

$$V_{co}^b = -\mathrm{Ad}_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b \tag{3}$$

where V_{wo}^b is the body velocity of the target object relative to Σ_w. We consider that it is the basic representation for the three coordinate frames of the visual feedback system. Roughly speaking, the relative rigid body motion g_{co} will depend on the difference between the camera velocity V_{wc}^b and the target object velocity V_{wo}^b, because V_{co}^b is defined as the body velocity of the relative rigid body motion g_{co}.

In the case of fixed camera configuration, i.e. $V_{wc}^b = 0$, the fundamental representation V_{co}^b can be rewritten as

$$V_{co}^b \quad = \quad V_{wo}^b. \tag{4}$$

Hence, the fundamental representation V_{co}^b equals the target object velocity V_{wo}^b, in the case of fixed camera configuration.

B. Estimation Error and Control Error Systems

The visual feedback control task requires information of the relative rigid body motion g_{co}. Since the measurable information is only the image information $f(g_{co})$ in the visual feedback system, we consider a nonlinear observer in order to estimate the relative rigid body motion from the image information $f(g_{co})$.

Firstly, using the basic representation (4), we choose estimates $\bar{g}_{co}$ and $\bar{V}_{co}^b$ of the relative rigid body motion and velocity, respectively as

$$\bar{V}_{co}^b \quad = \quad u_e \tag{5}$$

where u_e is the new input in order to converge the estimated value to the actual relative rigid body motion.

In order to establish the estimation error system, we define the estimation error between the estimated value $\bar{g}_{co}$ and the actual relative rigid body motion g_{co} as

$$g_{ee} = \bar{g}_{co}^{-1} g_{co}. \tag{6}$$

Using the notation $e_R(e^{\hat{\xi}\theta})$ in [18], the vector of the estimation error is given by $e_e := [p_{ee}^T \ e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$. Note that $e_e = 0$ iff $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I_3$. Therefore, if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\bar{g}_{co}$ equals the actual relative rigid body motion g_{co}. The estimation error vector e_e can be obtained from image information $f(g_{co})$ and the estimated value of the relative rigid body motion $\bar{g}_{co}$. In the same way as the basic representation for the visual feedback system, the estimation error system can be represented by

$$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \tag{7}$$

Moreover, let us consider the dual of the estimation error system, which we call the control error system, in order to establish the visual feedback system. We define the control error between g_d and $\bar{g}_{ho}$, which is called the control error, as follows

$$g_{ec} = g_d^{-1} \bar{g}_{ho}, \tag{8}$$

where $\bar{g}_{ho}$ is the estimated relative rigid body motion from Σ_h to Σ_o and obtained from $\bar{g}_{ho} = g_{ch}^{-1} \bar{g}_{co}$. Here, we assume that g_{ch} is calculated by using the known motion, i.e. g_{wc} and g_{wh}, exactly. The vector of the control error is defined as $e_c := [p_{ec}^T \ e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$. The control error system is described by

$$V_{ec}^b = -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{wh}^b + u_e \tag{9}$$

where V_{wh}^b is the body velocity of the hand relative to Σ_w. Combining (7) and (9), we construct the visual feedback system with fixed camera configuration as follows:

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b \tag{10}$$

where $u_{ce} := [(V_{wh}^b)^T \ u_e^T]^T$ denotes the control input.

Here, if the following condition (11) is satisfied, the passivity-based visual feedback system with eye-in-hand configuration [5] can be regarded as the special case of that with fixed camera configuration [7],

$$g_{ch} = I, \ V_{wh}^b = V_{wc}^b. \tag{11}$$

C. Dynamic Passivity based Visual Feedback System with Fixed Camera Configuration

The manipulator dynamics can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_d \tag{12}$$

where $M \in \mathcal{R}^{n\times n}$ is the inertia matrix, $C \in \mathcal{R}^{n\times n}$ is the Coriolis matrix, $g \in \mathcal{R}^n$ is the gravity terms, and q, $\dot{q}$ and $\ddot{q}$ are the joint angle, velocity and acceleration, respectively. τ is the vector of the input torque, and τ_d represents a disturbance input [19]. Now, we will construct a dynamic visual feedback system by connecting the visual feedback system (10) and the manipulator dynamics (12). The body velocity of the hand V_{wh}^b is given by

$$V_{wh}^b = J_b(q)\dot{q} \tag{13}$$

where $J_b(q)$ is the manipulator Jacobian [17].

Next, we propose the control law for the manipulator as

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + g(q) + J_b^T(q)\mathrm{Ad}_{(g_d^{-1})}^T e_c + u_\xi. \tag{14}$$

where $\dot{q}_d$ and $\ddot{q}_d$ represent the desired joint velocity and acceleration, respectively. The new input u_ξ is to be determined in order to achieve the control objective.

Let us define the error vector with respect to the joint velocity of the manipulator as $\xi := \dot{q} - \dot{q}_d$. Moreover, we design the reference of the joint velocity based on the relation between the hand velocity and the joint velocity (13) as $\dot{q}_d := J_b^\dagger(q) u_d$ where u_d is the desired body velocity of the hand which will be obtained from the visual feedback system. Thus, V_{wh}^b in (9) should be replaced by u_d.

Using (10)–(14), the visual feedback system with the manipulator dynamics with fixed camera configuration (we call the dynamic visual feedback system with fixed camera configuration) can be derived as follows:

$$\begin{bmatrix} \dot{\xi} \\ V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -M^{-1}C\xi + M^{-1}J_b^T \mathrm{Ad}_{(g_d^{-1})}^T e_c \\ -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} J_b \xi \\ 0 \end{bmatrix} + \begin{bmatrix} M^{-1} & 0 & 0 \\ 0 & -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u + \begin{bmatrix} M^{-1} & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} w \tag{15}$$

where $u := [u_\xi^T \ u_d^T \ u_e^T]^T$. We define the state and the disturbance of dynamic visual feedback system as $x := [\xi^T \ e_c^T \ e_e^T]^T$ and $w := [\tau_d^T \ (V_{wo}^b)^T]^T$, respectively.

D. Energy Function and Stabilizing Control Law

In previous work [5], the passivity of the visual feedback system (15) is derived by using the following energy function $V(x)$

$$V(x) = \frac{1}{2}\xi^T M(q)\xi + E(g_{ec}) + E(g_{ee}) \tag{16}$$

where $E(g) := \frac{1}{2}\|p\|^2 + \phi(e^{\hat{\xi}\theta})$, and $\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta})$ is the error function of the rotation matrix (see e.g. [18]). Here, we consider the following control input

$$u = -K\nu := u_k, \quad K := \begin{bmatrix} K_\xi & 0 & 0 \\ 0 & K_c & 0 \\ 0 & 0 & K_e \end{bmatrix}, \tag{17}$$

$$\nu := Nx := \begin{bmatrix} I & 0 & 0 \\ 0 & -\mathrm{Ad}_{(g_d^{-1})}^T & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})}^T & -I \end{bmatrix} x. \tag{18}$$

where $K_\xi := \mathrm{diag}\{k_{\xi 1}, \cdots, k_{\xi n}\} \in \mathcal{R}^{n\times n}$, $K_c := \mathrm{diag}\{k_{c1}, \cdots, k_{c6}\} \in \mathcal{R}^{6\times 6}$ and $K_e := \mathrm{diag}\{k_{e1}, \cdots, k_{e6}\} \in \mathcal{R}^{6\times 6}$ are positive gain matrices. By differentiating $V(x)$ along the trajectory of the system and using the control input u_k, the next equation is obtained if $w = 0$ is satisfied.

$$\dot{V} = \nu^T u = -x^T N^T K N x. \tag{19}$$

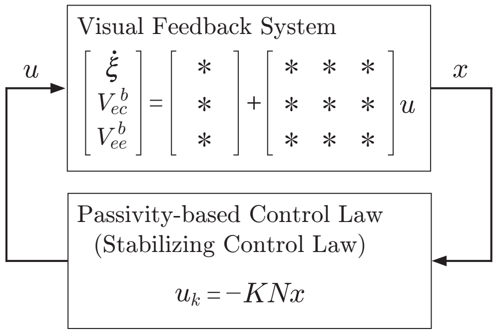

Fig. 2. Block diagram of the passivity based visual feedback control.

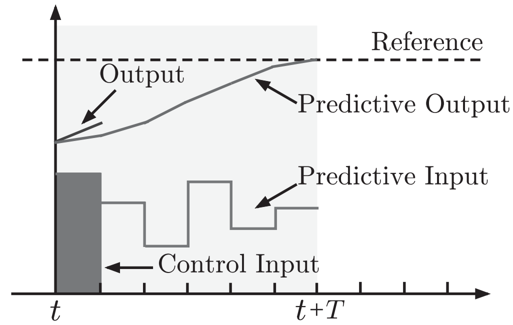

Fig. 3. Receding horizon scheme.

Therefore, if $w = 0$, the equilibrium point $x = 0$ for the closed-loop system (15) and (17) is asymptotic stable, i.e., u_k is a stabilizing control law for the system. The block diagram of the passivity based visual feedback control is shown in Fig. 2.

However, the stabilizing control law u_k (17) is not based on optimization, the desired control performance cannot be guaranteed explicitly. In the next section, a stabilizing receding horizon control based on optimal control theory is proposed.

III. Stabilizing Receding Horizon Control for Three Dimensional Visual Feedback System with Fixed Camera Configuration

The objective of this section is to propose a predictive visual feedback control based on optimal control theory. A camera can provide more information than the current derivation from a nominal position at the sample instant. This property can be exploited to predict the target's future position and improve the control performance. As a first step for a predictive visual feedback control, we propose a stabilizing receding horizon control based on optimization in this paper.

A. *Control Lyapunov Function*

In this section, the finite horizon optimal control problem (FHOCP) for the visual feedback system (15) is considered. The FHOCP at time t consists of the minimization with respect to the input $u(\tau, x(\tau)), \tau \in [t, t+T]$, of the cost function

$$J(x_0, u, T) = \int_t^{t+T} l(x(\tau), u(\tau))d\tau + F(x(t+T)) \quad (20)$$

$$l(x(t), u(t)) = x^T(t)Q(t)x(t) + u^T(t)R(t)u(t). \quad (21)$$

$$Q(t) \geq 0, \;\; R(t) > 0, \;\; F(x(t+T)) \geq 0$$

with the state $x(t) = x_0$. For a given initial condition x_0, we denote this solution of the FHOCP as $u^*(\tau, x(\tau)), \tau \in [t, t+T]$. In receding horizon control, at each sampling time δ, the resulting feedback control at state x_0 is obtained by solving the FHOCP and setting

$$u_{RH} := u^*(0, x_0). \quad (22)$$

Fig. 3 shows the receding horizon scheme.

The following lemma concerning a control Lyapunov function is important to prove a stabilizing receding horizon control. The definition for a control Lyapunov function $M(x)$ is given by

$$\inf_u \left[\dot{M}(x) + l(x, u)\right] \leq 0, \quad (23)$$

where $l(x, u)$ is a positive definite function [10].

Lemma 1: Suppose that the following matrix P is positive semi definite and $w = 0$.

$$P := \rho N^T KN - Q - N^T K^T RKN, \quad \rho > 0 \quad (24)$$

Then, the energy function $\rho V(x)$ of the visual feedback system (15) can be regarded as a control Lyapunov function.

Proof: In [5], we have already shown that the time derivative of V along the trajectory of the system (15) is formulated as (19). Using the positive definite function $l(x(t), u(t))$ (21) and the stabilizing control law u_k (17) for the system, Equation (23) can be transformed into

$$\begin{aligned} &\inf_u [\dot{M}(x) + l(x, u)] \\ &= \inf_u \left[\rho \dot{V} + x^T Qx + u^T Ru\right] \\ &= \inf_u \left[\rho x^T N^T u + x^T Qx + u^T Ru\right] \\ &= -x^T(\rho N^T KN - Q - N^T K^T RKN)x \\ &= -x^T Px. \end{aligned} \quad (25)$$

This completes the proof. ■

Lemma 1 shows the energy function $\rho V(x)$ of the visual feedback system (15) can be regarded as a control Lyapunov function in the case of $P \geq 0$. This assumption $P \geq 0$ is satisfied if ρ is chosen large enough.

B. *Stabilizing Receding Horizon Control for the 3D Visual Feedback System with fixed camera configuration*

Suppose that the terminal cost is the control Lyapunov function $\rho V(x)$, the following theorem concerning the stability of the receding horizon control holds.

Theorem 1: Consider the following cost function for the visual feedback system (15).

$$J(x_0, u, T) = \int_t^{t+T} l(x(\tau), u(\tau))d\tau + F(x(t+T)) \quad (26)$$

$$l(x(t), u(t)) = x^T(t)Q(t)x(t) + u^T(t)R(t)u(t) \quad (27)$$

$$Q(t) \geq 0, \ R(t) > 0$$

$$F(x) = \rho V(x). \quad \rho > 0 \quad (28)$$

Suppose that P (24) is positive semi definite and $w = 0$, then the receding horizon control for the visual feedback system is asymptotically stabilizing.

Proof: Our goal is to prove that $J(x^*(t), u_{RH}, T)$, which is the cost-to-go applying the receding optimal control u_{RH}, will qualify as a Lyapunov function for the closed loop system. Construct the following suboptimal control strategy for the time interval $[t+\delta, t+T+\delta]$

$$\tilde{u} = \begin{cases} u^*(\tau) & \tau \in [t+\delta, t+T] \\ u_k(\tau) & \tau \in [t+T, t+T+\delta] \end{cases} \quad (29)$$

where u_k is the stabilizing control law (17) for the system. The associated cost is

$$\begin{aligned} & J(x^*(t+\delta), \tilde{u}, T) \\ & = J(x(t), u^*, T) + \rho[V(x(t+T+\delta)) - V(x^*(t+T))] \\ & - \int_t^{t+\delta} l(x^*(\tau), u^*)d\tau + \int_{t+T}^{t+T+\delta} l(x^*(\tau+T), u_k)d\tau, \end{aligned} \quad (30)$$

where x^* is the optimal state trajectory. This cost, which is an upper bound for $J(x^*(t+\delta), u^*, T)$, satisfies

$$\begin{aligned} & J(x^*(t+\delta), u^*, T) - J(x^*(t), u^*, T) \\ & \leq \rho[V(x(t+T+\delta)) - V(x^*(t+T))] \\ & - \int_t^{t+\delta} l(x^*(\tau), u^*)d\tau + \int_{t+T}^{t+T+\delta} l(x^*(\tau+T), u_k)d\tau. \end{aligned} \quad (31)$$

Using the positive definite function $l(x(t), u(t))$ (21) and the stabilizing control law u_k (17) for the system, and dividing both sides by δ and taking the limit as $\delta \to 0$, Equation (31) can be transformed into

$$\begin{aligned} & \lim_{\delta \to 0} \frac{J(x^*(t+\delta), u^*, T) - J(x^*(t), u^*, T)}{\delta} \\ & \leq -x^{*T}(t+T)(\rho N^T KN - Q - N^T K^T RKN)x^*(t+T) \\ & \quad -x^{*T}(t)Qx^*(t) - u^{*T}Ru^* \\ & = -x^{*T}(t+T)Px^*(t+T) - x^{*T}(t)Qx^*(t) - u^{*T}Ru^*. \end{aligned} \quad (32)$$

Considering that the control input during first δ is $u_{RH} = u^*$, by the assumption $P \geq 0$, the derivative of $J(x^*(t), u_{RH}, T)$ is negative definite. Therefore, we have shown that $J(x^*(t), u_{RH}, T)$ qualifies as a Lyapunov function and asymptotic stability is guaranteed. ■

Theorem 1 guarantees the stability of the receding horizon control using a control Lyapunov function for the 3D visual feedback system (15) which is a highly nonlinear and relatively fast system. Since the stabilizing receding horizon control design is based on optimal control theory, the control performance should be improved compared to the simple passivity based control [7], under the condition of adequate gain assignment in the cost function. Moreover, compared with previous work [15], the main advantage of this approach is that the 3D dynamic visual feedback system with fixed camera configuration can treat not only three coordinate frames but also four coordinate frames which is used for typically visual feedback systems. This allows us to extend the technological application area. In this paper, as a first step, we propose unconstrained stabilizing receding horizon control schemes. In the near future, we will consider constraints which represent one of the advantages of receding horizon control, and develop it using level set, see [10]. The block diagram of the predictive visual feedback control is shown in Fig. 4.

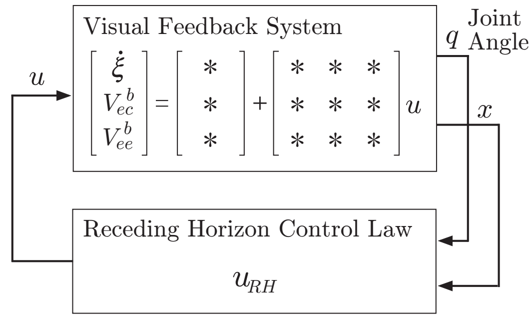

Fig. 4. Block diagram of the predictive visual feedback control.

Moreover, focused on the inverse optimality approach, the following corollary is derived.

Corollary 1: Consider the following weights of the cost function (26)–(28).

$$Q(t) = qN^T(t)KN(t) \quad q \geq 0 \quad (33)$$

$$R(t) = rK^{-1} \quad r > 0 \quad (34)$$

$$\rho = 2\sqrt{qr} \quad (35)$$

If $w = 0$, then the receding horizon control for the visual feedback system is asymptotically stabilizing, the receding horizon control law is

$$u_{RH} = -\sqrt{\frac{q}{r}} KNx \quad (36)$$

and the cost-to-go is given by

$$J(x_0, u_{RH}, T) = \rho V(x). \quad (37)$$

Proof: As a preliminary, the following calculation concerning the cost function which is set to (33)–(35) is needed

$$\begin{aligned} & \dot{F}(x) + l(x, u) \\ & = \rho \dot{V} + x^T Qx + u^T Ru \\ & = 2\sqrt{qr} x^T N^T u + qx^T N^T KNx + ru^T K^{-1} u \\ & = \left(\sqrt{r}u + \sqrt{q}KNx\right)^T K^{-1} \left(\sqrt{r}u + \sqrt{q}KNx\right) \\ & = \left\| \sqrt{r}u + \sqrt{q}KNx \right\|^2_{K^{-1}}. \end{aligned} \quad (38)$$

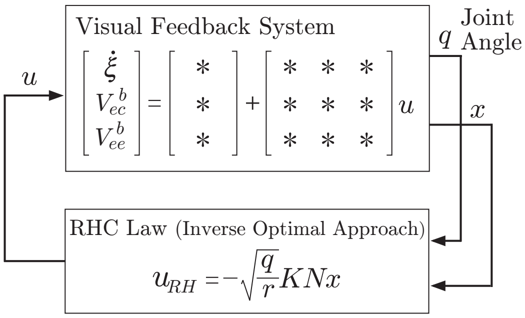

Fig. 5. Block diagram of the predictive visual feedback control focused on the inverse optimality approach.

Using the above equation (38), the cost function (26) can be transformed into

$$\begin{aligned}J(x_0,u,T)&=\int_t^{t+T}\left(x^TQx+u^TRu\right)d\tau+\rho V(x(t+T))\\&=\int_t^{t+T}\left(\left\|\sqrt{r}u+\sqrt{q}KNx\right\|_{K^{-1}}^2-\rho\dot{V}\right)d\tau\\&\qquad\qquad+\rho V(x(t+T))\\&=\int_t^{t+T}\left\|\sqrt{r}u+\sqrt{q}KNx\right\|_{K^{-1}}^2d\tau+\rho V(x(t)).\end{aligned}\tag{39}$$

Thus, the receding horizon control law

$$u_{RH}=-\sqrt{\frac{q}{r}}KNx\tag{40}$$

minimizes the cost function (26), and the cost-to-go is given by

$$J(x_0,u_{RH},T)=\rho V(x).\tag{41}$$

By regarding the cost-to-go $J(x_0,u_{RH},T)$ as a Lyapunov function and using the receding horizon control input (36), the time derivative of ρV along trajectory of the system (15) can be transformed into

$$\begin{aligned}\dot{J}(x_0,u_{RH},T)&=\rho\dot{V}(x)\\&=-\rho\sqrt{\frac{q}{r}}x^TN^TKNx.\end{aligned}\tag{42}$$

Therefore, the equilibrium point $x=0$ for the closed-loop system is asymptotic stable. ■

Corollary 1 results from the inverse optimality approach. If the weights of the cost function are set to (33)–(35), the controller that satisfies $\inf_u[\dot{M}(x)+l(x,u)]=0$ is analytically derived. The block diagram of the predictive visual feedback control focused on the inverse optimality approach is shown in Fig. 5.

IV. Conclusions

This paper proposes a stabilizing receding horizon control for a 3D visual feedback system with fixed camera configuration, which is a highly nonlinear and relatively fast system, as a first step for a predictive visual feedback control. It is shown that the stability of the receding horizon control scheme is guaranteed by using the terminal cost derived from an energy function of the visual feedback system. In this paper, the stabilizing receding controller was implemented for a low level inner loop, in the near future, we would like to tackle the implementation on a high level outer loop.

References

[1] S. Hutchinson, G. D. Hager and P. I. Corke, "A Tutorial on Visual Servo Control," *IEEE Trans. Robotics and Automation*, Vol. 12, No. 5, pp. 651–670, 1996.

[2] H. I. Christensen and P. Corke, "Editorial: Special Issue on Visual Servoing," *The Int. Journal of Robotics Research*, Vol. 22, No. 10–11, pp. 779–780, 2003.

[3] R. Kelly, R. Carelli, O. Nasisi, B. Kuchen and F. Reyes, "Stable Visual Servoing of Camera-in-Hand Robotic Systems," *IEEE Trans. Mechatronics*, Vol. 5, No. 1, pp. 39–48, 2000.

[4] N. J. Cowan, J. D. Weingarten and D. E. Koditschek, "Visual Servoing via Navigation Functions," *IEEE Trans. Robotics and Automation*, Vol. 18, No. 4, pp. 521–533, 2002.

[5] H. Kawai and M. Fujita, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L_2-gain Performance Analysis," *Proc. 2004 American Control Conference*, pp. 1522–1527, 2004.

[6] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L_2-gain Performance Analysis," *IEEE Trans. Control Systems Technology*, 2006(to appear).

[7] H. Kawai, T. Murao and M. Fujita, "Passivity-based Control and Estimation of Dynamic Visual Feedback Systems with a Fixed Camera," *Proc. of the 43rd IEEE Conf. on Decision and Control*, pp. 4022–4027, 2004.

[8] D. Q. Mayne, J. B. Rawlings, C. V. Rao and P. O. M. Scokaert, "Constrained Model Predictive Control: Stability and Optimality," *Automatica*, Vol. 36, No. 7, pp. 789–814, 2000.

[9] T. Parisini and R. Zoppoli, "A Receding-Horizon Regulator for Nonlinear Systems and a Neural Approximation," *Automatica*, Vol. 31, No. 10, pp. 1443–1451, 1995.

[10] A. Jadbabaie, J. Yu and J. Hauser, "Unconstrained Receding-Horizon Control of Nonlinear Systems," *IEEE Trans. Automatic Control*, Vol. 46, No. 5, pp. 776–783, 2001.

[11] J. Yu, A. Jadbabaie, J. Primbs and Y. Huang, "Comparison of Nonlinear Control Design Techniques on a Model of the Caltech Ducted Fan," *Automatica*, Vol. 37, No. 12, pp. 1971–1978, 2001.

[12] A. Jadbabaie and J. Hauser, "Control of a Thrust-Vectored Flying Wing: a Receding Horizon – LPV Approach," *International Journal of Robust and Nonlinear Control*, Vol. 12, No. 9, pp. 869–896, 2002.

[13] A. E. Hunt and A. C. Sanderson, "Vision-based Predictive Robotic Tracking of a Moving Target," *Technical Report*, Carnegie Mellon University, 1982.

[14] M. Fujita, Y. Kawai and Y. Nakaso, "An Experimental Study of Stabilizing Receding Horizon Control of Visual Feedback System with Planar Manipulators," *Proc. 2005 International Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control*, pp. 319–326, 2005.

[15] T. Murao, T. Yamada and M. Fujita, "Predictive Visual Feedback Control with Eye-in-Hand System via Stabilizing Receding Horizon Approach," *Proc. of the 45th IEEE Conference on Decision and Control*, 2006(submitted).

[16] S. Yu and B. J. Nelson, "Autonomous Injection of Biological Cells Using Visual Servoing," In:D. Rus and S. Singh (Eds), *Experimental Robotics VII*, Springer–Verlag, pp. 169–178, 2001.

[17] R. M. Murray, Z. Li and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, CRC Press, 1994.

[18] F. Bullo and A. D. Lewis, *Geometric Control of Mechanical Systems*, Springer–Verlag, 2004.

[19] M. W. Spong, S. Hutchinson and M. Vidyasagar, *Robot Modeling and Control*, John Wiley & Sons, 2006.

[20] T. Ohtsuka, "A Continuation/GMRES Method for Fast Computation of Nonlinear Receding Horizon Control," *Automatica*, Vol. 40, No. 4, pp. 563–574, 2004.