# Predictive Visual Feedback Control with Eye-in-Hand System via Stabilizing Receding Horizon Approach

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***Abstract*— This paper investigates vision based robot control based on a receding horizon control strategy, as a first step for a predictive visual feedback control. Firstly, the brief summary of the 3D dynamic visual feedback system with eye-in-hand configuration is given. Next, a stabilizing receding horizon control for the 3D dynamic visual feedback system, a highly nonlinear and relatively fast system, is proposed. The stability of the receding horizon control scheme is guaranteed by using the terminal cost derived from an energy function of the visual feedback system. Furthermore, simulation results are assessed with respect to the stability and the performance.**

## I. Introduction

Robotics and intelligent machines need sensory information to behave autonomously in dynamical environments. Visual information is particularly suited to recognize unknown surroundings. In this sense, vision is one of the highest sensing modalities that currently exist. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, is important when we consider a mechanical system working in dynamical environments [1], [2].

In previous works, for the problem of three dimensional (3D) visual servo control, Kelly $et\ al.$ [3] considered an image based controller under the assumption that the objects' depths are known. Cowan $et\ al.$ [4] addressed the field-of-view problem for 3D dynamic visual feedback system using navigation functions. Although good solutions to the set-point problem are reported in those papers, few results have been obtained for the tracking problem of moving target objects in the full 3D dynamic visual feedback system that include not only the position and the orientation but also the manipulator dynamics. The authors discussed passivity based control for a moving target object in 3D workspace with eye-in-hand configuration [5], [6]. However, the control law proposed in [5] is not based on optimization, the desired control performance cannot be guaranteed explicitly.

Receding horizon control, also recognized as model predictive control is a well-known control strategy in which the current control action is computed by solving, a finite horizon optimal control problem on-line [7]. A large number of industrial applications using model predictive control can be found in chemical industries where the processes have relatively slow dynamics. On the contrary, for nonlinear and relatively fast systems such as in robotics, few implementations of the receding horizon control have been reported. For the receding horizon control, many researchers have tackled the problem of stability guarantees. An approach proposed by Parisini $et\ al.$ [8] is based on using a quadratic endpoint penalty of the form $ax^T(t+T)Px(t+T)$ for some $a>0$, some positive definite matrix $P$ and a terminal state $x(t+T)$. Jadbabaie $et\ al.$ [9] showed that closed-loop stability is ensured through the use of a terminal cost consisting of a control Lyapunov function. Moreover, these results were applied to the Caltech Ducted Fan to perform aggressive maneuvers [10], [11]. Visual feedback, however, is not considered here. Predictive control could be of significant benefit when used in conjunction with visual servoing. With the incorporation of visual information, the system could anticipate the target's future position and be waiting there to intercept it [12]. In [13], the authors proposed stabilizing receding horizon control for the planar visual feedback system. However, the visual feedback system proposed in [13] is restricted to a planar manipulator, and this method can only treat the desired position problem.

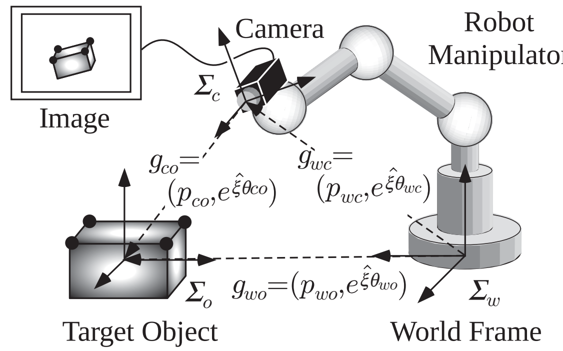


Fig. 1. Eye-in-hand visual feedback system.

In this paper, as a first step for a predictive visual feedback control, stabilizing receding horizon control is applied to the 3D visual feedback system in [5], a highly nonlinear and relatively fast system. Compared with previous work [13], this 3D dynamic visual feedback system can treat not only the position but also the orientation, so the possible application area should be undoubtedly increasing. This represents a first step towards high performance visual servoing targeting more aggressive maneuvers. The main idea is the use of the terminal cost derived from an energy function of the visual

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feedback system.

Firstly, the brief summary of our prior work [5], which is the 3D dynamic visual feedback system, is given. Next, a stabilizing receding horizon control for the 3D visual feedback system using a control Lyapunov function is proposed. Then, the control performance of the stabilizing receding horizon control scheme is evaluated through simulation results.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times 3}$ to represent the change of the principle axes of a frame $\Sigma_b$ relative to a frame $\Sigma_a$. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. For simplicity we use $\hat{\xi}\theta_{ab}$ to denote $\hat{\xi}_{ab}\theta_{ab}$. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$', i.e., $so(3) \to \mathcal{R}^3$. Recall that a skew-symmetric matrix corresponds to an axis of rotation (via the mapping $a \mapsto \hat{a}$). We use the $4\times 4$ matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \tag{1}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$ describing the configuration of a frame $\Sigma_b$ relative to a frame $\Sigma_a$. The adjoint transformation associated with $g_{ab}$ is denoted by $\mathrm{Ad}_{(g_{ab})}$ [14].

## II. Three Dimensional Dynamic Visual Feedback System with Eye-in-Hand Configuration

In this section, the brief summary of our prior work in [5] is given. An energy function and a stabilizing control law, which play an important role for a predictive visual feedback control, are derived.

### A. Basic Representation for Visual Feedback System

The visual feedback system considered in this paper has the camera mounted on the robot's end-effector as depicted in Fig. 1, where the coordinate frames $\Sigma_w$, $\Sigma_c$ and $\Sigma_o$ represent the world frame, the camera (end-effector) frame, and the object frame, respectively. Then, the relative rigid body motion from $\Sigma_c$ to $\Sigma_o$ can be represented by $g_{co}$. Similarly, $g_{wc}$ and $g_{wo}$ denote the rigid body motions from the world frame $\Sigma_w$ to the camera frame $\Sigma_c$ and from the world frame $\Sigma_w$ to the object frame $\Sigma_o$, respectively, as shown in Fig. 1.

The objective of visual feedback control is to bring the actual relative rigid body motion $g_{co}$ to a given reference $g_d$. The reference $g_d$ for the relative rigid body motion $g_{co}$ is assumed to be constant throughout this paper, because the camera can track the moving target object in this case.

The relative rigid body motion from $\Sigma_c$ to $\Sigma_o$ can be led by using the composition rule for rigid body transformations ([14], Chap. 2, pp. 37, eq. (2.24)) as follows:

$$g_{co} = g_{wc}^{-1} g_{wo}. \tag{2}$$

The relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [14]. We define the body velocity of the camera relative to the world frame $\Sigma_w$ as $V_{wc}^b = [v_{wc}^T \ \omega_{wc}^T]^T$, where $v_{wc}$ and $\omega_{wc}$ represent the velocity of the origin and the angular velocity from $\Sigma_w$ to $\Sigma_c$, respectively ([14] Chap. 2, eq. (2.55)).

Differentiating (2) with respect to time, the body velocity of the relative rigid body motion $g_{co}$ can be written as follows [5].

$$V_{co}^b = -\mathrm{Ad}_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b \tag{3}$$

where $V_{wo}^b$ is the body velocity of the target object relative to $\Sigma_w$. We consider that it is the basic representation for the three coordinate frames of the visual feedback system. Roughly speaking, the relative rigid body motion $g_{co}$ will depend on the difference between the camera velocity $V_{wc}^b$ and the target object velocity $V_{wo}^b$, because $V_{co}^b$ is defined as the body velocity of the relative rigid body motion $g_{co}$.

### B. Estimation Error and Control Error Systems

The visual feedback control task requires information of the relative rigid body motion $g_{co}$. Since the measurable information is only the image information $f(g_{co})$ in the visual feedback system, we consider a nonlinear observer in order to estimate the relative rigid body motion from the image information $f(g_{co})$.

Firstly, using the basic representation (3), we choose estimates $\bar{g}_{co}$ and $\bar{V}_{co}^b$ of the relative rigid body motion and velocity, respectively as

$$\bar{V}_{co}^b \;\; = \;\; -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e, \tag{4}$$

where $u_e$ is the new input in order to converge the estimated value to the actual relative rigid body motion.

In order to establish the estimation error system, we define the estimation error between the estimated value $\bar{g}_{co}$ and the actual relative rigid body motion $g_{co}$ as

$$g_{ee} = \bar{g}_{co}^{-1} g_{co}. \tag{5}$$

Using the notation $e_R(e^{\hat{\xi}\theta})$ in [15], the vector of the estimation error is given by $e_e := [p_{ee}^T \ e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$. Note that $e_e = 0$ iff $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I_3$. Therefore, if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\bar{g}_{co}$ equals the actual relative rigid body motion $g_{co}$. The estimation error vector $e_e$ can be obtained from image information $f(g_{co})$ and the estimated value of the relative rigid body motion $\bar{g}_{co}$. In the same way as the basic representation for the visual feedback system, the estimation error system can be represented by

$$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \tag{6}$$

Moreover, let us consider the dual of the estimation error system, which we call the control error system, in order to establish the visual feedback system. We define the control error as follows:

$$g_{ec} = g_d^{-1} \bar{g}_{co}, \tag{7}$$

which represents the error between the estimated value $\bar{g}_{co}$ and the reference of the relative rigid body motion $g_d$. The vector of the camera control error is defined as $e_c :=$

$[p_{ec}^T \ e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$. The camera control error system can be described by

$$V_{ec}^b = -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e. \tag{8}$$

Combining (6) and (8), we construct the visual feedback system as follows:

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b \tag{9}$$

where $u_{ce} := [(V_{wc}^b)^T \ u_e^T]^T$ denotes the control input.

### *C. Dynamic Passivity based Visual Feedback System with Eye-in-hand Configuration*

The manipulator dynamics can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_d \tag{10}$$

where $M \in \mathcal{R}^{n\times n}$ is the inertia matrix, $C \in \mathcal{R}^{n\times n}$ is the Coriolis matrix, $g \in \mathcal{R}^n$ is the gravity terms, and $q$, $\dot{q}$ and $\ddot{q}$ are the joint angle, velocity and acceleration, respectively. $\tau$ is the vector of the input torque, and $\tau_d$ represents a disturbance input [16]. Now, we will construct a dynamic visual feedback system by connecting the visual feedback system (9) and the manipulator dynamics (10). Since the camera is mounted on the end-effector of the manipulator in the eye-in-hand configuration, the body velocity of the camera $V_{wc}^b$ is given by

$$V_{wc}^b = J_b(q)\dot{q} \tag{11}$$

where $J_b(q)$ is the manipulator Jacobian [14].

Next, we propose the control law for the manipulator as

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + g(q) + J_b^T(q)\mathrm{Ad}_{(g_d^{-1})}^T e_c + u_\xi. \tag{12}$$

where $\dot{q}_d$ and $\ddot{q}_d$ represent the desired joint velocity and acceleration, respectively. The new input $u_\xi$ is to be determined in order to achieve the control objective.

Let us define the error vector with respect to the joint velocity of the manipulator as $\xi := \dot{q} - \dot{q}_d$. Moreover, we design the reference of the joint velocity based on the relation between the camera velocity and the joint velocity (11) as $\dot{q}_d := J_b^\dagger(q)u_d$ where $u_d$ is the desired body velocity of the camera which will be obtained from the visual feedback system. Thus, $V_{wc}^b$ in (8) should be replaced by $u_d$.

Using (9)–(12), the visual feedback system with the manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows:

$$\begin{bmatrix} \dot{\xi} \\ V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -M^{-1}C\xi + M^{-1}J_b^T\mathrm{Ad}_{(g_d^{-1})}^T e_c \\ -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} J_b \xi \\ 0 \end{bmatrix} + \begin{bmatrix} M^{-1} & 0 & 0 \\ 0 & -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} & I \\ 0 & 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u + \begin{bmatrix} M^{-1} & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} w \tag{13}$$

where $u := [u_\xi^T \ u_d^T \ u_e^T]^T$. We define the state and the disturbance of dynamic visual feedback system as $x := [\xi^T \ e_c^T \ e_e^T]^T$ and $w := [\tau_d^T \ (V_{wo}^b)^T]^T$, respectively.

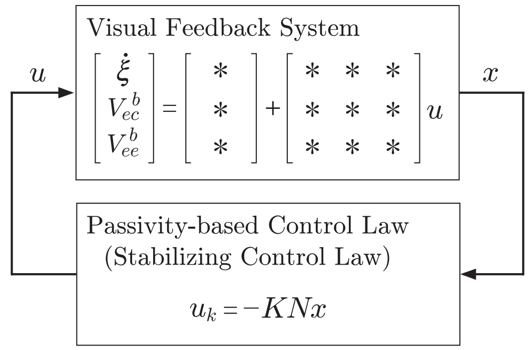


Fig. 2. Block diagram of the passivity based visual feedback control.

### *D. Energy Function and Stabilizing Control Law*

In previous work [5], the passivity of the visual feedback system (13) is derived by using the following energy function $V(x)$

$$V(x) = \frac{1}{2}\xi^T M(q)\xi + E(g_{ec}) + E(g_{ee}) \tag{14}$$

where $E(g) := \frac{1}{2}\|p\|^2 + \phi(e^{\hat{\xi}\theta})$ and $\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta})$ is the error function of the rotation matrix (see e.g. [15]). Here, we consider the following control input

$$u = -K\nu := u_k, \quad K > 0, \tag{15}$$

$$\nu := Nx := \begin{bmatrix} I & 0 & 0 \\ 0 & -\mathrm{Ad}_{(g_d^{-1})}^T & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})}^T & -I \end{bmatrix} x. \tag{16}$$

By differentiating $V(x)$ along the trajectory of the system and using the control input $u_k$, the next equation is obtained if $w = 0$ is satisfied.

$$\dot{V} = \nu^T u = -x^T N^T K N x. \tag{17}$$

Therefore, if $w = 0$, the equilibrium point $x = 0$ for the closed-loop system (13) and (15) is asymptotic stable, i.e., $u_k$ is a stabilizing control law for the system. The block diagram of the passivity based visual feedback control is shown in Fig. 2.

However, the stabilizing control law $u_k$ (15) is not based on optimization, the desired control performance cannot be guaranteed explicitly. In the next section, a stabilizing receding horizon control based on optimal control theory is proposed.

## III. Predictive Visual Feedback Control

The objective of this section is to propose a predictive visual feedback control based on optimal control theory. A camera can provide more information than the current derivation from a nominal position at the sample instant. This property can be exploited to predict the target's future position and improve the control performance. As a first step for a predictive visual feedback control, we propose a stabilizing receding horizon control based on optimization in this paper.

*A. Control Lyapunov Function*

In this section, the finite horizon optimal control problem (FHOCP) for the visual feedback system (13) is considered. The FHOCP for the visual feedback system (13) at time $t$ consists of the minimization with respect to the input $u(\tau, x(\tau)), \tau \in [t, t+T]$, of the following cost function

$$J(x_0, u, T) = \int_t^{t+T} l(x(\tau), u(\tau))d\tau + F(x(t+T)) \quad (18)$$

$$l(x(t), u(t)) = q_\xi(t)\|\xi(t)\|^2 + E_{qc}(g_{ec}(t)) + E_{qe}(g_{ee}(t)) + u^T(t)R(t)u(t) \quad (19)$$

$$F(x) = \rho V(x) \quad (20)$$

$$q_\xi(t) \geq 0,\ q_p(t) \geq 0,\ q_R(t) \geq 0,\ R(t) > 0,\ \rho > 0,$$

where $E_q(g(t)) := q_p(t)\|p(t)\|^2 + q_R(t)\phi(e^{\hat{\xi}\theta(t)})$, with the state $x(t) = x_0$. The speciality of the cost function (18)–(20) is that the terminal cost is derived from an energy function of the visual feedback system. Furthermore, the rotation error related part of the stage cost is derived from the error function $\phi(e^{\hat{\xi}\theta})$ instead of the commonly used quadratic form $\|e_R(e^{\hat{\xi}\theta})\|^2$. For a given initial condition $x_0$, we denote this solution of the FHOCP as $u^*(\tau, x(\tau)), \tau \in [t, t+T]$. In receding horizon control, at each sampling time $\delta$, the resulting feedback control at state $x_0$ is obtained by solving the FHOCP and setting

$$u_{RH} := u^*(\delta, x_0). \quad (21)$$

The following lemma concerning a control Lyapunov function is important to prove a stabilizing receding horizon control. The definition for a control Lyapunov function $S(x)$ is given by

$$\inf_u \left[\dot{S}(x) + l(x, u)\right] \leq 0, \quad (22)$$

where $l(x, u)$ is a positive definite function [9].

*Lemma 1:* Suppose that $w = 0$, $\|\theta_{ec}\| \leq \frac{\pi}{2}$, $\|\theta_{ee}\| \leq \frac{\pi}{2}$ and the design parameter $\rho$ satisfies

$$\rho^2 I \geq 4QN^{-1}RN^{-T}, \quad (23)$$

where $Q := \mathrm{diag}\{q_\xi I_n, q_{pc}I_3, q_{Rc}I_3, q_{pe}I_3, q_{Re}I_3\}$. Then, the energy function $\rho V(x)$ of the visual feedback system (13) can be regarded as a control Lyapunov function.

*Proof:* In [5], we have already shown that the time derivative of $V$ along the trajectory of the system (13) is formulated as (17). Using the positive definite function $l(x(t), u(t))$ (19) and the stabilizing control law $u_k$ (15) with $K = \frac{\rho}{2}R^{-1}$ for the system, Equation (22) can be transformed into

$$\begin{aligned}
&\inf_u[\dot{S}(x) + l(x, u)] \\
&= \inf_u \left[\rho\dot{V} + q_\xi\|\xi\|^2 + E_{qc}(g_{ec}) + E_{qe}(g_{ee}) + u^T Ru\right] \\
&= \inf_u \left[\rho x^T N^T u + q_\xi\|\xi\|^2 + E_{qc}(g_{ec}) + E_{qe}(g_{ee}) + u^T Ru\right] \\
&= \inf_u \left[\left(u + \frac{\rho}{2}R^{-1}Nx\right)^T R\left(u + \frac{\rho}{2}R^{-1}Nx\right)\right. \\
&\qquad \left. - \frac{\rho^2}{4}x^T N^T R^{-1}Nx + q_\xi\|\xi\|^2 + E_{qc}(g_{ec}) + E_{qe}(g_{ee})\right] \\
&= -\frac{\rho^2}{4}x^T N^T R^{-1}Nx + q_\xi\|\xi\|^2 + q_{pc}\|p_{ec}\|^2 \\
&\quad + q_{Rec}\phi(e^{\hat{\xi}\theta_{ec}}) + q_{pe}\|p_{ee}\|^2 + q_{Ree}\phi(e^{\hat{\xi}\theta_{ee}}) \\
&\leq -\frac{\rho^2}{4}x^T N^T R^{-1}Nx + q_\xi\|\xi\|^2 + q_{pc}\|p_{ec}\|^2 \\
&\quad + q_{Rec}\|e_R^T(e^{\hat{\xi}\theta_{ec}})\| + q_{pe}\|p_{ee}\|^2 + q_{Ree}\|e_R^T(e^{\hat{\xi}\theta_{ee}})\| \\
&= -x^T\left(\frac{\rho^2}{4}N^T R^{-1}N - Q\right)x,
\end{aligned} \quad (24)$$

where we have used the fact that $\phi(e^{\hat{\xi}\theta}) \leq \|e_R^T(e^{\hat{\xi}\theta})\|$ for all $\|\theta\| \leq \frac{\pi}{2}$. Therefore, the condition $\inf_u[\dot{S}(x) + l(x, u)] \leq 0$ will be satisfied, if the assumption $\rho^2 I \geq 4QN^{-1}RN^{-T}$. ■

Lemma 1 shows the energy function $\rho V(x)$ of the visual feedback system (13) can be regarded as a control Lyapunov function in the case of $\rho^2 I \geq 4QN^{-1}RN^{-T}$.

*B. Stabilizing Receding Horizon Control for the 3D Visual Feedback System*

Suppose that the terminal cost is the control Lyapunov function $\rho V(x)$, the following theorem concerning the stability of the receding horizon control holds.

*Theorem 1:* Consider the cost function (18)–(20) for the visual feedback system (13). Suppose that $w = 0$, $\|\theta_{ec}\| \leq \frac{\pi}{2}$, $\|\theta_{ee}\| \leq \frac{\pi}{2}$, and $\rho^2 I \geq 4QN^{-1}RN^{-T}$, then the receding horizon control for the visual feedback system is asymptotically stabilizing.

*Proof:* Our goal is to prove that $J(x^*(t), u_{RH}, T)$, which is the cost-to-go applying the receding optimal control $u_{RH}$, will qualify as a Lyapunov function for the closed loop system. Construct the following suboptimal control strategy for the time interval $[t+\delta, t+T+\delta]$

$$\tilde{u} = \begin{cases} u^*(\tau) & \tau \in [t+\delta, t+T] \\ u_k(\tau) = -\frac{\rho}{2}R^{-1}Nx & \tau \in [t+T, t+T+\delta] \end{cases} \quad (25)$$

where $u_k$ is the stabilizing control law (15) with $K = \frac{\rho}{2}R^{-1}$ for the visual feedback system. The associated cost is

$$\begin{aligned}
&J(x^*(t+\delta), \tilde{u}, T) \\
&\quad = J(x(t), u^*, T) + \rho[V(x(t+T+\delta)) - V(x^*(t+T))] \\
&\quad - \int_t^{t+\delta} l(x^*(\tau), u^*)d\tau + \int_{t+T}^{t+T+\delta} l(x^*(\tau+T), u_k)d\tau,
\end{aligned} \quad (26)$$

where $x^*$ is the optimal state trajectory. This cost, which is an upper bound for $J(x^*(t+\delta), u^*, T)$, satisfies

$$\begin{aligned}
&J(x^*(t+\delta), u^*, T) - J(x^*(t), u^*, T) \\
&\quad \leq \rho[V(x(t+T+\delta)) - V(x^*(t+T))] \\
&\quad - \int_t^{t+\delta} l(x^*(\tau), u^*)d\tau + \int_{t+T}^{t+T+\delta} l(x^*(\tau+T), u_k)d\tau.
\end{aligned} \quad (27)$$

Using the positive definite function $l(x(t), u(t))$ (19) and the stabilizing control law $u_k$ (15) for the system, and dividing

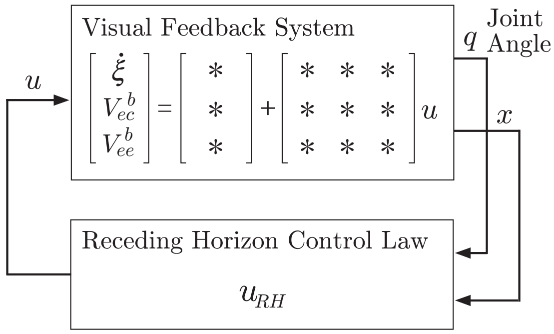


Fig. 3. Block diagram of the predictive visual feedback control.

both sides by $\delta$ and taking the limit as $\delta \to 0$, Equation (27) can be transformed into

$$\begin{aligned}
&\lim_{\delta \to 0} \frac{J(x^*(t+\delta), u^*, T) - J(x^*(t), u^*, T)}{\delta} \\
&\leq -\frac{\rho^2}{4} x^{*T}(t+T) N^T R^{-1} N x^*(t+T) \\
&\quad + q_\xi \|\xi^*(t+T)\|^2 + E_{qc}(g_{ec}^*(t+T)) + E_{qe}(g_{ee}^*(t+T)) \\
&\quad - x^{*T}(t) Q x^*(t) - u^{*T} R u^* \\
&\leq - x^{*T}(t+T) \left( \frac{\rho^2}{4} N^T R^{-1} N - Q \right) x^*(t+T) \\
&\quad - x^{*T}(t) Q x^*(t) - u^{*T} R u^*. \qquad (28)
\end{aligned}$$

Considering that the control input during first $\delta$ is $u_{RH} = u^*$, by the assumption $\rho^2 I \geq 4 Q N^{-1} R N^{-T}$, the derivative of $J(x^*(t), u_{RH}, T)$ is negative definite. Therefore, we have shown that $J(x^*(t), u_{RH}, T)$ qualifies as a Lyapunov function and asymptotic stability is guaranteed. ■

Theorem 1 guarantees the stability of the receding horizon control using a control Lyapunov function for the 3D visual feedback system (13) which is a highly nonlinear and relatively fast system. Since the stabilizing receding horizon control design is based on optimal control theory, the control performance should be improved compared to the simple passivity based control [5], under the condition of adequate gain assignment in the cost function. It should be noted that the error function $\phi(e^{\hat{\xi}\theta})$ of the rotation matrix can be directly used in the stage cost (19). Moreover, compared with previous work [13], the main advantage of this approach is that the 3D dynamic visual feedback system is not restricted to a planar manipulator, and can treat not only the position but also the orientation. This allows us to extend the technological application area. In this paper, as a first step, we propose unconstrained stabilizing receding horizon control schemes. In the near future, we will consider constraints which represent one of the advantages of receding horizon control, and develop it using level set, see [9]. The assumptions $\|\theta_{ec}\| \leq \frac{\pi}{2}$ and $\|\theta_{ee}\| \leq \frac{\pi}{2}$ will be considered by using constraints. The block diagram of the predictive visual feedback control is shown in Fig. 3.

In the next section, the stabilizing receding horizon control is applied to a 3D visual feedback system. It is expected that the control performance is improved using the receding horizon control.

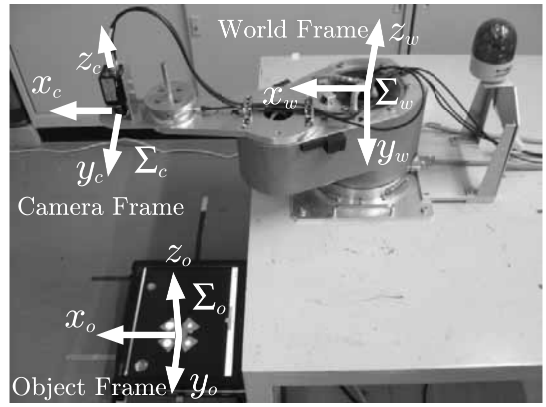


Fig. 4. Coordinated frames for 3D dynamic visual feedback systems with 2DOF manipulator.

## IV. Simulations

In this section, the validity of the proposed control law can be confirmed by comparing the simulation results. Although we only discuss simulation results in this paper, it should be noted that the model parameters were developed using actual visual servo system.

The simulation results on 2DOF manipulator as depicted in Fig. 4 are shown in order to understand our proposed method simply, though it is valid for 3D dynamic visual feedback systems. We defined the three coordinates which were described in Fig. 4. The target object has four feature points. The control objective is to bring the actual relative rigid body motion $g_{co}$ to a given reference $g_d$. In this paper, we present results for the stability analysis with a static target object. The simulation is carried out with the initial condition $q_1(0) = \pi/6$ [rad], $q_2(0) = -\pi/6$ [rad], $p_{wo} = [0.3986 \ 0 \ -0.9]^T$ [m], $\xi\theta_{wo} = [0 \ 0 \ -0.5087]^T$ [rad], $p_{wc} = [0.4732 \ 0.1 \ 0]^T$ [m], $\xi\theta_{wc} = [0 \ 0 \ 0]^T$ [rad]. The desired relative rigid body motion $g_d$ is $p_d = [0 \ 0 \ -0.9]^T$ [m], $\xi\theta_d = [0 \ 0 \ 0]^T$ [rad], and the initial error condition $x(0)$ is $\xi(0) = [0 \ 0]^T$ [rad/s], $p_{ec}(0) = [-0.0746 \ -0.1 \ 0]^T$ [m], $\xi\theta_{ec}(0) = [0 \ 0 \ -0.5077]^T$ [rad], $p_{ee}(0) = [0 \ 0 \ 0]^T$ [m], $\xi\theta_{ee}(0) = [0 \ 0 \ -0.001]^T$ [rad] in this simulation.

In this simulation, we compare the performance of the receding horizon control law proposed in Theorem 1 and the passivity based control law $u_k$ (15). The weights of the cost function (18)–(20) were selected as $q_\xi = 0.001$, $q_{pc} = 0.003$, $q_{Rc} = 0.001$, $q_{pe} = 0.0003$, $q_{Re} = 0.0001$, $R = \mathrm{diag}\{0.1, 3.2, 0.07, 50, 15, 15, 15, 50, 300, 300, 30, 30, 30, 300\}$ and $\rho = 1$ satisfy $\rho^2 I \geq 4 Q N^{-1} R N^{-T}$. The controller parameters for the passivity based control law $u_k$ (15) were empirically selected as $K = \mathrm{diag}\{10, 1, 2, 1, 0.2, 0.2, 0.2, 1, 1, 1, 1, 1, 1, 1\}$. To solve the real time optimization problem, the software C/GMRES [17] is utilized. The control input with the receding horizon control is updated every 20 [ms]. It must be calculated by the receding horizon controller within that period. The horizon was selected as $T = 0.02$ [s].

The simulation results are presented in Fig. 5. This figure shows the actual control error $e_r := [p_{er}^T \ e_R^T(e^{\hat{\xi}\theta_{er}})]^T$, for the

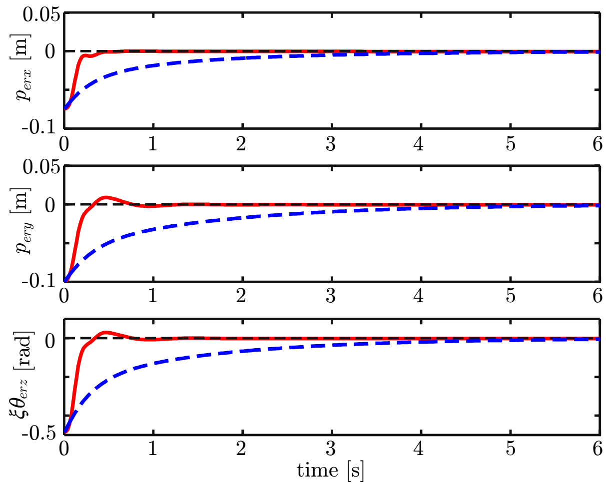


Fig. 5. Actual control error: solid: with stabilizing receding control law; dashed: with passivity based control law.

position based control, which is the error vector between $g_{co}$ and $g_d$. In Fig. 5, the solid lines denote the errors applying the proposed stabilizing receding horizon control, and the dashed lines denote those for the passivity based control law $u_k$ (15). We focus on the errors of the translations of $x$ and $y$ and the rotation of $z$, because the errors of the translation of $z$ and the rotations of $x$ and $y$ are zeros ideally on the defined coordinates in Fig. 4.

In Fig. 5, the asymptotic stability can be confirmed by steady state performance. Moreover, the rise time applying the receding horizon control is shorter than that for the passivity based control. The controller predicts the movement of the target object using the visual information, as a result, the manipulator moves more aggressively. This validates one of the expected advantages of the stabilizing receding horizon control for the visual feedback system.

The performance for parameter value $T$ and $\rho$ is compared in terms of the integral cost in Table I. Since the cost of the stabilizing receding horizon method is smaller than the passivity based control method under conditions of the adequate cost function, it can be easily verified that the control performance is improved. With increasing weight of the terminal cost from $\rho = 0.75$ to $\rho = 1.5$ the cost increases, too. With higher terminal cost the state value is reduced more strictly, using a large control input. In this simulation, since the weights of the control input are larger than those of the state, the cost increased consequently. As the horizon length increases from $T = 0.005$ to $T = 0.05$, the cost is reduced. In the case of $T = 0.2$, the calculation can not be completed within one sampling interval, due to limited computing power.

## V. Conclusions

This paper proposes a stabilizing receding horizon control for a 3D visual feedback system, which is a highly nonlinear and relatively fast system, as a first step for a predictive visual feedback control. It is shown that the stability of the receding horizon control scheme is guaranteed by using

TABLE I

Values of the integral cost

| Control Scheme | cost |
|---|---|
| Passivity based Control | 1208 |
| Receding Horizon Control ($T = 0.02$ [s], $\rho = 0.75$) | 9.61 |
| Receding Horizon Control ($T = 0.02$ [s], $\rho = 1$) | 31.5 |
| Receding Horizon Control ($T = 0.02$ [s], $\rho = 1.5$) | 682 |
| Receding Horizon Control ($T = 0.005$ [s], $\rho = 1$) | 32.4 |
| Receding Horizon Control ($T = 0.05$ [s], $\rho = 1$) | 30.1 |

the terminal cost derived from an energy function of the visual feedback system. In the simulation results, the control performance of the stabilizing receding horizon control is improved compared to that of the simple passivity based control. In this paper, the stabilizing receding controller was implemented for a low level inner loop, in the near future, we would like to tackle the implementation on a high level outer loop.

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