Open-loop Control for 2DOF Robot Manipulators with Antagonistic Bi-articular Muscles

Keisuke Sano, Hiroyuki Kawai, Toshiyuki Murao and Masayuki Fujita

***Abstract*— This paper investigates open-loop control, which does not need the joint angles and velocities, for two degree of freedom(2DOF) robot manipulators with antagonistic bi-articular muscles which are passing over adjacent two joints and acting the both joints simultaneously. The manipulator dynamics of three muscle torques, we call the bi-articular manipulator dynamics, is constructed in order to design the control input. Stability analysis with respect to our proposed control law is discussed based on the Lyapunov method. Our approach is inspired by the fact that humans do not measure the joint angles and velocities explicitly, Finally, simulation results are shown in order to confirm the proposed method and given a design procedure in order to assign the tuning parameter.**

I. Introduction

Recently, activity of robots in homes, workspaces, providing support in services, healthcare and assistance are expected. These robots are not expected to have high torque and high speed, but have safely and dependably. Robot manipulators which have actuators in each joint can move with high torque and high speed, so that we can consider that there might be an inflicting injury on human. These robots are not suitable as modern robots which interact human, i.e., rehabilitation, care, surgery, and so on [1].

On the other hand, neurons, muscles, bones, joints and ligaments are related to human motion. Recently, analysis of human motion and robot control using the mechanism of human body attract attention. For example, the configuration of the affected human limb(s) can be controlled at each joint with rehabilitation robots, so that missing motor synergies can now be compensated for severely disabled patients [2]. M. Kuschel $et\ al.$ [3] have proposed a mathematical model for visual-haptic perception of compliant objects based on psychophysical experiments. Wang $et\ al.$ [4] dealt with a neural network based inverse optimal neuromuscular electrical simulation controller to enable the lower limb to track a desired trajectory. One of the most important mechanisms of human motion is antagonistic bi-articular muscles, which are passing over adjacent two joints and acting the both joints simultaneously. Since robot manipulators with antagonistic bi-articular muscles do not need high torque, the inflicting injury on human decreases. Kumamoto $et\ al.$ give us the effects of the existence of antagonistic bi-articular muscles [5]–[7]. Oh $et\ al.$ have proposed two-degree-of-freedom control for robot manipulators with antagonistic bi-articular muscles [8][9]. While stability analysis has not been discussed in these works, we have proposed passivity-based control and discussed the stability analysis explicitly in [10]. However, the proposed control law utilize the joint angles and velocities, although humans do not measure these informations explicitly.

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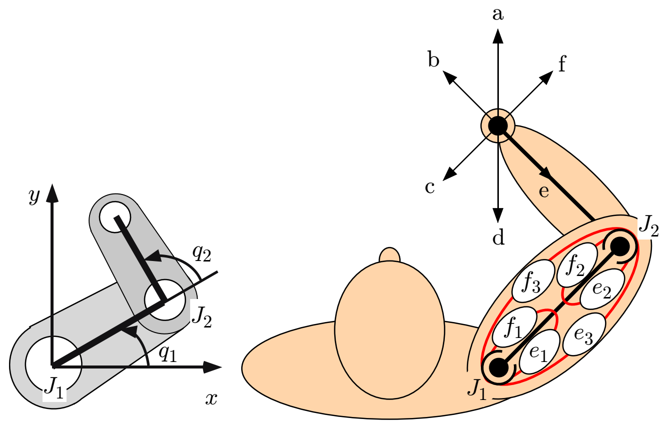

(a) 2DOF robot arm (b) Human arm model

Fig. 1. (a)2DOF robot arm. (b)Human arm model. Two couples of the antagonistic mono-articular muscles of f_1 and e_1, and of f_2 and e_2 are attached to the joints of J_1 and J_2, respectively. A couple of the antagonistic bi-articular muscles f_3 and e_3 are attached to both joints of J_1 and J_2.

This paper deals with open-loop control, which does not need the joint angles and velocities, for 2DOF robot manipulators by using antagonistic bi-articular muscles. Firstly, the brief summary of passivity-based control for 2DOF robot manipulators with antagonistic bi-articular muscles is given. Secondly, we propose an open-loop control law and discuss the stability analysis of the error dynamics based on the Lyapunov method. Finally, the simulation results show the validity of the proposed control law and give a design procedure in order to assign the tuning parameter.

II. Previous Works

A. Bi-articular Manipulator Dynamics

The dynamics of n-link rigid robot manipulators can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = T \quad (1)$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angle, velocity and acceleration, respectively. T is the vector of the input torque. $M(q) \in$

$\mathcal{R}^{n\times n}$ is the manipulator inertia matrix, $C(q,\dot{q}) \in \mathcal{R}^{n\times n}$ is the Coriolis matrix and $g(q) \in \mathcal{R}^n$ is the gravity vector [11]. In the case of 2DOF robot manipulators as shown in Fig. 1(a), the dynamics can be concretely represented as

$$\begin{bmatrix} M_1+2M_2+2RC_2 & 2M_2+RC_2 \\ 2M_2+RC_2 & 2M_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -RS_2\dot{q}_2 & -RS_2(\dot{q}_1+\dot{q}_2) \\ RS_2\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} g(m_1l_{g1}+m_2l_2)C_1+g(m_2l_{g2})C_{12} \\ g(m_2l_{g2})C_{12} \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}. \tag{2}$$

Human arm model can be simplified as three pairs of antagonistic muscles as depicted in Fig. 1. Generally, the joint torque T will be designed as control input directly in robot motion control. A couple of bi-articular muscles are attached to both joints as shown in Fig. 2. The joint torques are described as

$$\begin{aligned} T_i &= (F_{fi}-F_{ei})r_p + (F_{f3}-F_{e3})r_p \\ &= (u_{fi}-u_{ei})r_p - (u_{fi}+u_{ei})k_ir_p^2q_i - (u_{fi}+u_{ei})b_ir_p^2\dot{q}_i \\ &\quad +(u_{f3}-u_{e3})r_p - (u_{f3}+u_{e3})k_3r_p^2(q_1+q_2) \\ &\quad -(u_{f3}+u_{e3})b_3r_p^2(\dot{q}_1+\dot{q}_2). \quad (i=1,2) \end{aligned} \tag{3}$$

In Fig.2, F_{fj} and $F_{ej}(j=1,2,3)$ are output forces by flexor muscle and by extensor muscle. u_{fj} and u_{ej} represent contractile forces of flexor muscle and of extensor muscle. r_p, k_j and b_j are the radius of the joint, coefficients w.r.t. elastic and coefficients w.r.t. viscosity[5]. The contractile forces of flexor muscle and of extensor muscle have following relationship[6].

$$u_{fj}+u_{ej}=1 \quad (j=1,2,3). \tag{4}$$

Because the contractile force of flexor muscle u_{fj} can be decided by an actuator, muscle torques are defined as $\tau_i := (2u_{fi}-1)r_p$. Then the joint torques (3) can be transformed into

$$\begin{aligned} T_i &= \tau_i + \tau_3 - k_ir_p^2q_i - k_3r_p^2(q_1+q_2) \\ &\quad -b_ir_p^2\dot{q}_i - b_3r_p^2(\dot{q}_1+\dot{q}_2). \quad (i=1,2) \end{aligned} \tag{5}$$

In order to design the bi-articular muscle torque $\tau \in \mathcal{R}^3$ as the control input, we define the antagonistic bi-articular muscles torque as

$$\begin{aligned} \tau_3 &= \frac{1}{2}(m_2l_{g2}^2+\tilde{I}_2)(\ddot{q}_1+\ddot{q}_2) + g(m_2l_{g2})C_{12} \\ &\qquad +k_3r_p^2(q_1+q_2) + b_3r_p^2(\dot{q}_1+\dot{q}_2) \\ &= M_2(\ddot{q}_1+\ddot{q}_2) + g(m_2l_{g2})C_{12} \\ &\qquad +k_3r_p^2(q_1+q_2) + b_3r_p^2(\dot{q}_1+\dot{q}_2). \end{aligned} \tag{6}$$

From Eqs. (2), (5) and (6), the manipulator dynamics of

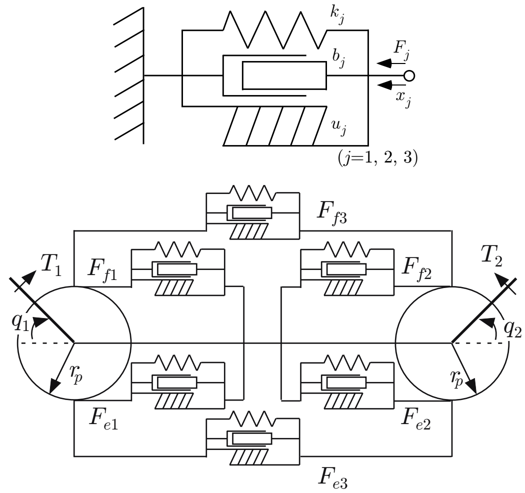

Fig. 2. Visco-elastic muscle model [6]. F_j: output force, u_j: contractile force, k_j: coefficient w.r.t. elastic, b_j: coefficient w.r.t. viscosity, x_j: contracting length, r_p: radius of the joint.

three antagonistic muscle torques can be represented as

$$\begin{bmatrix} M_1+M_2+2RC_2 & M_2+RC_2 & 0 \\ M_2+RC_2 & M_2 & 0 \\ 0 & 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_1+\ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -RS_2\dot{q}_2 & -RS_2(\dot{q}_1+\dot{q}_2) & 0 \\ RS_2\dot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_1+\dot{q}_2 \end{bmatrix} + \begin{bmatrix} g(m_1l_{g1}+m_2l_1)C_1 \\ 0 \\ g(m_2l_{g2})C_{12} \end{bmatrix} + K_r\begin{bmatrix} q_1 \\ q_2 \\ q_1+q_2 \end{bmatrix} + B_r\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_1+\dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{7}$$

where $M_1 = m_1l_{g1}^2 + m_2l_1^2 + \tilde{I}_1$, $M_2 = \frac{1}{2}(m_2l_{g2}^2 + \tilde{I}_2)$ and $R = m_2l_1l_{g2}$. S_i, C_i, S_{ij} and C_{ij} means $\sin q_i$, $\cos q_i$, $\sin(q_i+q_j)$ and $\cos(q_i+q_j)$ (i, j = 1, 2), respectively. m_i, l_i, l_{gi} and $\tilde{I}_i$ are the weight of the link i, the length of the link i, the distance from the center of a joint i to the center of the gravity point of the link i and the moment of inertia about an axis through the center of mass of the link i. $K_r := \mathrm{diag}\{k_1,k_2,k_3\}\, r_p^2 \in \mathcal{R}^{3\times3}$ and $B_r := \mathrm{diag}\{b_1,b_2,b_3\}\, r_p^2 \in \mathcal{R}^{3\times3}$ mean the matrices w.r.t. elastic and viscosity. Moreover, we define the extended joint angle vector for the manipulator dynamics of three antagonistic muscle torques as

$$\theta = \begin{bmatrix} q_1 \\ q_2 \\ q_1+q_2 \end{bmatrix}. \tag{8}$$

The manipulator dynamics with antagonistic bi-articular muscles, *we call the bi-articular manipulator dynamics[10]*, can be described as

$$M_b(\theta)\ddot{\theta} + C_b(\theta,\dot{\theta})\dot{\theta} + g_b(\theta) + K_r\theta + B_r\dot{\theta} = \tau \tag{9}$$

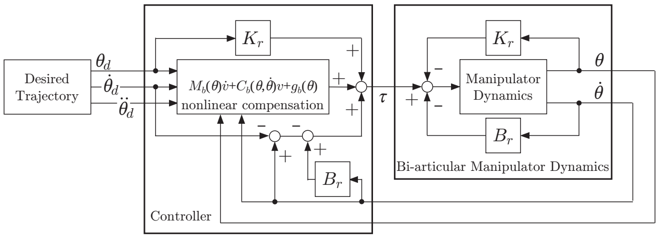

Fig. 3. Block diagram of the closed-loop system and the previous control law

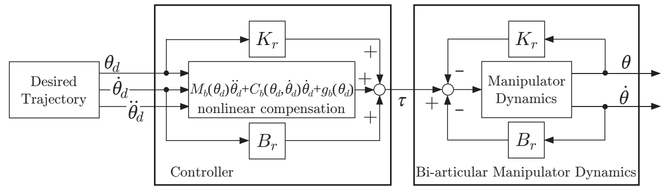

Fig. 4. Block diagram of the proposed control law.

where the elements of $M_b(\theta) \in \mathcal{R}^{3\times3}$, $C_b(\theta, \dot{\theta}) \in \mathcal{R}^{3\times3}$ and $g_b(\theta) \in \mathcal{R}^3$ are correspond to Eq. (7). Then the bi-articular manipulator dynamics has following important properties.

Property 1: Under the conditions $M_1 + M_2 > 2R$ and $M_1 M_2 > R^2$, the inertia matrix $M_b(\theta)$ preserves the positive definiteness [1].

Property 2: $\dot{M}_b(\theta) - 2C_b(\theta, \dot{\theta})$ is skew-symmetric.

Because Property 1 and 2 are very important factors for stability analysis, we construct the bi-articular manipulator dynamics (9) which satisfies them.

B. Passivity-based Control

The control objective of the manipulators with antagonistic bi-articular muscles is that both joint angle and the joint velocity coincide with the desired ones, respectively. For the bi-articular manipulator dynamics, we have considered the control law as

$$\tau = M_b(\theta)\dot{v} + C_b(\theta, \dot{\theta})v + g_b(\theta) - \dot{e} + K_r\theta_d + B_r\dot{\theta} \tag{10}$$

where v, e and a desired joint angle is defined as $v = \dot{\theta}_d - K_r e$, $e = \theta - \theta_d$ and $\theta_d = [q_{d1} \;\; q_{d2} \;\; q_{d1} + q_{d2}]^T$. Substituting Eq. (10) into Eq. (9), the closed-loop system can be obtained as

$$M_b(\theta)\dot{s} + C_b(\theta, \dot{\theta})s + s = 0 \tag{11}$$

where $s := \dot{e} + K_r e$. The block diagram of the closed-loop system is depicted in Fig.3.

Theorem 1: [10] The equilibrium point $(e, \dot{e}) = 0$ for the closed-loop system (11) is asymptotic stable.

Theorem 1 can be proved by using the following Lyapunov function

$$V = \frac{1}{2}s^T M_b(\theta)s + e^T K_r e \tag{12}$$

via Property 1 and 2 which are concerned with the passivity, although the passivity of the bi-articular manipulator dynamics can not be shown on account of antagonistic bi-articular muscles explicitly. Indeed, this control law is similar to one of passivity-based control laws for robot manipulators which is well-known as the Slotine and Li scheme [12]. More details are available in [10].

[1] In the ideal case with $l_1 = l_2 = 2l_{g1} = 2l_{g2}$, the conditions can be simple as $m_1 > 3/2m_2$.

III. Open-loop Control for Bi-articular Manipulator Dynamics

In this section, we present the analysis of the open-loop model of 2DOF robot manipulators with antagonistic bi-articular muscles.

A. Open-loop Control Law

Because it is inferred that humans do not measure the joint angles and velocities explicitly, we propose the following open-loop control law

$$\begin{aligned}\tau = M_b(\theta_d)\ddot{\theta}_d + C_b(\theta_d, \dot{\theta}_d)\dot{\theta}_d + g_b(\theta_d)\\ +K_r\theta_d + B_r\dot{\theta}_d.\end{aligned} \tag{13}$$

This is noted that θ and $\dot{\theta}$ are replaced with θ_d and $\dot{\theta}_d$ in Eq. (10). The block diagram of the proposed control law is depicted in Fig. 4 . Substituting Eq. (9) into Eq. (13), the error dynamics can be obtained as

$$M_b(\theta)\ddot{e} = -C_b(\theta, \dot{\theta})\dot{e} - K_r e - B_r\dot{e} - h\,(t, e, \dot{e}) \tag{14}$$

where the residual dynamics $h\,(t, e, \dot{e})$ is defined as follows:

$$\begin{aligned}h\,(t, e, \dot{e}) = \{M_b(\theta) - M_b(\theta_d)\}\,\ddot{\theta}_d\\ +\{C_b(\theta, \dot{\theta}) - C_b(\theta_d, \dot{\theta}_d)\}\dot{\theta}_d + g_b(\theta) - g_b(\theta_d).\end{aligned} \tag{15}$$

Then, the bi-articular manipulator dynamics satisfies the following properties like that of $n-$DOF robots manipulators [13].

Property 3: There exists a constant $k_M > 0$ such that

$$\|M_b(x)z - M_b(y)z\| \le k_M\|x - y\|z$$

for all vectors x, y, $z \in \mathcal{R}^3$.

Property 4: There exist numbers $k_{C1} > 0$, $k_{C2} > 0$ such that

$$\begin{aligned}\|C_b(x, z)w - C_b(y, v)w\| \le k_{C1}\|z - v\|\|w\|\\ +k_{C2}\|x - y\|\|w\|\|z\|\end{aligned}$$

for all vectors v, w, x, y, $z \in \mathcal{R}^3$.

Property 5: The vector $g_b(\theta)$ is Lipschitz, that is, there exists a nonnegative constant $k_g \ge \left\|\frac{\partial g_b(\theta)}{\partial\theta}\right\|$ such that

$$\|g_b(x) - g_b(y)\| \le k_g\|x - y\|$$

for all vectors x, $y \in \mathcal{R}^3$.

Property 6: There exist constants k_{h1}, $k_{h2} \ge 0$ such that the norm of the residual dynamics satisfies

$$\|h\,(t, e, \dot{e})\,\| \le k_{h1}\,\|\dot{e}\| + k_{h2}\,\|\mathrm{Sin}(e)\| \tag{16}$$

for all $e,\ \dot{e}\ \in\ \mathcal{R}^3$, where $\mathrm{Sin}(e)$ is following saturated function [14]

$$\mathrm{Sin}(x) = [\mathrm{Sin}(x_1), \mathrm{Sin}(x_2), \cdots, \mathrm{Sin}(x_n)]^T \tag{17}$$

$$\mathrm{Sin}(x_i) = \begin{cases} 1 & x_i \geq \frac{\pi}{2} \\ \sin(x_i) & |x_i| < \frac{\pi}{2} \\ -1 & x_i \leq -\frac{\pi}{2} \end{cases} \tag{18}$$

$$\frac{\partial \mathrm{Sin}(x)}{\partial x} = \mathrm{Cos}(x) = \mathrm{diag}\{\mathrm{Cos}(x_1), \cdots, \mathrm{Cos}(x_n)\} \tag{19}$$

for all vector $x \in \mathcal{R}^n$. Saturated function $\mathrm{Sin}(x)$ satisfies the following properties

- $\|\mathrm{Sin}(x)\| \leq \|x\|$
- $\|\mathrm{Sin}(x)\| \leq \sqrt{n}$
- $\|\mathrm{Sin}(x)\|^2 \leq \mathrm{Sin}(x)^T x$
- $\|\mathrm{Cos}(x)\dot{x}\| \leq \|\dot{x}\|$

for all $x,\ \dot{x} \in \mathcal{R}^n$.

B. Stability Analysis with Open-loop Control Law

Here, we define the state of the error dynamics with the bi-articular manipulator dynamics and the proposed control law as $x := [e^T\ \ \dot{e}^T]^T$. If the equilibrium point $x = 0$, then the joint angle and the joint velocity coincide with the desired ones and the control objective is achieved. We show the following theorem concerning the stability of the error dynamics.

Theorem 2: Given a positive scalar γ, assume

$$\lambda_{\min}\{B_r\} > k_{h1} + \gamma b \tag{20}$$

$$\lambda_{\min}\{K_r\} > \mathrm{Max}\left\{\gamma^2 \frac{\lambda^2_{\mathrm{Max}}\{\mathrm{M_b}(\theta)\}}{\lambda_{\min}\{\mathrm{M_b}(\theta)\}}, \frac{\{2\gamma a + k_{h2}\}^2}{4\gamma[\lambda_{\min}\{B_r\} - k_{h1} - \gamma b]} + k_{h2}\right\} \tag{21}$$

where the constant a and b are given by

$$a = \frac{1}{2}\left[-\lambda_{\min}\{B_r\} + k_{C1}\left\{\sup\left(\dot{\theta}_d\right)\right\} + k_{h1}\right],$$
$$b = \lambda_{\mathrm{Max}}\{M_b(\theta)\} + \sqrt{3}k_{C1}.$$

Then the equilibrium point $x = 0$ for the error dynamics is asymptotic stable,

Proof: Consider the following Lyapunov function candidate,

$$V(t, e, \dot{e}) = \frac{1}{2}\dot{e}^T M_b(\theta)\dot{e} + \frac{1}{2}e^T K_r e + \gamma \mathrm{Sin}(e)^T M_b(\theta)\dot{e} \tag{22}$$

where $\mathrm{Sin}(e)$ is defined as the saturated function (18). To show that the Lyapunov function candidate (22) is positive definite, we first observe that the third term in (22) satisfies

$$-\gamma \mathrm{Sin}(e)^T M_b(\theta)\dot{e} \geq -\gamma\lambda_{\mathrm{Max}}\{M_b(\theta)\}\|e\|\|\dot{e}\| \tag{23}$$

Therefore, the Lyapunov function candidate (22) satisfies the following inequality:

$$V(t, e, \dot{e}) \geq \frac{1}{2}\begin{bmatrix}\|e\| \\ \|\dot{e}\|\end{bmatrix}^T \begin{bmatrix}\lambda_{\min}\{K_r\} & -\gamma\lambda_{\mathrm{Max}}\{M_b(\theta)\} \\ -\gamma\lambda_{\mathrm{Max}}\{M_b(\theta)\} & \lambda_{\min}\{M_b(\theta)\}\end{bmatrix}\begin{bmatrix}\|e\| \\ \|\dot{e}\|\end{bmatrix} \tag{24}$$

and consequently, it happens to be positive definite and radially unbounded since by assumption, K_r is positive definite and we also supported that it is chosen so as to satisfy (21).

Using Property 2, the time derivative of the Lyapunov function candidate (22) yields

$$\begin{aligned}\dot{V}(t, e, \dot{e}) = & -\dot{e}^T B_r\dot{e} + \gamma\dot{e}^T \mathrm{Cos}(e)^T M_b(\theta)\dot{e} - \gamma \mathrm{Sin}(e)^T K_r e \\ & -\gamma \mathrm{Sin}(e)^T B_r \dot{e} + \gamma \mathrm{Sin}(e)^T C_b(\theta, \dot{\theta})^T \dot{e} \\ & -\dot{e}^T h(t, e, \dot{e}) - \gamma \mathrm{Sin}(e)^T h(t, e, \dot{e}).\end{aligned} \tag{25}$$

We now proceed to upper-bound $\dot{V}(t, e, \dot{e})$ by a negative definite function in terms of the states e and $\dot{e}$. Using the properties of the bi-articular manipulator dynamics, each term in Eq. (25) satisfy

$$-\dot{e}^T B_r \dot{e} \leq -\lambda_{\min}\{B_r\}\|\dot{e}\|^2 \tag{26}$$

$$\gamma\dot{e}\mathrm{Cos}(e)^T M_b(\theta)\dot{e} \leq \gamma\lambda_{\mathrm{Max}}\{M_b(\theta)\}\|\dot{e}\|^2 \tag{27}$$

$$-\gamma \mathrm{Sin}(e)^T K_r e \leq -\gamma\lambda_{\min}\{K_r\}\|\mathrm{Sin}(e)\|^2 \tag{28}$$

$$-\gamma \mathrm{Sin}(e)^T B_r \dot{e} \leq -\gamma\lambda_{\min}\{B_r\}\|\dot{e}\|\|\mathrm{Sin}(e)\| \tag{29}$$

$$\begin{aligned}\gamma \mathrm{Sin}(e)^T C_b(\theta, \dot{\theta})^T \dot{e} \leq & \gamma\sqrt{3}k_{C1}\|\dot{e}\|^2 \\ & +\gamma k_{C1}\left\{\sup\left(\dot{\theta}_d\right)\right\}\|\dot{e}\|\|\mathrm{Sin}(e)\|\end{aligned} \tag{30}$$

$$-\dot{e}^T h(t, e, \dot{e}) \leq k_{h1}\|\dot{e}\|^2 + k_{h2}\|\dot{e}\|\|\mathrm{Sin}(e)\| \tag{31}$$

$$\begin{aligned}-\gamma \mathrm{Sin}(e)^T h(t, e, \dot{e}) \leq & \gamma k_{h1}\|\dot{e}\|\|\mathrm{Sin}(e)\| \\ & +\gamma k_{h2}\|\mathrm{Sin}(e)\|^2.\end{aligned} \tag{32}$$

The bounds (26)–(32) yield that the time derivative $\dot{V}(t, e, \dot{e})$ in (25), satisfies

$$\dot{V}(t, e, \dot{e}) \leq -\gamma\begin{bmatrix}\|\mathrm{Sin}(e)\| \\ \|\dot{e}\|\end{bmatrix}^T R(\gamma)\begin{bmatrix}\|\mathrm{Sin}(e)\| \\ \|\dot{e}\|\end{bmatrix} \tag{33}$$

where the matrix $R(\gamma)$ is defined as

$$R(\gamma) = \begin{bmatrix}\lambda_{\min}\{K_r\} - k_{h2} & -a - \frac{1}{\gamma}\frac{k_{h2}}{2} \\ -a - \frac{1}{\gamma}\frac{k_{h2}}{2} & \frac{1}{\gamma}\{\lambda_{\min}\{B_r\} - k_{h1}\} - b\end{bmatrix}.$$

According to the theorem of Sylvester, in order for the matrix $R(\gamma)$ to be positive definite it is necessary and sufficient that the component R_{11} and $\det\{R(\gamma)\}$ be strictly positive. With respect to the first condition we stress that the matrix w.r.t. elastic K_r must satisfy

$$\lambda_{\min}\{K_r\} \geq k_{h2}. \tag{34}$$

On the other hand, the determinant of $R(\gamma)$ is given by

$$\begin{aligned}\det\{R(\gamma)\} = & \frac{1}{\gamma}[\lambda_{\min}\{K_r\} - k_{h2}][\lambda_{\min}\{B_r\} - k_{h1}] \\ & - [\lambda_{\min}\{K_r\} - k_{h2}]b - \left[a + \frac{1}{\gamma}\frac{k_{h2}}{2}\right]^2.\end{aligned}$$

The latter must be strictly positive for which it is necessary and sufficient that the matrix w.r.t. elastic K_r satisfies

$$\lambda_{\min}\{K_r\} > \frac{\{2\gamma a + k_{h2}\}^2}{4\gamma[\lambda_{\min}\{B_r\} - k_{h1} - \gamma b]} + k_{h2} \tag{35}$$

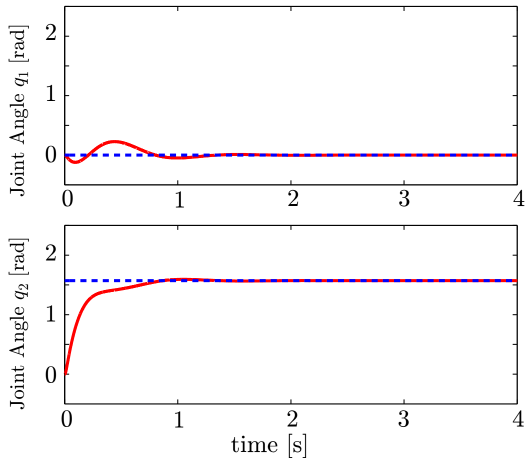

Fig. 5. Step response in the case of $q_{d1} = 0$, $q_{d2} = \frac{\pi}{2}$ with a open-loop ($\gamma = 0.1815$)

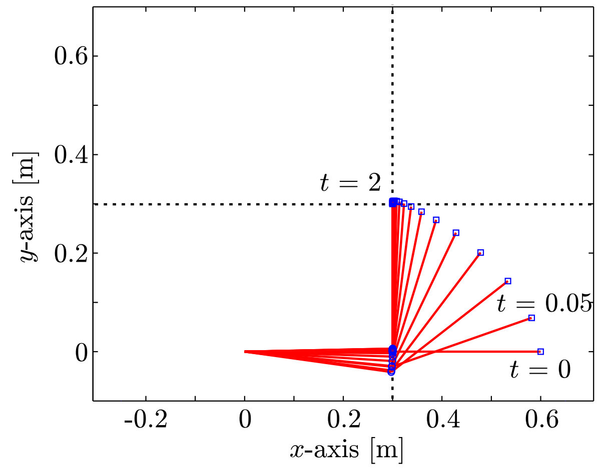

Fig. 6. Trajectory of the arm in the case of $q_{d1} = 0$, $q_{d2} = \frac{\pi}{2}$ with a open-loop ($\gamma = 0.1815$)

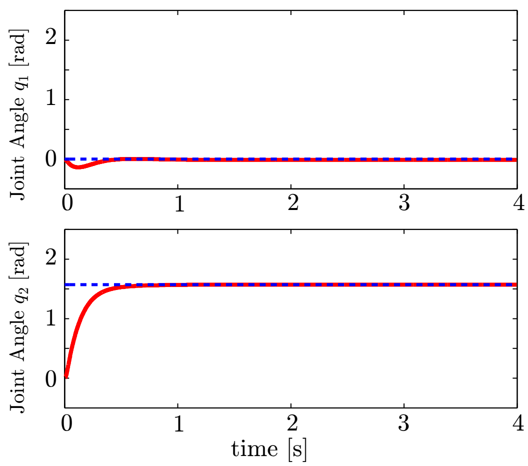

Fig. 7. Step response in the case of $q_{d1} = 0$, $q_{d2} = \frac{\pi}{2}$ with a closed-loop[10]

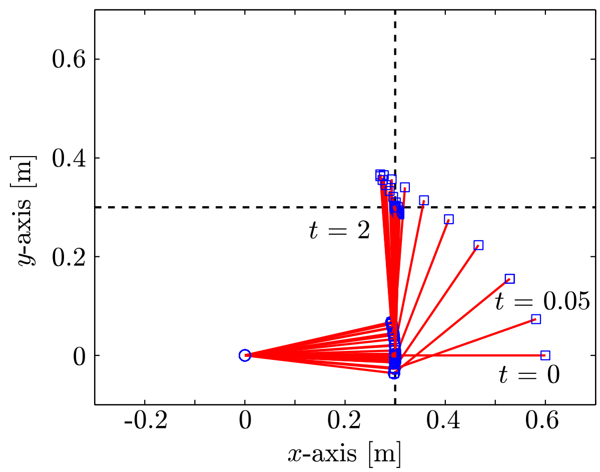

Fig. 8. Trajectory of the arm in the case of $q_{d1} = 0$, $q_{d2} = \frac{\pi}{2}$ with a closed-loop[10]

while it is sufficient that B_r satisfies

$$\lambda_{\min}\{B_r\} > k_{h1} + \gamma b \tag{36}$$

for the light-hand side of the inequality (35) to be positive. Observe that in this case the inequality (34) is trivially implied by (35).

Notice that the inequalities (35) and (36) correspond precisely to those in (20) and (21) as the tuning guidelines for the controller. This means that $R(\gamma)$ is positive definite and therefore, $\dot{V}(t, e, \dot{e})$ is negative definite.

According to the arguments above, given a positive constant γ we may determine the matrices K_r and B_r according to Eqs. (20) and (21) in a way that the function $V(t, e, \dot{e})$ given by (22) is positive definite while $\dot{V}(t, e, \dot{e})$ expressed as (33) is negative definite. For this reason, $V(t, e, \dot{e})$ is a strict Lyapunov function. According to Eqs. (22) and (33), the equilibrium point for the error dynamics (14) is asymptotic stable. ■

Theorem 2 shows the stability for the error dynamics combined by the bi-articular manipulator dynamics and the proposed open-loop control law via the Lyapunov method. It is interesting to note that the joint angles θ and the joint velocities $\dot{\theta}$ are exploited in the inner-loop only as depicted in Fig. 4.

IV. Simulation Results

We show the simulation results in order to confirm the proposed open-loop control law. The parameters of the bi-articular manipulator dynamics used in the simulation are $m_1 = 3.5$[kg], $m_2 = 1.75$[kg], $l_1 = 0.3$[m], $l_2 = 0.3$[m]. $l_{g1} = 0.15$[m], $l_{g2} = 0.15$[m], $\tilde{I}_1 = 0.026$[kg$\cdot$m^2], $\tilde{I}_2 = 0.013$[kg$\cdot$m^2] and $r_p = 0.05$[m]. The coefficients w.r.t. viscosity are $b_1 = b_2 = b_3 = 400$[Ns/m] and the coefficients w.r.t. elastic are $k_1 = 3000$[N/m], $k_2 = 2000$[N/m] and $k_3 = 4000$[N/m] for $\gamma = 0.1815$. We consider set-point problems with the following initial values $q_1(0) = 0$[rad], $q_2(0) = 0$[rad], $\dot{q}_1 = 0$[rad/s] and $\dot{q}_2 = 0$[rad/s]. The simulation results for the set-point problem are shown in Figs. 5– 10. Figs. 5 and 6 depict the joint angles in the case of $q_{d1} = 0$ and $q_{d2} = \frac{\pi}{2}$ with the proposed open-loop control law (13). For comparison, Figs. 7 and 8 depict the joint angles in the case of $q_{d1} = 0$ and $q_{d2} = \frac{\pi}{2}$ with the

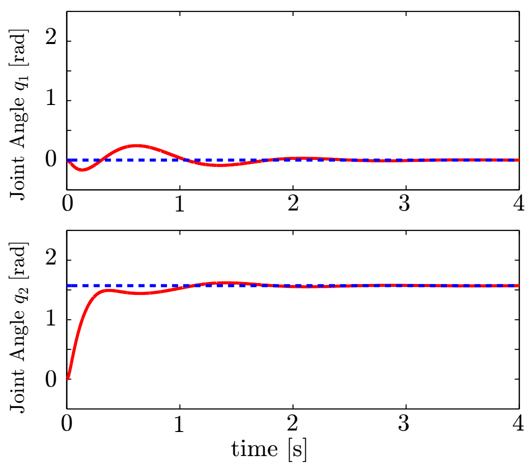

Fig. 9. Step response in the case of $q_{d1} = 0$, $q_{d2} = \frac{\pi}{2}$ ($\gamma = 0.1283$)

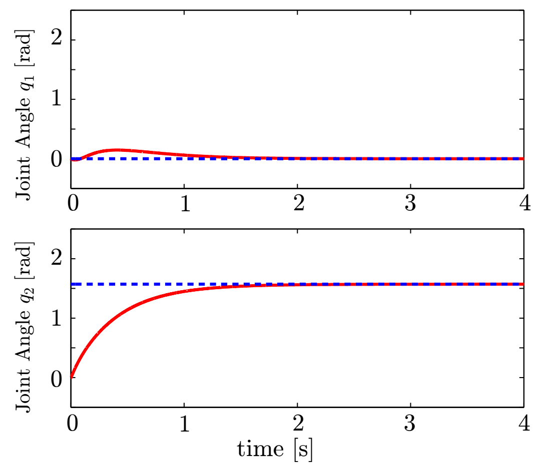

Fig. 10. Step response in the case of $q_{d1} = 0$, $q_{d2} = \frac{\pi}{2}$ ($\gamma = 0.2567$)

previous control law[10]. In Figs. 5 and 7, solid lines are the joint angles and dashed lines are desired ones. In Figs. 6 and 8, solid lines are the link and markers are the joints. Although the step response with a open-loop is inferior to the proposed closed-loop control law, these simulation results suggest that 2DOF robot manipulators with antagonistic bi-articular muscles could be controlled with the proposed open-loop control law.

Here, we give a design procedure in order to assign the matrices K_r and B_r in the following.

Step 1) The matrix B_r satisfying the condition (20) is chosen for a given γ.

Step 2) The matrix K_r satisfying the condition (21) is decided.

Based on the design procedure, we design the matrices B_r and K_r for the following two given γ.

$\gamma = 0.1283$:$K_r = \mathrm{diag}\{1500, 1000, 2000\}r_p^2$, $B_r = \mathrm{diag}\{200, 200, 200\}r_p^2$;

$\gamma = 0.2567$:$K_r = \mathrm{diag}\{6000, 4000, 8000\}r_p^2$, $B_r = \mathrm{diag}\{2000, 2000, 2000\}r_p^2$.

Figs. 9 and 10 depict the simulation results in the case of $\gamma = 0.1283$ and $\gamma = 0.2567$, respectively. The rise time is small for the smaller values of γ from Figs. 5, 9 and 10, while the time response is oscillatory. These simulations show how the method can be used to adjust the rise time and the overshoot for the proposed open-loop control system, where the parameter γ together with the matrices K_r and B_r can be tuned for such purpose.

V. Conclusions

This paper considers open-loop control for the 2DOF robot manipulators with antagonistic bi-articular muscles. The stability analysis of the error dynamics has been discussed based on the Lyapunov method. The simulation results show the validity of the proposed open-loop control law and the design procedure for the matrices K_r and B_r. This approach does not deal with the uncertainties of the bi-articular manipulator dynamics. In our future work, we will consider vision-based control for the bi-articular manipulator dynamics in order to overcome this drawback and verify the validity of the proposed open-loop control law with experiments.

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