

# Visual Motion Observer-Based Bilateral Control for Eye-in-Hand Mobile Robot Teleoperation

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**Abstract**—This paper investigates, as a first step for visual feedback bilateral control, visual motion observer-based bilateral attitude and position control for eye-in-hand mobile robot teleoperation systems. Firstly, visual motion observers for eye-in-hand mobile robots are presented. Then, a visual motion observer-based bilateral and coordination control laws for attitude maneuvers are proposed. Moreover, visual motion observer-based bilateral position control and stability analysis are presented. The main contribution of this paper is to show that the bilateral control framework in which teleoperation systems utilize visual information to obtain the relative poses of the master and slave robots can be provided based on passivity. Finally, the effectiveness of the proposed framework is verified through computer simulations and experiments.

## I. INTRODUCTION

Visual feedback control, which can be regarded as a cross-point between control theory and computer vision, is now a very flexible and useful method in robot control [1]. Nowadays, there are several works related to the visual feedback control, in particular, for a mobile robot [2]–[6]. Gans *et al.* [2] presented a method to track multiple objects and keep them in the camera field of view without the need of a goal image or a goal feature trajectory. In [3], a novel 2-1/2-D visual servoing strategy that does not need homography/fundamental matrix estimation or decomposition was developed for nonholonomic mobile robots. Cherubini and Chaumette [4] proposed an appearance-based visual navigation framework that avoids obstacles sensed by an on-board range scanner in outdoor environments. The authors have been proposed passivity-based visual feedback control for three-dimensional (3-D) target tracking via an image space and obstacle avoidance navigation functions in [5] and [6], respectively.

On the other hand, over the past 50 years, a considerable number of research papers have been reported for bilateral teleoperation. One of the goals is that a slave robot, which exists at the remote location, mimics the movement of a master robot that the human operates. Another is that environmental force with physical contact at the slave side is reflected to the operator [7]. Among bilateral teleoperation researches from a control theoretic point of view, mobile

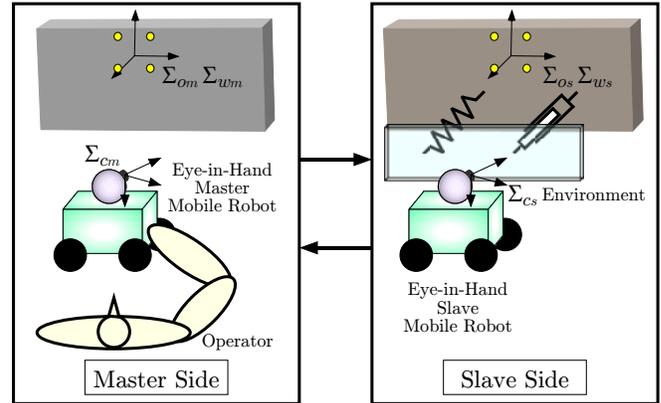


Fig. 1. Eye-in-hand mobile robot teleoperation system.

robots have emerged as a relatively new target of them with applications that include rescue and surveillance, operation in hazardous environments, and exploration of wide areas. Martínez-Palafox *et al.* [8] proposed bilateral teleoperation of a wheeled mobile robot over communication channel with constant time delays. The teleoperation system in [8] consists of a 2-degrees-of-freedom master haptic device and a slave wheeled mobile robot. In [9], the framework in [8] was extended by adding a virtual image robot for master haptic device operation. Farkhatdinov and Ryu [10] presented a teleoperation controller that modifies a force feedback gain online in order to handle a mobile robot in small and/or cluttered workspaces. In control frameworks [8]–[10], the slave robot is restricted to a ground vehicle only with a 1-directional linear velocity and a 1-directional heading angle which the human can operate based on a car-driving metaphor.

Recently, multi-slave teleoperation systems have received a lot of interest from the robotics and control community. Rodríguez-Seda *et al.* [11] presented a bilateral control framework for a single-master-multi-slave teleoperation system. This control method achieves position tracking, force reflection, formation control and collision-free trajectories through the use of passifying PD-based control and avoidance functions. In [12], a decentralized control strategy for bilateral teleoperation of heterogeneous groups of mobile robots were developed. These bilateral teleoperation approaches for mobile robots in [8]–[12] use a sonar sensor, encoders, a motion capture system or an external tracking system in order to measure the slave mobile robot pose or the distance to the environment. Although most slave mobile robots have a camera to monitor surrounding condition, the visual information is not strongly utilized to obtain the robot

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poses and to generate the inputs to the master and slave robots.

In this paper, as a first step for visual feedback bilateral control, we present visual motion observer-based bilateral attitude and position control for eye-in-hand mobile robot teleoperation systems as shown in Fig. 1. Firstly, visual motion observers through a pinhole camera model for eye-in-hand mobile robots are discussed. Secondly, we propose two visual motion observer-based control laws for attitude maneuvers. One is a bilateral control law, which is valid in the case where direction of rotation is constant. The other is a coordination control law, which is valid in the case of no physical contact. Then, we develop a visual motion observer-based bilateral position control law for teleoperation systems of a mobile robot. Stability analysis of the proposed control laws is provided. Our proposed bilateral control laws are based on the frameworks [13], [14], which are bilateral control for manipulator teleoperation without visual information. In this approach, we aim at integrating passivity-based visual feedback control into teleoperation systems to enable bilateral control without additional sensors or systems used in [8]–[12]. Finally, control performance of the proposed control scheme is evaluated through simulation and experimental results.

## II. VISUAL MOTION OBSERVERS

Eye-in-hand mobile robot teleoperation systems use six coordinate frames which consist of the world frames  $\Sigma_{w_*}$  ( $*$  =  $m$  or  $s$ ), the camera frames  $\Sigma_{c_*}$  and the target object frames  $\Sigma_{o_*}$  as shown in Fig. 1. The subscript ‘‘m’’ and ‘‘s’’ denote the master and slave indexes. In this paper, the target objects are static, and the world frames  $\Sigma_{w_m}$  and  $\Sigma_{w_s}$  are set on the target object frames  $\Sigma_{o_m}$  and  $\Sigma_{o_s}$ , respectively. Let  $p_{c_*o_*} \in \mathcal{R}^3$  and  $e^{\hat{\xi}_{c_*o_*}\theta_{c_*o_*}} \in SO(3)$  be position vectors and rotation matrices from the camera frames  $\Sigma_{c_*}$  to the target object frames  $\Sigma_{o_*}$ , where the notation ‘ $\wedge$ ’ (wedge) is the skew-symmetric operator [15]. In contrast, the notation ‘ $\vee$ ’ (vee) denotes the inverse operator to ‘ $\wedge$ ’. For simplicity we define the elements of rotation as  $\zeta_{c_*o_*} := \xi_{c_*o_*}\theta_{c_*o_*} \in \mathcal{R}^3$ , where  $\xi_{c_*o_*} \in \mathcal{R}^3$  specify the directions of rotation and  $\theta_{c_*o_*} \in \mathcal{R}$  are the angles of rotation. Then, relative poses from  $\Sigma_{c_*}$  to  $\Sigma_{o_*}$  can be represented by  $g_{c_*o_*} = (p_{c_*o_*}, e^{\hat{\xi}_{c_*o_*}\theta_{c_*o_*}}) \in SE(3)$ . Hereafter,  $g_{co_*} = (p_{co_*}, e^{\hat{\xi}_{co_*}})$  mean  $g_{c_*o_*}$  for simplicity. The relative poses  $g_{co_*}$  can be represented by using the composition rule for rigid body transformations [15] as follows:

$$g_{co_*} = g_{w_*c_*}^{-1} g_{wo_*}. \quad (1)$$

The relative pose involves a velocity of each rigid body. We define body velocities of the cameras relative to the world frames  $\Sigma_{w_*}$  as  $V_{w_*c_*}^b = [(v_{w_*c_*}^b)^T (\omega_{w_*c_*}^b)^T]^T \in \mathcal{R}^6$ , where  $v_{w_*c_*}^b$  and  $\omega_{w_*c_*}^b$  represent velocities of origins and angular velocities from  $\Sigma_{w_*}$  to  $\Sigma_{c_*}$ , respectively [15]. Differentiating Eq. (1) with respect to time, the body velocities of the relative poses  $g_{co_*}$  can be written as follows (see [16]):

$$V_{co_*}^b = -\text{Ad}_{(g_{co_*}^{-1})} V_{wc_*}^b, \quad (2)$$

where  $\text{Ad}_{(g_{co_*})}$  are the adjoint transformations associated with  $g_{co_*}$  [15]. Here, we exploited the fact that the target objects are static, i.e.,  $V_{wo_*}^b = 0$ . In the eye-in-hand teleoperation system, we use a pinhole camera model with a perspective projection. In this paper, we set  $m_*(\geq 4)$  feature points on the each target object. The perspective projection of the  $i$ -th feature point onto the image plane gives us image features  $f_{co_*} \in \mathcal{R}^{2m_*}$  [16].

Next, we design nonlinear observers (we call them visual motion observers) in order to estimate the relative poses  $g_{co_*}$  from the image features  $f_{co_*}$ . We define the estimation errors  $g_{ee_*}$  between the relative poses  $g_{co_*}$  and the estimated values  $\bar{g}_{co_*}$  as

$$g_{ee_*} = \bar{g}_{co_*}^{-1} g_{co_*}. \quad (3)$$

We next define error vectors of the rotation matrices  $e^{\hat{\xi}_{ei_*}}$  as  $r_{ei_*} := \text{sk}(e^{\hat{\xi}_{ei_*}})^\vee \in \mathcal{R}^3$ , where  $\text{sk}(e^{\hat{\xi}_{ei_*}}) := \frac{1}{2}(e^{\hat{\xi}_{ei_*}} - e^{-\hat{\xi}_{ei_*}})$ . Using this notations, the vectors of the estimation errors are given by  $e_{e_*} := [p_{ee_*}^T \ r_{ee_*}^T]^T \in \mathcal{R}^6$ . The vectors of the estimation errors  $e_{e_*}$  can be obtained by exploiting the image features  $f_{co_*}$  and the estimated values  $\bar{g}_{co_*}$  as

$$e_{e_*} = J_e^\dagger(\bar{g}_{co_*})(f_{co_*} - \bar{f}_{co_*}). \quad (4)$$

where  $\bar{f}_{co_*} \in \mathcal{R}^{2m_*}$  are estimated values of the image features and  $J(\bar{g}_{co_*}) \in \mathcal{R}^{2m_* \times 6}$  are image Jacobian-like matrices [16]. Here, we establish the following dynamical models in order to estimate the target objects using Eq. (2):

$$\bar{V}_{co_*}^b = -\text{Ad}_{(\bar{g}_{co_*}^{-1})} V_{wc_*}^b + K_{e_*} e_{e_*}, \quad (5)$$

where  $\bar{V}_{co_*}^b \in \mathcal{R}^6$  are the estimated values of  $V_{co_*}^b$ , and  $K_{e_*} := \text{diag}\{k_{e_*1}, \dots, k_{e_*6}\}$  are the positive gain matrices of  $x$ ,  $y$  and  $z$  axes of the translations and the rotations for the estimation errors. Differentiating Eq. (3) with respect to time, the estimation error systems can be represented by

$$V_{ee_*}^b = -\text{Ad}_{(g_{ee_*}^{-1})} K_{e_*} e_{e_*}. \quad (6)$$

The stability property is summarized in the following theorem:

*Theorem 1 ([16]):* The equilibrium points  $e_{e_*} = 0$  for the estimation error systems (6) are asymptotically stable.

Theorem 1, which shows Lyapunov stability for the closed-loop systems, can be proved by using the following positive definite functions:

$$V_{e_*} = \frac{1}{2} \|p_{ee_*}\|^2 + \phi(e^{\hat{\xi}_{ee_*}}), \quad (7)$$

where  $\phi(e^{\hat{\xi}_{ei_*}}) := \frac{1}{2} \text{tr}(I - e^{\hat{\xi}_{ei_*}})$  are error functions of the rotation matrices and  $I$  is the identity matrix.

Thanks to the visual motion observers, the relative poses  $g_{co_*}$  can be calculated by using the estimated relative poses  $\bar{g}_{co_*}$  and the estimation error vectors  $e_{e_*} = [p_{ee_*}^T \ r_{ee_*}^T]^T$  as follows:

$$g_{co_*} = \bar{g}_{co_*} g_{ee_*}, \quad \zeta_{ee_*} = \frac{\sin^{-1} \|r_{ee_*}\|}{\|r_{ee_*}\|} r_{ee_*}, \quad (8)$$

although  $g_{co_*}$  cannot be measured directly (see [17] for more details). Since the poses  $g_{co_*} = (p_{co_*}, e^{\hat{\xi}_{co_*}})$  are generated

from the image features  $f_{co^*}$  in this way, the master robot can communicate its poses to the slave robot, and vice versa, in eye-in-hand mobile robot teleoperation systems without additional sensors. In this paper, we do not explicitly consider communication delays similar to [10] and [12].

### III. ATTITUDE CONTROL FOR EYE-IN-HAND TELEOPERATION SYSTEMS

In this section, we propose two attitude controllers for eye-in-hand mobile robot teleoperation systems. The dynamics of the attitudes of the eye-in-hand mobile robots can be written as follows:

$$M_{m_r} \dot{\omega}_{w_{c_m}}^b + C_{m_r} \omega_{w_{c_m}}^b = \tau_{w_{c_m}}^b + \tau_{op}^b, \quad (9)$$

$$M_{s_r} \dot{\omega}_{w_{c_s}}^b + C_{s_r} \omega_{w_{c_s}}^b = \tau_{w_{c_s}}^b - \tau_{en}^b, \quad (10)$$

where  $\tau_{op}^b, \tau_{en}^b \in \mathcal{R}^6$  are the human/environmental torques,  $\tau_{w_{c_*}}^b \in \mathcal{R}^6$  are the control torque inputs,  $M_{*r} = I_* \in \mathcal{R}^{3 \times 3}$  are the symmetric and positive-definite inertia matrices for the attitudes,  $C_{*r} = -(I_* \omega_{w_{c_*}}^b)^\wedge \in \mathcal{R}^{3 \times 3}$  are the centrifugal and Coriolis matrices for the attitudes, and  $I_* \in \mathcal{R}^{3 \times 3}$  are the inertia tensors [15]. We assume that, following the procedures in [11], the gravity terms are either included in the forcing terms  $\tau_{op}^b, \tau_{en}^b$  or precompensated by active control. It should be noted that the above nonlinear dynamic representations satisfy the well known passivity properties, i.e.,  $\dot{M}_{*r} - 2C_{*r}$  are skew-symmetric matrices.

#### A. Bilateral Attitude Control for Eye-in-Hand Teleoperation Systems

In this subsection, we assume that the directions of rotation  $\xi_{co^*}$  are constant. In this case, the angular velocities can be transformed as

$$\omega_{co^*}^b = e^{-\hat{\zeta}_{co^*}} \dot{\zeta}_{co^*} = \dot{\zeta}_{co^*} \quad (11)$$

since  $\dot{e}^{\hat{\zeta}_{co^*}} = \hat{\zeta}_{co^*} e^{\hat{\zeta}_{co^*}} = e^{\hat{\zeta}_{co^*}} \hat{\zeta}_{co^*}$  [15]. By using this property, we design a visual motion observer-based bilateral attitude controller for eye-in-hand teleoperation systems in this subsection.

1) *Bilateral Attitude Control Inputs:* In this paper, we assume that the human operator applies constant force/torque on the master robot, and the remote environment can be modelled as a passive system that is represented as a linear spring-damper system similar to [13], [14]. Under these assumptions, the human operator and the remote environment for the attitude are described as follows:

$$\tau_{op}^b = \tilde{\tau}_{op}^b, \quad (12)$$

$$\tau_{en}^b = B_{s_r} \omega_{w_{c_s}}^b + K_{s_r} \zeta_{w_{c_s}}, \quad (13)$$

where  $\tilde{\tau}_{op}^b \in \mathcal{R}^3$  is the finite constant torque and  $B_{s_r}, K_{s_r} \in \mathcal{R}^{3 \times 3}$  are positive semi-definite matrices.

The control objectives in this subsection can be stated as follows:

*Control Objective A1:* (Coupled Stability for Attitude) the attitude teleoperation system is stable under any finite constant operation and any environmental torque inputs.

*Control Objective A2:* (Static Torque Reflection) the static contact torque in the slave side is accurately reflected to the operator in the master side under steady state conditions, i.e.,

$$\tau_{op}^b = \tau_{en}^b, \quad (14)$$

*Control Objective A3:* (Attitude Coordination) if  $\tau_{op}^b = \tau_{en}^b = 0$ ,

$$e^{\hat{\zeta}_{co_m}} = e^{\hat{\zeta}_{co_s}}. \quad (15)$$

To achieve these control objectives, we propose the bilateral torque inputs for the master and slave robots  $\tau_{w_{c_m}}^b, \tau_{w_{c_s}}^b$  in Eqs. (9) and (10) to be

$$\tau_{w_{c_m}}^b = -K_\omega (\omega_{w_{c_m}}^b - \omega_{w_{c_s}}^b) + K_\zeta (\zeta_{co_m} - \zeta_{co_s}), \quad (16)$$

$$\tau_{w_{c_s}}^b = -K_\omega (\omega_{w_{c_s}}^b - \omega_{w_{c_m}}^b) + K_\zeta (\zeta_{co_s} - \zeta_{co_m}), \quad (17)$$

where  $K_\zeta := \text{diag}\{k_{\zeta_x}, k_{\zeta_y}, k_{\zeta_z}\}$  and  $K_\omega := \text{diag}\{k_{\omega_x}, k_{\omega_y}, k_{\omega_z}\}$  are the positive gain matrices of  $x, y$  and  $z$  axes for the attitude and angular velocity errors, respectively. It should be remarked that the attitudes  $\zeta_{co^*}$  and the camera angular velocities  $\omega_{w_{c_*}}^b$  can be measured by utilizing the rotation matrices  $e^{\hat{\zeta}_{co^*}}$  obtained from the visual motion observers as follows:

$$\zeta_{co^*} = \frac{\sin^{-1} \|r_{co^*}\|}{\|r_{co^*}\|} r_{co^*}, \quad r_{co^*} = \frac{1}{2} (e^{\hat{\zeta}_{co_s}} - e^{-\hat{\zeta}_{co_s}})^\vee, \quad (18)$$

$$\omega_{w_{c_*}}^b = -(\dot{e}^{\hat{\zeta}_{co^*}} e^{-\hat{\zeta}_{co^*}})^\vee, \quad (19)$$

in the case that  $\|\theta_{co^*}\| \leq \frac{\pi}{2}$ .

2) *Stability Analysis:* To carry out the analysis of the system behaviour, we derive the closed-loop dynamics of the attitude teleoperation system. The equilibrium points of the attitudes of the master and the slave are defined as  $\tilde{\zeta}_{co_m}, \tilde{\zeta}_{co_s} \in \mathcal{R}^3$  such that

$$K_\zeta (\tilde{\zeta}_{co_m} - \tilde{\zeta}_{co_s}) + \tilde{\tau}_{op}^b = 0, \quad (20)$$

$$K_\zeta (\tilde{\zeta}_{co_s} - \tilde{\zeta}_{co_m}) - K_{s_r} \tilde{\zeta}_{w_{c_s}} = 0, \quad (21)$$

where  $\tilde{\zeta}_{w_{c_s}}$  is the equilibrium point of the slave from  $\Sigma_{w_s}$  to  $\Sigma_{c_s}$ . Note that  $\tilde{\zeta}_{w_{c_s}} = -\tilde{\zeta}_{co_s}$  from the relationship between the coordinate frames. We define the following attitude variables with the equilibrium points:

$$\zeta_{em} = \zeta_{co_m} - \tilde{\zeta}_{co_m}, \quad (22)$$

$$\zeta_{es} = \zeta_{co_s} - \tilde{\zeta}_{co_s}. \quad (23)$$

The closed-loop systems for the attitude teleoperation system can now be obtained by using Eqs. (9), (10), (12), (13), (16), (17) and (20)–(23) as follows:

$$M_{m_r} \dot{\omega}_{w_{c_m}}^b + C_{m_r} \omega_{w_{c_m}}^b = -K_\omega (\omega_{w_{c_m}}^b - \omega_{w_{c_s}}^b) + K_\zeta (\zeta_{em} - \zeta_{es}), \quad (24)$$

$$M_{s_r} \dot{\omega}_{w_{c_s}}^b + C_{s_r} \omega_{w_{c_s}}^b = -K_\omega (\omega_{w_{c_s}}^b - \omega_{w_{c_m}}^b) + K_\zeta (\zeta_{es} - \zeta_{em}) - B_{s_r} \omega_{w_{c_s}}^b + K_{s_r} \zeta_{es}, \quad (25)$$

where we have used the fact that  $\zeta_{w_{c_s}} = -\zeta_{co_s}$ . Our main result in this subsection follows:

*Theorem 2:* Suppose that the directions of rotation  $\xi_{co^*}$  are constant, then the equilibrium points  $\zeta_{em} = \zeta_{es} =$

$\omega_{wc_*}^b = 0$  for the attitude teleoperation closed-loop systems (24) (25) are asymptotically stable.

*Proof:* We first propose the following Lyapunov function candidate:

$$V_\zeta = \frac{1}{2}(\omega_{wc_m}^b)^T M_{m_r} \omega_{wc_m}^b + \frac{1}{2}(\omega_{wc_s}^b)^T M_{s_r} \omega_{wc_s}^b + \frac{1}{2}(\zeta_{em} - \zeta_{es})^T K_\zeta (\zeta_{em} - \zeta_{es}) + \frac{1}{2} \zeta_{es}^T K_{s_r} \zeta_{es}. \quad (26)$$

Using the property (11), the time derivative of  $V_\zeta$  along Eqs. (22)–(25) is given by

$$\begin{aligned} \dot{V}_\zeta &= (\omega_{wc_m}^b)^T M_{m_r} \dot{\omega}_{wc_m}^b + \frac{1}{2}(\omega_{wc_m}^b)^T \dot{M}_{m_r} \omega_{wc_m}^b \\ &\quad + (\omega_{wc_s}^b)^T M_{s_r} \dot{\omega}_{wc_s}^b + \frac{1}{2}(\omega_{wc_s}^b)^T \dot{M}_{s_r} \omega_{wc_s}^b \\ &\quad + \frac{1}{2}(\zeta_{em} - \zeta_{es})^T K_\zeta (\dot{\zeta}_{em} - \dot{\zeta}_{es}) + \frac{1}{2} \zeta_{es}^T K_{s_r} \dot{\zeta}_{es} \\ &= \frac{1}{2}(\omega_{wc_m}^b)^T (\dot{M}_{m_r} - 2C_{m_r}) \omega_{wc_m}^b \\ &\quad + \frac{1}{2}(\omega_{wc_s}^b)^T (\dot{M}_{s_r} - 2C_{s_r}) \omega_{wc_s}^b \\ &\quad + (\omega_{wc_m}^b)^T (-K_\omega (\omega_{wc_m}^b - \omega_{wc_s}^b) + K_\zeta (\zeta_{em} - \zeta_{es})) \\ &\quad + (\omega_{wc_s}^b)^T (-K_\omega (\omega_{wc_s}^b - \omega_{wc_m}^b) + K_\zeta (\zeta_{es} - \zeta_{em})) \\ &\quad + (\omega_{wc_s}^b)^T (-B_{s_r} \omega_{wc_s}^b + K_{s_r} \zeta_{es}) \\ &\quad + (\zeta_{em} - \zeta_{es})^T K_\zeta (\zeta_{em} - \zeta_{es}) + \zeta_{es}^T K_{s_r} \dot{\zeta}_{es} \\ &= -(\omega_{wc_m}^b - \omega_{wc_s}^b)^T K_\omega (\omega_{wc_m}^b - \omega_{wc_s}^b) - (\omega_{wc_s}^b)^T B_{s_r} \omega_{wc_s}^b \\ &\quad + (\zeta_{em} - \zeta_{es})^T K_\zeta (\dot{\zeta}_{wc_m} - \dot{\zeta}_{wc_s}) + \zeta_{es}^T K_{s_r} \dot{\zeta}_{wc_s} \\ &\quad + (\zeta_{em} - \zeta_{es})^T K_\zeta (\dot{\zeta}_{co_m} - \dot{\zeta}_{co_s}) + \zeta_{es}^T K_{s_r} \dot{\zeta}_{co_s} \\ &= -(\omega_{wc_m}^b - \omega_{wc_s}^b)^T K_\omega (\omega_{wc_m}^b - \omega_{wc_s}^b) - (\omega_{wc_s}^b)^T B_{s_r} \omega_{wc_s}^b. \end{aligned} \quad (27)$$

From Eqs. (26) and (27), it is easy to see that  $\omega_{wc_*}^b = 0$ . Due to the closed-loop systems (24) (25), we have that

$$K_{s_r} \zeta_{es} = 0, \quad (28)$$

and hence  $\zeta_{em} = \zeta_{es} = 0$  through Eq. (24). Therefore, from LaSalle's theorem [18], asymptotic stability of the origins can be shown. ■

The following corollaries for Control Objectives A2 and A3 are derived from Theorem 2.

*Corollary 1:* Consider the attitude teleoperation closed-loop systems (24) (25). The static torque reflection is achieved, i.e.,  $\tau_{op}^b = \tau_{en}^b$ .

*Proof:* By using Eqs. (12) (20) and (21), we have

$$\tau_{op}^b = \tilde{\tau}_{op}^b = K_{s_r} \tilde{\zeta}_{wc_s}. \quad (29)$$

From the fact that  $\omega_{wc_*} = 0$ ,  $\zeta_{co_*} = \tilde{\zeta}_{co_*}$  and  $\zeta_{wc_*} = -\zeta_{co_*}$ , Eq. (13) can be transformed into

$$\tau_{en}^b = K_{s_r} \zeta_{wc_s} = K_{s_r} \tilde{\zeta}_{wc_s}. \quad (30)$$

Therefore, it can be concluded that  $\tau_{op}^b = \tau_{en}^b$  from Eqs. (29) and (30). ■

*Corollary 2:* Consider the attitude teleoperation closed-loop systems (24) (25). If  $\tau_{op}^b = \tau_{en}^b = 0$ , then the master-slave attitude coordination is achieved, i.e.,  $e^{\hat{\zeta}_{co_m}} = e^{\hat{\zeta}_{co_s}}$ .

*Proof:* From Eq. (20) with  $\tau_{op}^b = 0$ , we can show that

$$K_\zeta (\tilde{\zeta}_{co_m} - \tilde{\zeta}_{co_s}) = 0. \quad (31)$$

Thus, it can be easily shown that  $e^{\hat{\zeta}_{co_m}} = e^{\hat{\zeta}_{co_s}}$  through  $\zeta_{em} = \zeta_{es} = 0$ . ■

From Theorem 2 and Corollaries 1 and 2, it can be verified that Control Objectives A1–A3 are achieved. In the case of that the human provides the constant torque input, the equilibrium points of the attitudes of the master and the slave  $\tilde{\zeta}_{co_m}$ ,  $\tilde{\zeta}_{co_s}$  can be calculated. Using Eqs. (20) and (21), they are derived as follows:

$$\tilde{\zeta}_{co_m} = -(K_{s_r}^{-1} + K_\zeta^{-1}) \tilde{\tau}_{op}^b, \quad (32)$$

$$\tilde{\zeta}_{co_s} = -K_{s_r}^{-1} \tilde{\tau}_{op}^b. \quad (33)$$

Thus, the gain  $K_\zeta$  determines the static attitude error between the master and the slave in contact with the environment. Thanks to the visual motion observers, bilateral attitude control can be applied to eye-in-hand mobile robot teleoperation systems with only pinhole cameras.

### B. Attitude Coordination Control for Eye-in-Hand Teleoperation Systems

In this subsection, we present attitude coordination control for eye-in-hand teleoperation systems. Unlike the proposed bilateral controller, the coordination controller can remove the assumptions for the directions of rotation  $\zeta_{co_*}$ . Instead, it cannot deal explicitly with the torques of the operator and the environment. In this subsection, we consider no physical contact case; hence, the torques of them are regarded as disturbances.

1) *Attitude Coordination Control Inputs:* The control objective under consideration in this subsection is only Control Objective A3, i.e., master-slave attitude coordination. Define the attitude coordination errors between the master and slave robots by

$$e^{\hat{\zeta}_{e_m s}} = e^{-\hat{\zeta}_{co_s}} e^{\hat{\zeta}_{co_m}}, \quad (34)$$

$$e^{\hat{\zeta}_{e_s m}} = e^{-\hat{\zeta}_{co_m}} e^{\hat{\zeta}_{co_s}}, \quad (35)$$

and the error vectors by  $r_{e_m s} := \text{sk}(e^{\hat{\zeta}_{e_m s}})^\vee$  and  $r_{e_s m} := \text{sk}(e^{\hat{\zeta}_{e_s m}})^\vee$ . Note that  $r_{e_m s} = -r_{e_s m}$  since  $e^{\hat{\zeta}_{e_m s}} = e^{-\hat{\zeta}_{e_s m}}$ . Differentiating Eqs. (34) and (35) with respect to time, the attitude coordination error systems can be derived as

$$\omega_{e_m s}^b = -e^{-\hat{\zeta}_{co_m}} (\omega_{wc_m}^b - \omega_{wc_s}^b), \quad (36)$$

$$\omega_{e_s m}^b = -e^{-\hat{\zeta}_{co_s}} (\omega_{wc_s}^b - \omega_{wc_m}^b). \quad (37)$$

In order for the slave side to track the master side, we propose the torque inputs as follows:

$$\tau_{wc_m}^b = -K_\omega \omega_{wc_m}^b + K_r \left( e^{\hat{\zeta}_{co_s}} r_{e_m s} - e^{\hat{\zeta}_{co_m}} r_{e_s m} \right), \quad (38)$$

$$\tau_{wc_s}^b = -K_\omega \omega_{wc_s}^b + K_r \left( e^{\hat{\zeta}_{co_m}} r_{e_s m} - e^{\hat{\zeta}_{co_s}} r_{e_m s} \right), \quad (39)$$

where  $K_r := k_r I_3$  and  $K_{\omega_*} := \text{diag}\{k_{\omega_* x}, k_{\omega_* y}, k_{\omega_* z}\}$  are the positive gain matrices of  $x$ ,  $y$  and  $z$  axes for the attitude coordination errors and the angular velocities, respectively. It should be noted that the proposed control torque inputs (38) (39) can be generated through Eqs. (19), (34) and (35) by using the visual motion observers.

2) *Stability Analysis*: Substitution of Eqs. (38) and (39) in the dynamics (9) (10) yields the following attitude coordination closed-loop systems:

$$M_{m_r} \dot{\omega}_{w_{c_m}}^b + C_{m_r} \omega_{w_{c_m}}^b = -K_{\omega_m} \omega_{w_{c_m}}^b + K_r \left( e^{\hat{\zeta}_{c_{o_s}}} r_{e_{m_s}} - e^{\hat{\zeta}_{c_{o_m}}} r_{e_{s_m}} \right) + \tau_{op}^b, \quad (40)$$

$$M_{s_r} \dot{\omega}_{w_{c_s}}^b + C_{s_r} \omega_{w_{c_s}}^b = -K_{\omega_s} \omega_{w_{c_s}}^b + K_r \left( e^{\hat{\zeta}_{c_{o_m}}} r_{e_{s_m}} - e^{\hat{\zeta}_{c_{o_s}}} r_{e_{m_s}} \right) - \tau_{en}^b. \quad (41)$$

We show the stability analysis concerning the master-slave coordination as follows:

*Theorem 3*: Suppose that the human operator and slave environment torques  $\tau_{op}^b = \tau_{en}^b = 0$ , then the equilibrium points  $r_{e_{m_s}} = r_{e_{s_m}} = \omega_{w_{c_*}}^b = 0$  for the attitude coordination closed-loop systems (40) (41) are asymptotically stable.

*Proof*: Consider the following positive definite function:

$$V_r = \frac{1}{2} (\omega_{w_{c_m}}^b)^T M_{m_r} \omega_{w_{c_m}}^b + \frac{1}{2} (\omega_{w_{c_s}}^b)^T M_{s_r} \omega_{w_{c_s}}^b + k_r \phi(e^{\hat{\zeta}_{e_{m_s}}}) + k_r \phi(e^{\hat{\zeta}_{e_{s_m}}}). \quad (42)$$

Evaluating the time derivative of  $V_r$  along the trajectories of Eqs. (36), (37), (40) and (41) gives us

$$\begin{aligned} \dot{V}_r &= (\omega_{w_{c_m}}^b)^T M_{m_r} \dot{\omega}_{w_{c_m}}^b + \frac{1}{2} (\omega_{w_{c_m}}^b)^T \dot{M}_{m_r} \omega_{w_{c_m}}^b \\ &\quad + (\omega_{w_{c_s}}^b)^T M_{s_r} \dot{\omega}_{w_{c_s}}^b + \frac{1}{2} (\omega_{w_{c_s}}^b)^T \dot{M}_{s_r} \omega_{w_{c_s}}^b \\ &\quad + k_r r_{e_{m_s}}^T e^{\hat{\zeta}_{e_{m_s}}} \dot{e}_{e_{m_s}}^b + k_r r_{e_{s_m}}^T e^{\hat{\zeta}_{e_{s_m}}} \dot{e}_{e_{s_m}}^b \\ &= \frac{1}{2} (\omega_{w_{c_m}}^b)^T (\dot{M}_{m_r} - 2C_{m_r}) \omega_{w_{c_m}}^b \\ &\quad + \frac{1}{2} (\omega_{w_{c_s}}^b)^T (\dot{M}_{s_r} - 2C_{s_r}) \omega_{w_{c_s}}^b \\ &\quad + (\omega_{w_{c_m}}^b)^T (-K_{\omega_m} \omega_{w_{c_m}}^b + K_r (e^{\hat{\zeta}_{c_{o_s}}} r_{e_{m_s}} - e^{\hat{\zeta}_{c_{o_m}}} r_{e_{s_m}})) \\ &\quad + (\omega_{w_{c_s}}^b)^T (-K_{\omega_s} \omega_{w_{c_s}}^b + K_r (e^{\hat{\zeta}_{c_{o_m}}} r_{e_{s_m}} - e^{\hat{\zeta}_{c_{o_s}}} r_{e_{m_s}})) \\ &\quad + k_r r_{e_{m_s}}^T e^{\hat{\zeta}_{e_{m_s}}} (-e^{-\hat{\zeta}_{c_{o_m}}} \omega_{w_{c_m}}^b + e^{-\hat{\zeta}_{c_{o_s}}} \omega_{w_{c_s}}^b) \\ &\quad + k_r r_{e_{s_m}}^T e^{\hat{\zeta}_{e_{s_m}}} (-e^{-\hat{\zeta}_{c_{o_s}}} \omega_{w_{c_s}}^b + e^{-\hat{\zeta}_{c_{o_m}}} \omega_{w_{c_m}}^b) \\ &= -(\omega_{w_{c_m}}^b)^T K_{\omega_m} \omega_{w_{c_m}}^b - (\omega_{w_{c_s}}^b)^T K_{\omega_s} \omega_{w_{c_s}}^b. \end{aligned} \quad (43)$$

From Eqs. (42) and (43),  $\dot{V}_r = 0$  means  $\omega_{w_{c_*}}^b = 0$  and hence the invariant set of Eq. (40) consists of all states  $(r_{e_{m_s}}, r_{e_{s_m}}, \omega_{w_{c_*}} = 0)$  that satisfy

$$e^{\hat{\zeta}_{c_{o_m}}} r_{e_{m_s}} = 0, \quad (44)$$

where we have used the fact that  $e^{\hat{\zeta}_{e_{m_s}}} r_{e_{m_s}} = r_{e_{m_s}}$  and  $r_{e_{m_s}} = -r_{e_{s_m}}$ . By following LaSalle's invariance theorem [18], it can be concluded that  $r_{e_{m_s}} = r_{e_{s_m}} = 0$ . Therefore, we have shown that asymptotic stability is guaranteed. ■

Theorem 3 guarantees the stability of the attitude coordination control for eye-in-hand teleoperation systems using a Lyapunov function. From Theorem 3, it can be shown that the attitude coordination between the master mobile robot and the slave one is achieved in non-contact with the environment.

In this section, we proposed two attitude controllers, i.e., the bilateral controller and the coordination controller. Both controllers achieve the master-slave attitude coordination for

eye-in-hand teleoperation systems. Here, we suppose that the attitude controller makes the attitudes of the master mobile robot and the slave robot synchronize before the positions of them converges. Under the condition, we propose bilateral position control for eye-in-hand teleoperation systems in the next section.

#### IV. BILATERAL POSITION CONTROL FOR EYE-IN-HAND TELEOPERATION SYSTEMS

##### A. Bilateral Position Control Inputs

The Euler-Lagrange equations for the positions of the eye-in-hand mobile robots can be written as follows:

$$M_{m_p} \dot{v}_{w_{c_m}}^b + C_{m_p} v_{w_{c_m}}^b = f_{w_{c_m}}^b + f_{op}^b, \quad (45)$$

$$M_{s_p} \dot{v}_{w_{c_s}}^b + C_{s_p} v_{w_{c_s}}^b = f_{w_{c_s}}^b - f_{en}^b, \quad (46)$$

where  $f_{op}^b, f_{en}^b \in \mathcal{R}^6$  are the human/environmental forces,  $f_{w_{c_*}}^b \in \mathcal{R}^6$  are the control force inputs,  $M_{*p} = m_* I_3 \in \mathcal{R}^{3 \times 3}$  are the symmetric and positive-definite inertia matrices for the positions,  $C_{*p} = -m_* \hat{\omega}_{w_{c_*}}^b \in \mathcal{R}^{3 \times 3}$  are the centrifugal and Coriolis matrices for the positions, and  $m_* \in \mathcal{R}$  are the masses of the rigid bodies [15]. The gravity terms and the passivity properties of the dynamics of the positions are similar to that of the attitude in Sec. III.

Under the aforementioned assumptions that the human operator applies constant force/torque and the remote environment can be modelled as a passive system, the human operator and the remote environment for the position are described as follows:

$$f_{op}^b = \tilde{f}_{op}^b, \quad (47)$$

$$f_{en}^b = B_{s_p} v_{w_{c_s}}^b + K_{s_p} e^{-\hat{\zeta}_{w_{c_s}}} p_{w_{c_s}}, \quad (48)$$

where  $\tilde{f}_{op}^b \in \mathcal{R}^3$  is the finite constant force and  $B_{s_p}, K_{s_p} \in \mathcal{R}^{3 \times 3}$  are positive semi-definite matrices.

For the position teleoperation system, we identify the following control objectives:

*Control Objective P1*: (Coupled Stability for Position) the position teleoperation system is stable under any finite constant operation and any environmental force inputs.

*Control Objective P2*: (Static Force Reflection) the static contact force in the slave side is accurately reflected to the operator in the master side under steady state conditions, i.e.,

$$f_{op}^b = f_{en}^b. \quad (49)$$

*Control Objective P3*: (Position Coordination) if  $f_{op}^b = f_{en}^b = 0$ ,

$$p_{c_{o_m}} = p_{c_{o_s}}. \quad (50)$$

To guarantee the safe interaction and coupled stability, we propose the following control force inputs:

$$f_{w_{c_m}}^b = -K_v (v_{w_{c_m}}^b - v_{w_{c_s}}^b) + K_p (p_{c_{o_m}} - p_{c_{o_s}}), \quad (51)$$

$$f_{w_{c_s}}^b = -K_v (v_{w_{c_s}}^b - v_{w_{c_m}}^b) + K_p (p_{c_{o_s}} - p_{c_{o_m}}), \quad (52)$$

where  $K_p := \text{diag}\{k_{p_x}, k_{p_y}, k_{p_z}\}$  and  $K_v := \text{diag}\{k_{v_x}, k_{v_y}, k_{v_z}\}$  are the positive gain matrices of

$x$ ,  $y$  and  $z$  axes for the position and velocity errors, respectively. Since the target objects are static, the camera translational velocities  $v_{wc*}$  can be measured as follows:

$$v_{wc*}^b = -\dot{p}_{co*} + \hat{p}_{co*} \omega_{wc*}^b. \quad (53)$$

Hence, similar to the control torque inputs, the control force inputs (51) (52) can be generated through Eq. (19) by using the visual motion observers.

### B. Stability Analysis

Similar to the attitude case, the closed-loop systems for the position teleoperation system are constructed as follows:

$$\begin{aligned} M_{m_p} \dot{v}_{wc_m}^b + C_{m_p} v_{wc_m}^b &= -K_v (v_{wc_m}^b - v_{wc_s}^b) + K_p (p_{em} - p_{es}), \quad (54) \\ M_{s_p} \dot{v}_{wc_s}^b + C_{s_p} v_{wc_s}^b &= -K_v (v_{wc_s}^b - v_{wc_m}^b) \\ &+ K_p (p_{es} - p_{em}) - B_{s_p} v_{wc_s}^b - K_{s_p} e^{-\hat{\zeta}_{wc_s}} (p_{wc_s} - \tilde{p}_{wc_s}), \quad (55) \end{aligned}$$

where the position variables with the equilibrium points  $p_{e*}$  are defined as

$$p_{em} = p_{co_m} - \tilde{p}_{co_m}, \quad (56)$$

$$p_{es} = p_{co_s} - \tilde{p}_{co_s}. \quad (57)$$

Note that the equilibrium points of the positions of the master and the slave are defined as  $\tilde{p}_{co_m}, \tilde{p}_{co_s} \in \mathcal{R}^3$  such that

$$K_p (\tilde{p}_{co_m} - \tilde{p}_{co_s}) + \tilde{f}_{op}^b = 0, \quad (58)$$

$$K_p (\tilde{p}_{co_s} - \tilde{p}_{co_m}) - K_{s_p} e^{-\hat{\zeta}_{wc_s}} \tilde{p}_{wc_s} = 0, \quad (59)$$

where  $\tilde{p}_{wc_s}$  is the equilibrium point of the slave from  $\Sigma_{w_s}$  to  $\Sigma_{c_s}$ . Note that  $\tilde{p}_{wc_s} = e^{\hat{\zeta}_{co_s}} \tilde{p}_{co_s}$ .

Suppose that the attitude coordination has been achieved with the proposed attitude controller in Sec. III, the following theorem concerning the stability for teleoperation of the position holds.

*Theorem 4:* Suppose that  $e^{\hat{\zeta}_{wc_m}} = e^{\hat{\zeta}_{wc_s}}$  and  $\omega_{wc*}^b = 0$ , then the equilibrium points  $p_{em} = p_{es} = v_{wc*}^b = 0$  for the position teleoperation closed-loop systems (54) (55) are asymptotically stable.

*Proof:* A positive definite function for the closed-loop system is defined as

$$\begin{aligned} V_p &= \frac{1}{2} (v_{wc_m}^b)^T M_{m_p} v_{wc_m}^b + \frac{1}{2} (v_{wc_s}^b)^T M_{s_p} v_{wc_s}^b \\ &+ \frac{1}{2} (p_{em} - p_{es})^T K_p (p_{em} - p_{es}) + \frac{1}{2} p_{es}^T K_{s_p} p_{es}. \quad (60) \end{aligned}$$

Along Eqs. (54)–(57), the time derivative of  $V_p$  becomes

$$\begin{aligned} \dot{V}_p &= (v_{wc_m}^b)^T M_{m_p} \dot{v}_{wc_m}^b + \frac{1}{2} (v_{wc_m}^b)^T \dot{M}_{m_p} v_{wc_m}^b \\ &+ (v_{wc_s}^b)^T M_{s_p} \dot{v}_{wc_s}^b + \frac{1}{2} (v_{wc_s}^b)^T \dot{M}_{s_p} v_{wc_s}^b \\ &+ \frac{1}{2} (p_{em} - p_{es})^T K_p (\dot{p}_{em} - \dot{p}_{es}) + \frac{1}{2} p_{es}^T K_{s_p} \dot{p}_{es} \\ &= \frac{1}{2} (v_{wc_m}^b)^T (\dot{M}_{m_p} - 2C_{m_p}) v_{wc_m}^b \\ &+ \frac{1}{2} (v_{wc_s}^b)^T (\dot{M}_{s_p} - 2C_{s_p}) v_{wc_s}^b \\ &+ (v_{wc_m}^b)^T (-K_v (v_{wc_m}^b - v_{wc_s}^b) + K_p (p_{em} - p_{es})) \\ &+ (v_{wc_s}^b)^T (-K_v (v_{wc_s}^b - v_{wc_m}^b) + K_p (p_{es} - p_{em})) \end{aligned}$$

$$\begin{aligned} &+ (v_{wc_s}^b)^T (-B_{s_p} v_{wc_s}^b - K_{s_p} e^{-\hat{\zeta}_{wc_s}} (p_{wc_s} - \tilde{p}_{wc_s})) \\ &+ (p_{em} - p_{es})^T K_p (\dot{p}_{co_m} - \dot{p}_{co_s}) + p_{es}^T K_{s_p} \dot{p}_{co_s} \\ &= -(v_{wc_m}^b - v_{wc_s}^b)^T K_v (v_{wc_m}^b - v_{wc_s}^b) - (v_{wc_s}^b)^T B_{s_p} v_{wc_s}^b \\ &+ (v_{wc_m}^b - v_{wc_s}^b)^T K_p (p_{em} - p_{es}) + (v_{wc_s}^b)^T K_{s_p} (p_{co_s} - \tilde{p}_{co_s}) \\ &- (p_{em} - p_{es})^T K_p (v_{wc_m}^b - v_{wc_s}^b) - p_{es}^T K_{s_p} v_{wc_s}^b \\ &= -(v_{wc_m}^b - v_{wc_s}^b)^T K_v (v_{wc_m}^b - v_{wc_s}^b) - (v_{wc_s}^b)^T B_{s_p} v_{wc_s}^b, \quad (61) \end{aligned}$$

where Eq. (53),  $e^{\hat{\zeta}_{co*}} = e^{\hat{\zeta}_{wc*}}$  and  $p_{co*} = -e^{-\hat{\zeta}_{wc*}} p_{wc*}$  were used. From Eqs. (60) and (61), it is shown that  $\dot{V}_p = 0$ . Moreover, from the closed-loop systems (54) (55), we obtain

$$K_{s_p} e^{-\hat{\zeta}_{wc_s}} (p_{wc_s} - \tilde{p}_{wc_s}) = 0. \quad (62)$$

By using the fact that  $e^{\hat{\zeta}_{co*}} = e^{\hat{\zeta}_{wc*}}$  and  $p_{co*} = -e^{-\hat{\zeta}_{wc*}} p_{wc*}$ , it can be concluded that  $p_{em} = p_{es} = 0$  through LaSalle's theorem [18]. Thus, asymptotic stability of the origins can be shown. ■

Similar to Corollaries 1 and 2, the following corollaries for Control Objectives P2 and P3 are satisfied.

*Corollary 3:* Consider the position teleoperation closed-loop systems (54) (55). If  $e^{\hat{\zeta}_{wc_m}} = e^{\hat{\zeta}_{wc_s}}$  and  $\omega_{wc*}^b = 0$ , then the static force reflection is achieved, i.e.,  $f_{op}^b = f_{en}^b$ .

*Corollary 4:* Consider the position teleoperation closed-loop systems (54) (55). If  $e^{\hat{\zeta}_{wc_m}} = e^{\hat{\zeta}_{wc_s}}$ ,  $\omega_{wc*}^b = 0$  and  $f_{op}^b = f_{en}^b = 0$ , then the master-slave position coordination is achieved, i.e.,  $p_{co_m} = p_{co_s}$ .

The equilibrium points of the positions of the master and the slave with the human input force are calculated as follows:

$$\tilde{p}_{co_m} = -(K_{s_p}^{-1} + K_p^{-1}) \tilde{f}_{op}^b, \quad (63)$$

$$\tilde{p}_{co_s} = -K_{s_p}^{-1} \tilde{f}_{op}^b. \quad (64)$$

Theorem 4 and Corollaries 3 and 4 guarantee the asymptotic stability, the static force reflection and the master-slave position coordination, respectively. The block diagram of the visual motion observer-based bilateral control for eye-in-hand mobile robot teleoperation is shown in Fig. 2.

Our proposed approach that deals explicitly with the estimation problem from the visual features, can be applied to 3-D teleoperation systems that have a camera to monitor surrounding condition. Thus, the proposed method, which integrates the visual motion observer-based pose control and the bilateral pose control without additional sensors based on passivity, allows us to extend technological application area. The main contribution of this paper is to provide the bilateral control framework in which teleoperation systems utilize visual information through a pinhole camera model in order to obtain the relative poses. Thanks to the visual motion observers, bilateral control inputs for not only teleoperation of the position but also of the attitude can be designed.

In our passivity-based visual feedback control approach, the image Jacobian-like matrix  $J(\cdot)$  of the pinhole camera model is exactly the same form as that of the panoramic camera model [19]. Therefore, the proposed approach can be applied to teleoperation systems with a panoramic camera using the image Jacobian-like matrix  $J(\cdot)$  in [19].

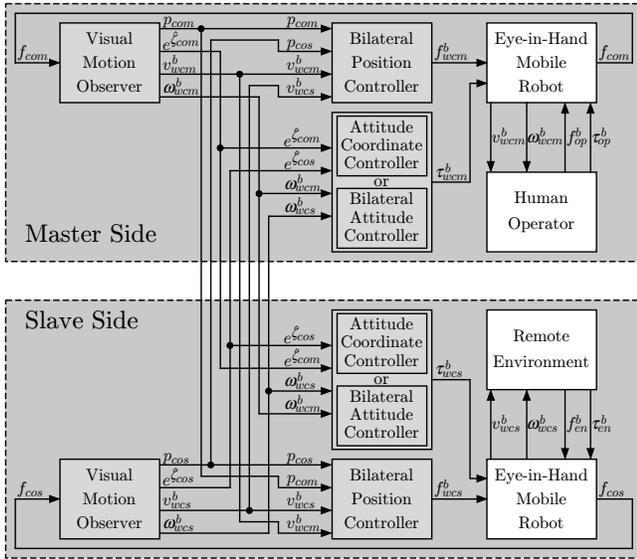


Fig. 2. Block diagram of visual motion observer-based bilateral control for eye-in-hand mobile robot teleoperation.

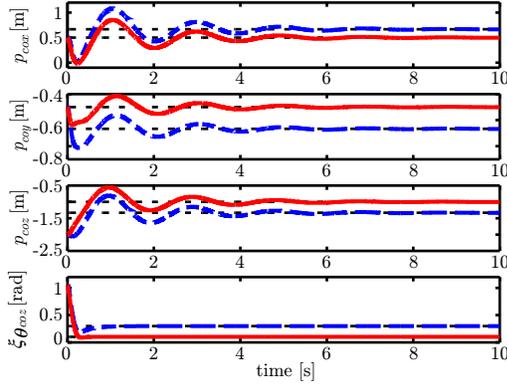


Fig. 3. Relative rigid pose  $g_{co}$ : Dashed (Blue); with master robot: Solid (Red); with slave one.

## V. SIMULATIONS

In this section, we simulate the teleoperation system using quadrotor aerial vehicles with pinhole cameras, which have  $xyz$ -translational and  $z$ -rotational motion, as eye-in-hand mobile robots. In the simulations, we perform the bilateral control proposed in Sec. III-A for the attitude.

Firstly, we present some results for stability analysis and force reflection in the case of contacting the environment, i.e., for Control Objectives A1, A2, P1 and P2. The simulation is carried out with the initial condition  $p_{co_m} = p_{co_s} = [0.5 \ -0.5 \ -2]^T$  m,  $\zeta_{co_m} = \zeta_{co_s} = [0 \ 0 \ \pi/3]^T$  rad. The constant force and torque are set as  $f_{op} = [-5 \ 5 \ 10]^T$  N,  $\tilde{\tau}_{op} = [0 \ 0 \ 1]^T$  N·m. The gains are selected to make the attitude converge faster than the position as follows:  $K_{s_p} = K_{s_r} = 10I$ ,  $B_{s_p} = B_{s_r} = \text{diag}\{0, 0, 1\}$ ,  $K_p = 30I$ ,  $K_v = 20I$ ,  $K_\zeta = 5I$ ,  $K_\omega = 10I$ . Under the above conditions, the equilibrium points are calculated by using Eqs. (32), (33), (63) and (64) as  $\tilde{p}_{co_m} = [0.667 \ -0.667 \ -1.333]^T$  m,  $\tilde{p}_{co_s} = [0.5 \ -0.5 \ -1]^T$  m,  $\tilde{\zeta}_{co_m} = [0 \ 0 \ 0.3]^T$  rad,  $\tilde{\zeta}_{co_s} = [0 \ 0 \ 0.1]^T$  rad.

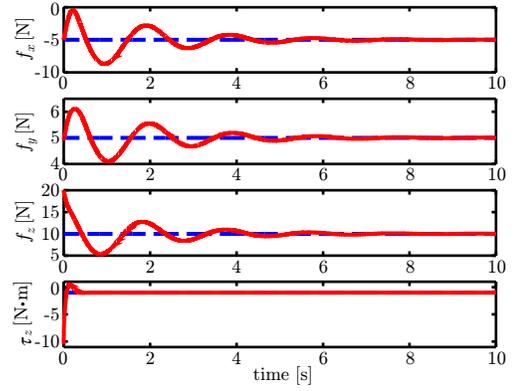


Fig. 4. Force  $f_{wc}^b$  and torque  $\tau_{wc}^b$ : Dashed (Blue); with master robot: Solid (Red); with slave one.

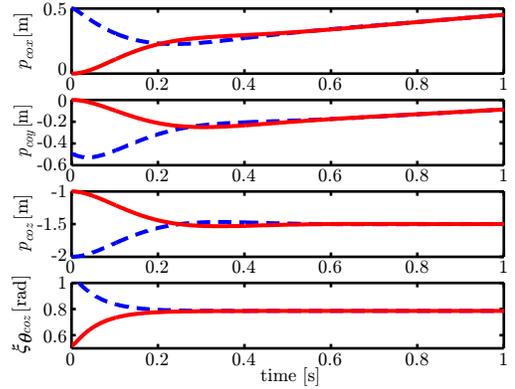


Fig. 5. Relative rigid pose  $g_{co}$ : Dashed (Blue); with master robot: Solid (Red); with slave one.

The simulation results for the stability analysis and force reflection are presented in Figs. 3 and 4. In Fig. 3, the dashed (blue) lines and the solid (red) lines denote the relative pose of the master robot  $g_{co_m}$  and the slave one  $g_{co_s}$ , respectively. Since the relative poses converges to the equilibrium points of them  $\tilde{p}_{co_*}$ ,  $\tilde{\zeta}_{co_*}$ , the asymptotic stability can be confirmed. Figure 4 shows the human force/torque  $f_{op}^b, \tau_{op}^b$  with the dashed (blue) lines and the environmental force/torque  $f_{en}^b, \tau_{en}^b$  with the solid (red) lines. From Fig. 4, it is clear that the static force reflection is achieved by steady state performance.

Next, we perform the simulation for the pose coordination i.e. for Control Objectives A3 and P3. From the aforementioned conditions, we change one of them as follows:  $p_{co_s} = [0 \ 0 \ -1]^T$  m,  $\zeta_{co_s} = [0 \ 0 \ \pi/6]^T$  rad,  $\tilde{f}_{op} = \tilde{\tau}_{op} = 0$ ,  $K_{s_p} = K_{s_r} = 0$ ,  $B_{s_p} = B_{s_r} = 0$ ,  $K_\zeta = 25I$ . We show the relative poses  $g_{co_m}, g_{co_s}$  in Fig. 5. From Fig. 5, it is concluded that the pose coordination is achieved in the case of  $f_{op}^b = \tau_{op}^b = f_{en}^b = \tau_{en}^b = 0$ .

## VI. EXPERIMENT

In this section, we present an experimental result applying the attitude coordination control proposed in Sec. III-B and the position bilateral control proposed in Sec. IV. The experimental environment is shown in Fig. 6. The experimental setup is composed by two Parrot AR.Drones which are wifi

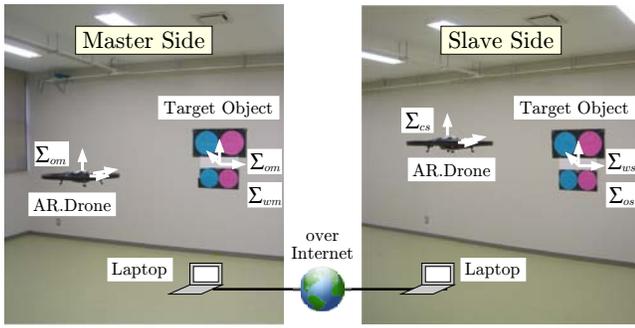


Fig. 6. Experimental environment.

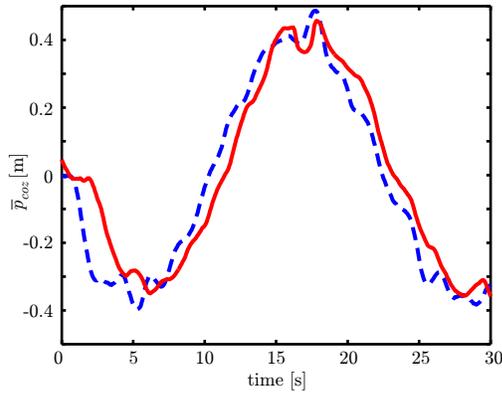


Fig. 7. Z-axis estimated relative position  $\bar{p}_{coz}$ : Dashed (Blue); with master robot: Solid (Red); with slave one.

quadrotor aerial vehicles with two cameras and two laptops. The master and slave controllers communicate to each other through the UDP sockets over Internet. The time delays from the master robot to the slave one and from the slave robot to the master one are found to be 0.54 s and 0.88 s, respectively. In this paper, we perform only  $z$ -axis translational movement with a contact-free case study.

The experiment is carried out with the initial condition  $p_{com} = p_{cos} = [0 \ 3 \ 0]^T$  m,  $\zeta_{com} = \zeta_{cos} = [0 \ 0 \ 0]^T$  rad. The gain matrices are set as  $K_p = K_v = \text{diag}\{150, 150, 120\}$ ,  $k_r = 1$ ,  $K_{\omega_m} = 0.25I$ ,  $K_{\omega_s} = 2.5I$ . We apply the human force  $f_{op}^b$  to make the master robot move up and down. The result is shown in Fig. 7. In Fig. 7, the dashed (blue) lines and the solid (red) lines denote the  $z$ -axis estimated relative position of the master robot  $\bar{g}_{com}$  and the slave one  $\bar{g}_{cos}$ , respectively. From Fig. 7, it is confirmed that the slave robot tracks the master one for  $z$ -axis translational movement. The experimental movie is available at <http://wwwr.kanazawa-it.ac.jp/kawai/research/ARDrone/ARDronevideos.html>.

## VII. CONCLUSIONS

This paper has proposed visual motion observer-based bilateral attitude and position control for eye-in-hand mobile robot teleoperation systems. The main contribution of this paper is to show that the bilateral control framework in which teleoperation systems utilize visual information through a pinhole camera model in order to obtain the relative poses

of the master and slave robots can be provided based on passivity. Thanks to the visual motion observers, bilateral control can be applied to not only teleoperation systems for the position but also for the attitude. Simulation and experimental results have been presented to verify the control performance of the proposed control scheme.

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