Passivity-based Control and Estimation of Dynamic Visual Feedback Systems with a Fixed Camera

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Abstract— **This paper deals with the control and the estimation of dynamic visual feedback systems with a fixed camera. Specifically, we consider the target tracking problem of dynamic visual feedback systems in the three dimensional(3D) workspace. Firstly the visual feedback system with four coordinate frames is established by using the homogeneous representation and adjoint transformation. Secondly we derive the passivity of the dynamic visual feedback system by combining the manipulator dynamics and the visual feedback system. Based on the passivity, stability and L_2-gain performance analysis are discussed. Finally simulation results are shown to verify the stability and L_2-gain performance of the dynamic visual feedback system.**

I. Introduction

Robotics and intelligent machines need many information to behave autonomously under dynamical environments. Visual information is undoubtedly suited to recognize unknown surroundings. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments [1], [2]. Recently, technological fields which need visual feedback control are undoubtedly increasing, such as the autonomous injection of biological cells [3], the laparoscopic surgery [4] and so on. Control will be more important for intelligent machines as future applications.

Classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop. However, as stated in [5], this assumption is invalid for high speed tasks, while it holds for kinematic control problems. Kelly [5] considered the set-point problems with a static target for the dynamics visual feedback system which includes the manipulator dynamics. In [6], Bishop $et\ al.$ proposed an inverse dynamics based control law for the position tracking and the camera calibration problems of the dynamics visual feedback system. Recently, Zergeroglu $et\ al.$ developed an adaptive control law for the position tracking and the camera calibration problems of the dynamics visual feedback system with parametric uncertainties in [7]. Although these control laws guarantee the stability of the system based on the Lyapunov method and are effective for the dynamics visual feedback system, robot manipulators are unfortunately limited to the planar type.

On the other hand, Kelly $et\ al.$ [8] considered a simple image-based controller for dynamic visual feedback system in the three dimensional(3D) workspace under the assumption that the objects' depths are known. Cowan $et\ al.$ [9] addressed the problems of the target tracking and the field of view for the 3D dynamic visual feedback system by using the navigation functions. More recently, the authors proposed the passivity-based dynamic visual feedback control for the 3D target tracking problem with the Eye-in-Hand configuration in [13]. However, this configuration has only three coordinate frames, while visual feedback systems typically use four coordinate frames which consist of a world frame Σ_w, a target object frame Σ_o, a camera frame Σ_c and a hand (end-effector) frame Σ_h as in Fig. 1. Because the camera is attached to the end-effector of robots, the camera frame represents the hand one in Eye-in-Hand configuration.

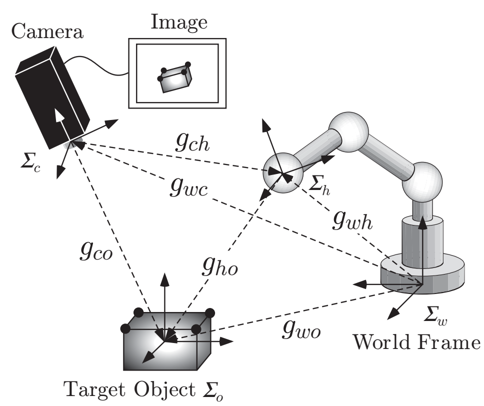

Fig. 1. Visual feedback system with a fixed camera

In this paper, we deal with the control and the estimation problems for dynamic visual feedback systems with the four coordinate frames. Extending the number of the coordinate frames from three to four, we propose the dynamic visual feedback systems with a fixed camera in the 3D workspace, based on our previous works [10], [11], [12], [13]. In this framework, we can design the control gain and the observer gain separately from each other, while the control problem and the estimation problem of the visual feedback system are considered in the same strategy. Moreover, we can derive that the dynamic visual feedback system preserves the passivity of the visual feedback system which is obtained

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in our previous works. Stability and L_2-gain performance analysis for the dynamic visual feedback system will be discussed based on passivity with an energy function.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times 3}$ to represent the change of the principle axes of a frame Σ_b relative to a frame Σ_a. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathcal{R}^3$. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. Here $\hat{\xi}\theta_{ab}$ denotes $\hat{\xi}_{ab}\theta_{ab}$ for the simplicity of notation. We use the 4×4 matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \tag{1}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$ which is the description of the configuration of a frame Σ_b relative to a frame Σ_a. The adjoint transformation associated with g_{ab} is denoted by $\mathrm{Ad}_{(g_{ab})}$ [14]. Let us define the vector form of the rotation matrix as $e_R(e^{\hat{\xi}\theta_{ab}}) := \mathrm{sk}(e^{\hat{\xi}\theta_{ab}})^\vee$ where $\mathrm{sk}(e^{\hat{\xi}\theta_{ab}})$ denotes $\frac{1}{2}(e^{\hat{\xi}\theta_{ab}} - e^{-\hat{\xi}\theta_{ab}})$.

II. Passivity-based Visual Feedback System

A. Fundamental Representation for Visual Feedback System

Visual feedback systems typically use four coordinate frames which consist of a world frame Σ_w, a target object frame Σ_o, a camera frame Σ_c and a hand (end-effector) frame Σ_h as in Fig. 1. Then, $g_{wh} = (p_{wh}, e^{\hat{\xi}\theta_{wh}}) \in SE(3)$, $g_{wc} = (p_{wc}, e^{\hat{\xi}\theta_{wc}}) \in SE(3)$ and $g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}}) \in SE(3)$ denote the rigid body motion from Σ_w to Σ_h, from Σ_w to Σ_c and from Σ_w to Σ_o, respectively. Similarly, the relative rigid body motion from Σ_c to Σ_h, from Σ_c to Σ_o and from Σ_h to Σ_o can be represented by $g_{ch} = (p_{ch}, e^{\hat{\xi}\theta_{ch}}) \in SE(3)$, $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}}) \in SE(3)$ and $g_{ho} = (p_{ho}, e^{\hat{\xi}\theta_{ho}}) \in SE(3)$, respectively. as shown in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion $g_{ho} = (p_{ho}, e^{\hat{\xi}\theta_{ho}})$ to a given reference $g_d = (p_d, e^{\hat{\xi}\theta_d})$. Our goal is to determine the hand's motion using the visual information for this purpose.

Hence we will extend the passivity-based approach which is proposed for the visual feedback system with Eye-in-Hand configuration in [13] to the visual feedback system with the four coordinates. In other words, we consider the relative rigid body motion from Σ_c to Σ_o, i.e. $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$, and the control problem of the relative rigid body motion from Σ_h to Σ_o, i.e. $g_{ho} = (p_{ho}, e^{\hat{\xi}\theta_{ho}})$.

However, the relative rigid body motion $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ can not be immediately obtained in the visual feedback system, because the target object velocity is unknown and furthermore can not be measured directly. Hence, we consider the estimation problem of the relative rigid body motion g_{co}.

The relative rigid body motion from Σ_c to Σ_o can be led by using the composition rule for rigid body transformations ([14], Chap. 2, pp. 37, eq. (2.24)) as follows

$$g_{co} = g_{wc}^{-1} g_{wo}. \tag{2}$$

The relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [14]. Now, we define the body velocity of the camera relative to the world frame Σ_w as

$$\hat{V}_{wc}^b = g_{wc}^{-1}\dot{g}_{wc} = \begin{bmatrix} \hat{\omega}_{wc} & v_{wc} \\ 0 & 0 \end{bmatrix} \quad V_{wc}^b = \begin{bmatrix} v_{wc} \\ \omega_{wc} \end{bmatrix} \tag{3}$$

where v_{wc} and ω_{wc} represent the velocity of the origin and the angular velocity from Σ_w to Σ_c, respectively ([14] Chap. 2, eq. (2.55)). Similarly, the body velocity of the target object relative to Σ_w will be denoted as

$$\hat{V}_{wo}^b = g_{wo}^{-1}\dot{g}_{wo} = \begin{bmatrix} \hat{\omega}_{wo} & v_{wo} \\ 0 & 0 \end{bmatrix} \quad V_{wo}^b = \begin{bmatrix} v_{wo} \\ \omega_{wo} \end{bmatrix} \tag{4}$$

where v_{wo} and ω_{wo} are the velocity of the origin and the angular velocity from Σ_w to Σ_o, respectively.

Then, the fundamental representation V_{co}^b for the three coordinate frames Σ_w, Σ_c and Σ_o is described as follows [13].

$$V_{co}^b = -\mathrm{Ad}_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b \tag{5}$$

where $V_{co}^b := [v_{co}^T \ \omega_{co}^T]^T$ and $\hat{V}_{co}^b := g_{co}^{-1}\dot{g}_{co}$. The notation $\mathrm{Ad}_{(g_{ab})}$ means the adjoint transformation associated with g_{ab} [14]. In the case of the fixed camera configuration, i.e. $V_{wc}^b = 0$, the fundamental representation V_{co}^b can be rewritten as

$$V_{co}^b = V_{wo}^b. \tag{6}$$

Roughly speaking, if both the camera and the target object move, then the relative rigid body motion $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ will be derived from the difference between the camera velocity V_{wc}^b and the target object velocity V_{wo}^b. Hence, the fundamental representation V_{co}^b equals the target object velocity V_{wo}^b, in the case of the fixed camera configuration.

B. Nonlinear Observer and Estimation Error System

To estimate the relative rigid body motion g_{co} using visual information provided by a computer vision system.

The visual information f is defined in [13] which includes the relative rigid body motion can be exploited, while the relative rigid body motion g_{co} can not be obtained directly in the visual feedback system. Hence, we consider a nonlinear observer in order to estimate the relative rigid body motion from the image information.

We shall consider the following model which just comes from the actual fundamental representation (6).

$$\bar{V}_{co}^b = u_e \tag{7}$$

where $\bar{V}_{co}^b := [\bar{v}_{co}^T \ \bar{\omega}_{co}^T]^T$ and $\hat{\bar{V}}_{co}^b := \bar{g}_{co}^{-1}\dot{\bar{g}}_{co}$ mean the estimated body velocity. Here, $\bar{g}_{co} = (\bar{p}_{co}, e^{\hat{\bar{\xi}}\bar{\theta}_{co}})$ denotes the estimated relative rigid body motion. The new input u_e is to be determined in order to converge the estimated value to the actual relative rigid body motion. Because the design of u_e needs a property of the whole visual feedback system, we will propose u_e in Section III-B

In order to establish the estimation error system, we define the estimation error between the estimated value $\bar{g}_{co}$ and the actual relative rigid body motion g_{co} as

$$g_{ee} = \bar{g}_{co}^{-1} g_{co}, \tag{8}$$

in other words, $p_{ee} = e^{-\hat{\bar{\xi}}\bar{\theta}_{co}}(p_{co} - \bar{p}_{co})$ and $e^{\hat{\xi}\theta_{ee}} = e^{-\hat{\bar{\xi}}\bar{\theta}_{co}} e^{\hat{\xi}\theta_{co}}$. Note that $p_{co} = \bar{p}_{co}$ and $e^{\hat{\xi}\theta_{co}} = e^{-\hat{\bar{\xi}}\bar{\theta}_{co}}$ iff $g_{ee} = I_4$, i.e. $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I_3$.

Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the estimation error is given by $e_e := [p_{ee}^T \ e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$. Note that $e_e = 0$ iff $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I_3$. Therefore, if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\bar{g}_{co}$ equals the actual relative rigid body motion g_{co}.

We can derive the vector of the estimation error e_e from image information f and the estimated value of the relative rigid body motion $(\bar{p}_{co}, e^{\hat{\bar{\xi}}\bar{\theta}_{co}})$,

$$e_e = J^{\dagger}(\bar{g}_{co})(f - \bar{f}) \tag{9}$$

where $J(\bar{g}_{co})$ and $\bar{f}$ are the image Jacobian and the estimated image information, and $\dagger$ denotes the pseudo-inverse [13]. Therefore the estimation error e_e can be exploited in the 3D visual feedback control law using image information f obtained from the camera. Hence, the estimation input u_e which can be determined from e_e in (9) with an estimation gain in Section III-B.

The estimation error system will be derived in the same way as the fundamental representation for the visual feedback system. The estimation error system is described as follows [13].

$$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \tag{10}$$

This system should be noted that if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\bar{g}_{co}$ equals the actual one g_{co}.

C. Control Error System

Let us derive the control error system. Similarly to (2), the relative rigid body motion from Σ_h to Σ_o is described as

$$g_{ho} = g_{ch}^{-1} g_{co}. \tag{11}$$

Because g_{co} can not be obtained directly, we represent the relative rigid body motion from Σ_h to Σ_o with the estimated one $\bar{g}_{co}$ as

$$\bar{g}_{ho} = g_{ch}^{-1} \bar{g}_{co}. \tag{12}$$

Here $g_{ch} = g_{wc}^{-1} g_{wh}$ can be obtained directly, because the rigid body motion $g_{wc} = (p_{wc}, e^{\hat{\xi}\theta_{wc}})$ from Σ_w to Σ_c and $g_{wh} = (p_{wh}, e^{\hat{\xi}\theta_{wh}})$ from Σ_w to Σ_h is known by the structure of the system and the angle of the manipulator. It is supposed that the relative rigid body motion from Σ_c to Σ_h, i.e. g_{ch}, can be measured exactly. Since the problem of the camera calibration is one of important research topics and good solutions to it are reported in some papers (see, e.g., [6], [7]), we will not consider the error of the camera calibration in this paper.

Then, the relative rigid body motion $\bar{g}_{ho}$ will be obtained in the same way as (5).

$$\begin{aligned} \bar{V}_{ho}^b &= -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{ch}^b + \bar{V}_{co}^b \\ &= -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{wh}^b + u_e \end{aligned} \tag{13}$$

where we exploit (7) and $V_{ch}^b = V_{wh}^b$ which is derived from $g_{ch} = g_{wc}^{-1} g_{wh}$. Here we define the control error between the estimated value $\bar{g}_{ho}$ and the reference of the relative rigid body motion g_d as

$$g_{ec} = g_d^{-1} \bar{g}_{ho}. \tag{14}$$

It should be remarked that the estimated relative rigid body motion equals the reference one if and only if the control error is equal to the identity matrix in matrix form, i.e. $p_d = \bar{p}_{ho}$ and $e^{\hat{\xi}\theta_d} = e^{\hat{\bar{\xi}}\bar{\theta}_{ho}}$ iff $g_{ec} = I_4$. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the control error is defined as $e_c := [p_{ec}^T \ e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$. Note that $e_c = 0$ iff $p_{ec} = 0$ and $e^{\hat{\xi}\theta_{ec}} = I_3$.

Similarly to (10), the control error can be obtained as

$$\begin{aligned} V_{ec}^b &= -\mathrm{Ad}_{(g_{ec}^{-1})} V_d^b + \bar{V}_{ho}^b \\ &= -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{wh}^b + u_e - \mathrm{Ad}_{(g_{ec}^{-1})} V_d^b \end{aligned} \tag{15}$$

where $V_d^b := [v_d^T \ \omega_d^T]^T$ and $\hat{V}_d^b := g_d^{-1} \dot{g}_d$.

D. Property of Visual Feedback System

Combining (10) and (15), we construct the visual feedback system as follows

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} \begin{bmatrix} V_{wh}^b \\ u_e \end{bmatrix} + \begin{bmatrix} -\mathrm{Ad}_{(g_{ec}^{-1})} \\ 0 \end{bmatrix} V_d^b + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b. \tag{16}$$

Using the relation of the adjoint transformation, i.e. $\mathrm{Ad}_{(g_{ec}^{-1})} = \mathrm{Ad}_{(\bar{g}_{ho}^{-1})} \mathrm{Ad}_{(g_d)}$, the above equation (16) can be rewritten as

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b \tag{17}$$

where

$$u_{ce} := \begin{bmatrix} V_{wh}^b + \mathrm{Ad}_{(g_d)} V_d^b \\ u_e \end{bmatrix} \tag{18}$$

denotes the control input for the visual feedback system. Let us define the error vector of the visual feedback system as $e := \begin{bmatrix} e_c^T & e_e^T \end{bmatrix}^T$ which contains of the control error vector e_c and the estimation error vector e_e. It should be noted that if the vectors of the control error and the estimation error are equal to zero, then the estimated relative rigid body motion $\bar{g}_{ho}$ equals the reference one g_d and the estimated one $\bar{g}_{co}$ equals the actual one g_{co}, respectively. Moreover, the error and the error vector between $\bar{g}_{ho}$ and g_{ho} can be also represented as g_{ee} and e_e by (8), (11) and (12), while g_{ee} and e_e are defined as the error and the error vector between $\bar{g}_{co}$ and g_{co} in (8). Therefore, the actual relative rigid body motion g_{ho} tends to the reference one g_d when $e \to 0$.

Now, we show an important lemma concerning a relation between the input and the output of the visual feedback system.

Lemma 1: If $V_{wo}^b = 0$, then the visual feedback system (17) satisfies

$$\int_0^T u_{ce}^T \nu_{ce} d\tau \geq -\beta_{ce}, \quad \forall T > 0 \tag{19}$$

where ν_{ce} is defined as

$$\nu_{ce} := \begin{bmatrix} -\mathrm{Ad}^T_{(g_d^{-1})} & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{bmatrix} e \tag{20}$$

and β_{ce} is a positive scalar.

This proof is not difficult using the same approach as in [13], and is omitted here due to space limitations.

Let us take u_{ce} as the input and ν_{ce} as its output. Then, Lemma 1 would suggest that the visual feedback system (17) is *passive* from the input u_{ce} to the output ν_{ce} just formally as in the definition in [15].

III. Dynamic Visual Feedback Control

A. Dynamic Visual Feedback System

The manipulator dynamics can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_d \tag{21}$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angles, velocities and accelerations, respectively. τ is the vector of the input torques and τ_d represents a disturbance input.

The body velocity of the hand V_{wh}^b is given by

$$V_{wh}^b = J_b(q)\dot{q} \tag{22}$$

where $J_b(q)$ is the manipulator body Jacobian [14]. We define the reference of the joint velocities as $\dot{q}_d := J_b^{\dagger}(q)u_d$ where u_d represents the desired body velocity of the hand.

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as $\xi := \dot{q} - \dot{q}_d$. Here, we define the weight matrices $W_c := \mathrm{diag}\{w_{pc}I_3,\, w_{rc}I_3\} \in \mathcal{R}^{6\times 6}$ and $W_e := \mathrm{diag}\{w_{pe}I_3,\, w_{re}I_3\} \in \mathcal{R}^{6\times 6}$ where w_{pc}, w_{rc}, w_{pe}, $w_{re} \in \mathcal{R}$ are positive. Now, we consider the passivity–based dynamic visual feedback control law as follows.

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + g(q) + J_b^T(q)\mathrm{Ad}^T_{(g_d^{-1})} W_c e_c + u_\xi. \tag{23}$$

The new input u_ξ is to be determined in order to achieve the control objectives.

Using (17), (21) and (23), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows

$$\begin{bmatrix} \dot{\xi} \\ V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -M^{-1}C\xi + M^{-1}J_b^T\mathrm{Ad}^T_{(g_d^{-1})}W_c e_c \\ -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})}J_b\xi \\ 0 \end{bmatrix} + \begin{bmatrix} M^{-1} & 0 & 0 \\ 0 & -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u + \begin{bmatrix} M^{-1} & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tau_d \\ V_{wo}^b \end{bmatrix}$$

$$z = \begin{bmatrix} \varepsilon x \\ \rho u \end{bmatrix} \tag{24}$$

where

$$x := \begin{bmatrix} \xi \\ e_c \\ e_e \end{bmatrix}, \quad u := \begin{bmatrix} u_\xi \\ u_d + \mathrm{Ad}_{(g_d)}V_d^b \\ u_e \end{bmatrix}.$$

ε and ρ are weight matrices for the state and the input, respectively. We define the disturbance of dynamic visual feedback system as $w := \left[\tau_d^T \ (V_{wo}^b)^T\right]^T$. Before constructing the dynamic visual feedback control law, we derive an important lemma.

Lemma 2: If $w = 0$, then the dynamic visual feedback system (24) satisfies

$$\int_0^T u^T \nu d\tau \geq -\beta, \quad \forall T > 0 \tag{25}$$

where

$$\nu := Nx, \ N := \begin{bmatrix} I & 0 & 0 \\ 0 & -\mathrm{Ad}^T_{(g_d^{-1})}W_c & 0 \\ 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})}W_c & -W_e \end{bmatrix}.$$

Proof: Consider the following positive definite function

$$V(x) = \frac{1}{2}\xi^T M \xi + \frac{1}{2}w_{pc}\|p_{ec}\|^2 + w_{rc}\phi(e^{\hat{\xi}\theta_{ec}}) + \frac{1}{2}w_{pe}\|p_{ee}\|^2 + w_{re}\phi(e^{\hat{\xi}\theta_{ee}}). \tag{26}$$

where $\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta})$ is the error function of the rotation matrix and has the following properties (see e.g. [16]). Differentiating (26) with respect to time yields

$$\dot{V} = \frac{1}{2}\xi^T \dot{M}\xi + x^T \begin{bmatrix} M(q) & 0 & 0 \\ 0 & W_c\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & 0 \\ 0 & 0 & W_e\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} \end{bmatrix} \begin{bmatrix} \dot{\xi} \\ V_{ec}^b \\ V_{ee}^b \end{bmatrix}. \tag{27}$$

Observing that the skew-symmetry of the matrices $\hat{p}_{ec}$ and $\hat{p}_{ee}$, i.e., $p_{ec}^T\hat{p}_{ec}e^{-\hat{\xi}\theta_d}\omega_{wh} = -p_{ec}^T(e^{-\hat{\xi}\theta_d}\omega_{wh})^\wedge p_{ec} = 0$, $p_{ee}^T\hat{p}_{ee}\omega_{we} = -p_{ee}^T\hat{\omega}_{we}p_{ee} = 0$, the above equation along the trajectories of the system (24) can be transformed into

$$\dot{V} = x^T \begin{bmatrix} I & 0 & 0 \\ 0 & -W_c\mathrm{Ad}_{(g_d^{-1})} & W_c\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} \\ 0 & 0 & -W_e \end{bmatrix} u. \tag{28}$$

Integrating (28) from 0 to T, we can obtain

$$\int_0^T u^T \nu d\tau = V(x(T)) - V(x(0)) \geq -V(x(0)) := -\beta \quad (29)$$

where β is the positive scalar which only depends on the initial states of ξ, g_{ec} and g_{ee}. ∎

Remark 1: Similarly to Lemma 1, Lemma 2 would suggest that the dynamic visual feedback system is *passive* from the input u to the output ν just formally. From Lemma 2, we can state that the dynamic visual feedback system (24) preserves the passivity of the visual feedback system (17). This is one of main contributions of this work.

B. Stability Analysis for Dynamic Visual Feedback System

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

$$u = -K\nu = -KNx, \ K := \begin{bmatrix} K_\xi & 0 & 0 \\ 0 & K_c & 0 \\ 0 & 0 & K_e \end{bmatrix} \quad (30)$$

where $K_\xi := \mathrm{diag}\{k_{\xi 1}, \cdots, k_{\xi n}\}$ denotes the positive gain matrix for each joint axis. $K_c := \mathrm{diag}\{k_{c1}, \cdots, k_{c6}\}$ and $K_e := \mathrm{diag}\{k_{e1}, \cdots, k_{e6}\}$ are the positive gain matrices of x, y and z axes of the translation and the rotation for the control error and the estimation error, respectively. The result with respect to asymptotic stability of the proposed control input (30) can be established as follows.

Theorem 1: If $w = 0$, then the equilibrium point $x = 0$ for the closed-loop system (17) and (30) is asymptotic stable.

Proof: In the proof of Lemma 2, we have already derived that the time derivative of V along the trajectory of the system (24) is formulated as (28). Using the control input (30), (28) can be transformed into

$$\dot{V} = -x^T N^T K N x \quad (31)$$

This completes the proof. ∎
Considering the manipulator dynamics, Theorem 1 shows the stability via Lyapunov method for the full 3D dynamic visual feedback system. It is interesting to note that stability analysis is based on the passivity as described in (25).

C. L_2-gain Performance Analysis for Dynamic Visual Feedback System

Based on the dissipative systems theory, we consider L_2-gain performance analysis for the dynamic visual feedback system (24) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

$$P := N^T K N - \frac{1}{2\gamma^2} W - \frac{1}{2}\|\varepsilon\|^2 - \frac{1}{2}\|\rho K N\|^2$$

where $\gamma \in \mathcal{R}$ is positive and $W := \mathrm{diag}\{I, 0, W_e^2\}$. Then we have the following theorem.

Theorem 2: Given a positive scalar γ and consider the control input (30) with the weight matrices ε, ρ, W_c and W_e and the gains K_ξ, K_c and K_e such that the matrix P is positive semi-definite, then the closed-loop system (24) and (30) has L_2-gain $\leq \gamma$.

Proof: By differentiating the positive definite function V defined in (26) along the trajectory of the closed-loop system and completing the squares, it holds that

$$\dot{V} + \frac{1}{2}\|z\|^2 - \frac{\gamma^2}{2}\|w\|^2 \leq -x^T P x \leq 0 \quad (32)$$

if P is positive semi-definite. Integrating (32) from 0 to T and noticing $V(T) \geq 0$, we have

$$\int_0^T \|z\|^2 dt \leq \gamma^2 \int_0^T \|w\|^2 dt + 2V(0), \ \forall T > 0. \quad (33)$$

This completes the proof. ∎
Theorem 2 can be proved using the energy function (26) as a storage function for L_2-gain performance analysis. γ represents a disturbance attenuation level for the dynamic visual feedback system. Theorem 1 and 2 can be proved using the energy function (26) as a Lyapunov function and a storage function, respectively. Therefore, the passivity of the dynamic visual feedback system is particularly important in our framework.

IV. Simulation

The simulation results on the two degree-of-freedom manipulator as depicted in Fig. 2 are shown in order to understand our proposed method simply, though it is valid for 3D visual feedback systems. The target object has four feature points and moves for $t = 4.8$ [s] moves along a straight line ($0 \leq t < 2$) and a "Figure 8" motion ($2 \leq t < 4.8$) as depicted in Fig. 3 and Fig. 4, respectively. Specifically, we use the reference of the relative rigid body motion as a constant value, i.e. $p_d = [0\ 0\ -0.81]^T$, $e^{\hat{\xi}\theta_d} = I$ and $V_d^b = 0$, for the tracking problems in the simulation.

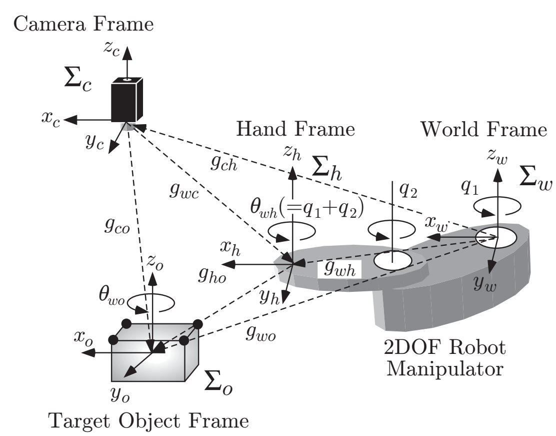

Fig. 2. Coordinate frames for dynamic visual feedback system with two degree of freedom manipulator

Firstly, we design the weight matrices concerning controlled output as $\varepsilon = \mathrm{diag}\{0.4,\ 0.4,\ 1,\ 1,\ 0.25,\ 0.25,\ 0.25,$

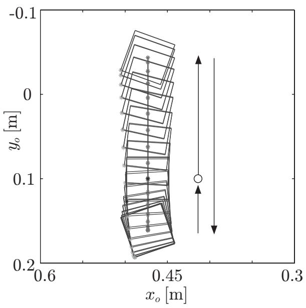

Fig. 3. Trajectory of target object along the straight line in $0 \leq t < 2$

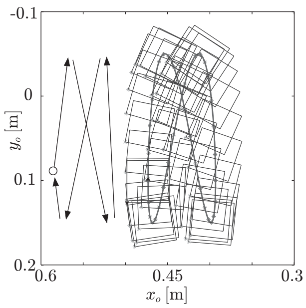

Fig. 4. Trajectory of target object along the "Figure 8" in $2 \leq t < 4.8$

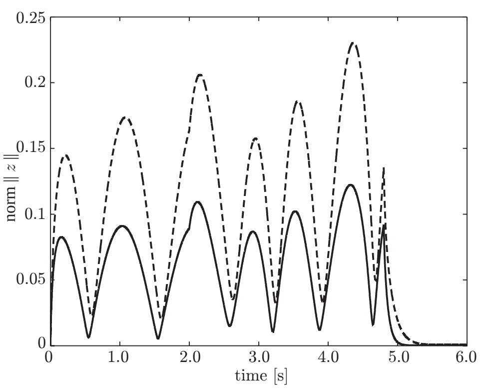

Fig. 5. Euclid norms of z(Gain A: dashed, Gain B: solid)

1, 2.5, 2.5, 0.025, 0.025, 0.025, 2.5}, $\rho = \mathrm{diag}\{200,\ 200,\ 2,\ 2,\ 1,\ 1,\ 1,\ 2,\ 0.1,\ 0.1,\ 0.001,\ 0.001,\ 0.001,\ 0.1\} \times 10^{-3}$. Also, we select the weight matrices for control design as $W_c = 0.1I$ and $W_e = I$. Control gain of manipulator is chosen as $K_\xi = \mathrm{diag}\{20, 20\}$ and gains K_c and K_e are chosen as follows

$$\text{Gain A} : K_c = \mathrm{diag}\{100, 100, 50, 50, 50, 100\},\ K_e = 25I$$
$$\text{Gain B} : K_c = \mathrm{diag}\{2, 2, 1, 1, 1, 2\} \times 10^2,\ K_e = 50I.$$

Then, the closed-loop system (24) and (30) with gain A has $\gamma = 0.229$ and with gain B has $\gamma = 0.159$.

In Fig. 5, dashed line and solid line show the norm of z in the case of $\gamma = 0.229$ and $\gamma = 0.159$, respectively. In the case of static target object, i.e. after $t = 4.8$ [s], all errors in Fig. 5 tend to zero. It can be concluded that the equilibrium point is asymptotically stable if the target object is static. In the case of $\gamma = 0.159$, the performance is improved as compared to the case of $\gamma = 0.229$. After all, the simulation results show that L_2-gain is adequate for the performance measure of the dynamic visual feedback control.

V. Conclusions

This paper dealt with the control and the estimation of dynamic visual feedback systems with a fixed camera. The main contribution of this work is that the dynamic visual feedback system with four coordinate frames is constructed in order to extend our previous works. In this framework, we can design the control gain and the observer gain separately from each other, while the control problem and the estimation problem of the visual feedback system are considered in the same strategy. Stability and L_2-gain performance analysis for the dynamic visual feedback system have been discussed based on passivity with the energy function. The experimental results is omitted due to space limitations, the reader is referred to [17] for more details.

In our future work, we will establish the more general framework by combining Eye-in-Hand configuration with the four coordinate frames. Additionally, we consider that the reference velocity V_d^b will play a role in the trajectory planning of the dynamic visual feedback systems.

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