# Passivity-Based Iterative Learning Control for 2DOF Robot Manipulators with Antagonistic Bi-Articular Muscles

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*Abstract*— This paper investigates iterative learning control based on passivity for two-degree-of-freedom (2DOF) robot manipulators with antagonistic bi-articular muscles. Firstly, a brief summary of dynamics of 2DOF robot manipulators with antagonistic bi-articular muscles is given. Next, an error dynamics of the bi-articular manipulator for iterative learning control that has an output strictly passivity property is constructed. Then, we propose an iterative learning control law for the biarticular manipulator. The proposed torque input does not need the parameters for the accurate models. Convergence analysis of the closed-loop system is carried out based on passivity. Finally, simulation results are presented in order to confirm the effectiveness of the proposed control law.

#### I. INTRODUCTION

Medical robots, rehabilitation and health care robots and domestic robots are expected to become a major market in near future [1]. Compared to classical industrial robots, these robots in the relatively new fields do not need to have high torque and high speed, but need strong safety and reliability so as never to hurt human. In other words, it is indicated that classical robots are not the best choice for the next generation of robots with physical interaction. In addition, for robot application to rehabilitation therapy, it is known that the slight difference between a human body and a rehabilitation robot bears disadvantages to patients [2]. In these background, robot control based on the mechanism of human body and/or analysis of human motion has lately received considerable attention. M. Kuschel et al. [3] have presented a mathematical model for visual-haptic perception of compliant objects via psychophysical experiments. Wang et al. [4] have developed a neural network-based inverse optimal neuromuscular electrical stimulation controller to enable the lower limb to track a desired trajectory.

Antagonistic bi-articular muscles, which are passing over adjacent two joints and acting the both joints simultaneously, are also known as one of the most important mechanisms of human motion. The two-degree-of-freedom (2DOF) robot manipulator with antagonistic bi-articular muscles, which is inspired by this mechanism, has three low-power actuators whereas the conventional one needs two high-power actuators. This leads to safety with respect to humans who interact closely with robots. Kumamoto *et al.* discuss the effects

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of the existence of antagonistic bi-articular muscles through robot arm experiments [5], [6]. Oh and Hori *et al.* have proposed some control methods for 2DOF bi-articular manipulators in a series of papers [7]–[10]. Although efficient control solutions based on the physical characteristics of antagonistic bi-articular muscles have been reported, stability analysis has not been discussed in these works. The authors have proposed passivity-based and open-loop control laws for 2DOF bi-articular manipulators [11], [12]. Even though the closed-loop stability for these control laws is guaranteed, the input torque needs the parameters for the accurate models.

On the other hand, iterative learning control has also been an attractive control method for improving the transient response and tracking performance of uncertain dynamic systems that operate repetitively [13]. Recent examples include not only classical robots, but also robots in the relatively new fields such as surgical assistant robots [14] and upper-limb stroke rehabilitation robots [15]. Needless to say, in the new robot fields, there exist various situations in which robots execute the same task multiple times. However, iterative learning control has not been applied to robot manipulators with antagonistic bi-articular muscles before.

In this paper, we propose an iterative learning control law based on passivity for 2DOF robot manipulators with antagonistic bi-articular muscles. Firstly, a brief summary of dynamics of 2DOF robot manipulators with antagonistic biarticular muscles is given. Secondly, we design a torque input so that an error dynamics of the bi-articular manipulator for iterative learning control satisfies an output strictly passivity property. Next, we propose an iterative learning control law based on Arimoto-type iterative learning control [16] that is much efficient for motion control of classical robot systems. Convergence analysis of the closed-loop system is discussed based on passivity. Compared with our previous works [11], [12], the proposed control law is robust in the sense that the input torque does not need the parameters for the accurate models. Also, the proposed scheme can improve control performance by incorporating prior error information into the control for subsequent iterations. Finally, control performance of the proposed scheme is evaluated through simulation results.

#### II. BI-ARTICULAR MANIPULATOR DYNAMICS

In this section, we review the dynamics of 2DOF robot manipulators with antagonistic bi-articular muscles [11], [12]. The dynamics of n-link rigid robot manipulators can



Fig. 1. (a) 2DOF robot arm. (b) Human arm model. Two couples of the antagonistic mono-articular muscles of  $f_1$  and  $e_1$ , and of  $f_2$  and  $e_2$  are attached to the joints of  $J_1$  and  $J_2$ , respectively. A couple of the antagonistic bi-articular muscles  $f_3$  and  $e_3$  are attached to both joints of  $J_1$  and  $J_2$ .

be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = T, \tag{1}$$

where  $q, \dot{q}$  and  $\ddot{q}$  are the joint angle, velocity and acceleration, respectively.  $M(q) \in \mathcal{R}^{n \times n}$  is the manipulator inertia matrix,  $C(q, \dot{q}) \in \mathcal{R}^{n \times n}$  is the Coriolis matrix,  $g(q) \in \mathcal{R}^n$  is the gravity vector and  $T \in \mathcal{R}^n$  is the input torque [17]. In the case of 2DOF robot manipulators as depicted in Fig. 1 (a), the manipulator dynamics can be given as

$$\begin{bmatrix} M_{1} + 2M_{2} + 2RC_{2} & 2M_{2} + RC_{2} \\ 2M_{2} + RC_{2} & 2M_{2} \end{bmatrix} \begin{bmatrix} \ddot{q}_{1} \\ \ddot{q}_{2} \end{bmatrix} \\ + \begin{bmatrix} -RS_{2}\dot{q}_{2} & -RS_{2}(\dot{q}_{1} + \dot{q}_{2}) \\ RS_{2}\dot{q}_{1} & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_{1} \\ \dot{q}_{2} \end{bmatrix} \\ + \begin{bmatrix} g(m_{1}l_{g1} + m_{2}l_{2})C_{1} + g(m_{2}l_{g2})C_{12} \\ g(m_{2}l_{g2})C_{12} \end{bmatrix} = \begin{bmatrix} T_{1} \\ T_{2} \end{bmatrix},$$
(2)

where  $M_1 = m_1 l_{g1}^2 + m_2 l_1^2 + \tilde{I}_1$ ,  $M_2 = \frac{1}{2}(m_2 l_{g2}^2 + \tilde{I}_2)$  and  $R = m_2 l_1 l_{g2}$ .  $m_i$ ,  $l_i$ ,  $l_{gi}$  and  $\tilde{I}_i$  are the weight of the link i, the length of the link i, the distance from the center of a joint i to the center of the gravity point of the link i and the moment of inertia about an axis through the center of mass of the link i.  $S_i$ ,  $C_i$ ,  $S_{12}$  and  $C_{12}$  denote  $\sin q_i$ ,  $\cos q_i$ ,  $\sin(q_1 + q_2)$  and  $\cos(q_1 + q_2)$ , respectively.

Next, we consider a human arm model, which can be simplified as three pairs of antagonistic muscles, as shown in Fig. 1 (b). In standard robot motion control, the joint torque T will be designed as a control input directly. On the other hand, since a couple of bi-articular muscles are attached to both joints as depicted in Fig. 2, the joint torques  $T_1$  and  $T_2$  are described as

$$T_{i} = (F_{fi} - F_{ei})r_{p} + (F_{f3} - F_{e3})r_{p}$$
  
=  $(u_{fi} - u_{ei})r_{p} - (u_{fi} + u_{ei})k_{i}r_{p}^{2}q_{i} - (u_{fi} + u_{ei})b_{i}r_{p}^{2}\dot{q}_{i}$   
+  $(u_{f3} - u_{e3})r_{p} - (u_{f3} + u_{e3})k_{3}r_{p}^{2}(q_{1} + q_{2})$   
-  $(u_{f3} + u_{e3})b_{3}r_{p}^{2}(\dot{q}_{1} + \dot{q}_{2}), \quad i = 1, 2.$  (3)

In Fig. 2,  $F_{fj}$  and  $F_{ej}$ , j = 1, 2, 3 are output forces by flexor muscles and by extensor muscles, respectively.  $u_{fj}$ 



Fig. 2. Visco-elastic muscle model [5].  $F_j$ : output force,  $u_j$ : contractile force,  $k_j$ : coefficient w.r.t. elastic,  $b_j$ : coefficient w.r.t. viscosity,  $x_j$ : contracting length,  $r_p$ : radius of the joint.

and  $u_{ej}$  represent contractile forces of flexor muscles and of extensor muscles, respectively.  $r_p$  is the radius of the joint.  $k_j$  and  $b_j$  are coefficients w.r.t. elastic and viscosity, respectively [5]. The contractile forces of flexor muscles and of extensor muscles have following relationship [5]:

$$u_{fj} + u_{ej} = 1, \quad j = 1, 2, 3.$$
 (4)

Because each contractile force of flexor muscle  $u_{fj}$  can be decided by each actuator, muscle torques are defined as  $\tau_j := (2u_{fj} - 1)r_p$ , j = 1, 2, 3. Then the joint torques (3) can be transformed into

$$T_{i} = \tau_{i} + \tau_{3} - k_{i}r_{p}^{2}q_{i} - k_{3}r_{p}^{2}(q_{1} + q_{2}) -b_{i}r_{p}^{2}\dot{q}_{i} - b_{3}r_{p}^{2}(\dot{q}_{1} + \dot{q}_{2}), \quad i = 1, 2.$$
(5)

Here, we define the antagonistic bi-articular muscle torque  $\tau_3$  as follows:

From Eqs. (2), (5) and (6), the manipulator dynamics with antagonistic bi-articular muscles, we call the *bi-articular manipulator dynamics*, can be derived as

$$M_b(\theta)\ddot{\theta} + C_b(\theta,\dot{\theta})\dot{\theta} + g_b(\theta) + K_b\theta + B_b\dot{\theta} = \tau, \quad (7)$$

where

$$\begin{split} M_b(\theta) &= \left[ \begin{array}{ccc} M_1 + M_2 + 2RC_2 & M_2 + RC_2 & 0 \\ M_2 + RC_2 & M_2 & 0 \\ 0 & 0 & M_2 \end{array} \right], \\ C_b(\theta, \dot{\theta}) &= \left[ \begin{array}{ccc} -RS_2 \dot{\theta}_2 & -RS_2 \dot{\theta}_3 & 0 \\ RS_2 \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right], \\ g_b(\theta) &= \left[ \begin{array}{ccc} g(m_1 l_{g1} + m_2 l_1) C_1 \\ 0 \\ g(m_2 l_{g2}) C_3 \end{array} \right], \quad \theta = \left[ \begin{array}{ccc} q_1 \\ q_2 \\ q_1 + q_2 \end{array} \right], \end{split}$$

 $K_b := \text{diag} \{k_1, k_2, k_3\} r_p^2$  and  $B_b := \text{diag} \{b_1, b_2, b_3\} r_p^2$ . The bi-articular manipulator dynamics (7) has following important properties [11], [12]:



Fig. 3. Block diagram of bi-articular manipulator dynamics.

Property 1: Under the conditions  $M_1 + M_2 > 2R$  and  $M_1M_2 > R^2$ , the inertia matrix  $M_b(\theta)$  preserves the positive definiteness.

Property 2:  $\dot{M}_b(\theta) - 2C_b(\theta, \dot{\theta})$  is skew-symmetric.

In this paper, we construct the bi-articular manipulator dynamics (7) which satisfies  $M_1 + M_2 > 2R$  and  $M_1M_2 > R^2$ . Fig. 3 shows a block diagram of the bi-articular manipulator dynamics.

### III. PASSIVITY OF ERROR DYNAMICS FOR ITERATIVE LEARNING CONTROL

Iterative learning control can make it possible to improve the transient response performance of the system that operates repetitively [13]. In this section, we design a torque input so that an error dynamics of the bi-articular manipulator for iterative learning control satisfies an output strictly passivity property.

# A. Error Dynamics for Iterative Learning Control

Let us derive the error dynamics of the bi-articular manipulator for iterative learning control. Suppose the reference joint angle  $\theta_d \in \mathcal{R}^3$  is given, we consider the problem of set-point tracking. We define the joint angle error  $e \in \mathcal{R}^3$ as

$$e = \theta - \theta_d. \tag{8}$$

Here, we assume that the initial joint angle and velocity are equal to the reference ones, i.e.,  $\theta(0) = \theta_d(0)$  and  $\dot{\theta}(0) = \dot{\theta}_d(0)$ .

Next, we consider that the torque input of the bi-articular manipulator  $\tau$  has an ideal one  $\tau_d$  which can achieve to bring the joint angle  $\theta$  to the reference one  $\theta_d$ . Substituting  $\theta$  into  $\theta_d$  in Eq. (7), the ideal torque input  $\tau_d$  can be represented as follows:

$$M_b(\theta_d)\ddot{\theta}_d + C_b(\theta_d, \dot{\theta}_d)\dot{\theta}_d + g_b(\theta_d) + K_b\theta_d + B_b\dot{\theta}_d = \tau_d.$$
(9)

Note that it is difficult to implement the ideal torque input  $\tau_d$  for the robot system in general because of the presence of model uncertainty and frictional influence.

We now propose the torque input as

$$\tau = -K_y y + \tau_l,\tag{10}$$

where

$$y = \dot{e} + k_p \operatorname{Sin}(e), \tag{11}$$

and  $K_y := \text{diag}\{k_{y1}, k_{y2}, k_{y3}\}$  is a positive gain matrix and  $k_p \in \mathcal{R}$  is a positive scalar gain. Sin(e) represents the saturated function (See Appendix A). The new torque input  $\tau_l$  is designed later by an iterative learning control scheme. Substituting Eq. (10) into Eq. (7), we have that

$$M_b(\theta)\ddot{\theta} + C_b(\theta,\dot{\theta})\dot{\theta} + g_b(\theta) + K_b\theta + B_b\dot{\theta} + K_yy = \tau_l.$$
(12)

It should be noted that  $\tau = \tau_l \rightarrow \tau_d$  from Eqs. (10) and (12) when  $\theta \rightarrow \theta_d$ .

Here, we define the ideal torque input error as follows:

$$\tau_e = \tau_l - \tau_d. \tag{13}$$

Using Eqs. (9), (12) and (13), the error dynamics of the bi-articular manipulator for iterative learning control can be derived as

$$M_b(\theta)\ddot{e} + C_b(\theta,\dot{\theta})\dot{e} + K_be + B_b\dot{e} + K_yy + h\left(e,\dot{e}\right) = \tau_e,$$
(14)

where the residual dynamics  $h(e, \dot{e})$  is defined as follows:

$$h(e, \dot{e}) = \{M_b(\theta) - M_b(\theta_d)\} \dot{\theta}_d + \{C_b(\theta, \dot{\theta}) - C_b(\theta_d, \dot{\theta}_d)\} \dot{\theta}_d + g_b(\theta) - g_b(\theta_d).$$
(15)

The error dynamics of the bi-articular manipulator for iterative learning control (14) has some properties as well as standard manipulator dynamics. These properties are addressed in more detail in Appendix B.

## B. Passivity Property of Error Dynamics

Next, we show an important relation between the input  $\tau_e$  and the output y of the error dynamics of the bi-articular manipulator for iterative learning control.

Lemma 1: Given a positive scalar  $k_p$ , assume

$$\lambda_{\min}\{B_b\} > \alpha_{h1} + k_p b,$$
(16)  
$$\lambda_{\min}\{K_b\} > \max\{k_p^2 \lambda_{\max}\{M_b(\theta)\}, \frac{(2k_p a + \alpha_{h2})^2}{4k_p (\lambda_{\min}\{B_b\} - \alpha_{h1} - k_p b)} + \alpha_{h2} \},$$
(17)

where the constant a and b are given by

$$a = \frac{1}{2} \left( -\lambda_{\min} \{ B_b \} + \alpha_{C1} \sup \left\| \dot{\theta}_d \right\| + \alpha_{h1} \right),$$
  
$$b = \lambda_{\max} \{ M_b(\theta) \} + \sqrt{3} \alpha_{C1},$$

where  $\lambda_{\min}\{\cdot\}$  and  $\lambda_{Max}\{\cdot\}$  denote the minimum and maximum eigenvalues of a matrix, respectively. Then, the error dynamics of the bi-articular manipulator for iterative learning control (14) satisfies

$$\int_0^T \tau_e^T y dt \ge -\beta + \int_0^T y^T K_y y dt, \quad \forall T > 0, \quad (18)$$

where  $\beta$  is a positive scalar.

*Proof:* Consider the following function:

$$V(e, \dot{e}) = \frac{1}{2} \dot{e}^T M_b(\theta) \dot{e} + \frac{1}{2} e^T K_b e + k_p \text{Sin}(e)^T M_b(\theta) \dot{e}.$$
(19)

This function (19) can be transformed as

$$V(e, \dot{e}) = \frac{1}{2} (\dot{e} + k_p \operatorname{Sin}(e))^T M_b(\theta) (\dot{e} + k_p \operatorname{Sin}(e)) - \frac{1}{2} k_p^2 \operatorname{Sin}(e)^T M_b(\theta) \operatorname{Sin}(e) + \frac{1}{2} e^T K_b e \geq \frac{1}{2} (\dot{e} + k_p \operatorname{Sin}(e))^T M_b(\theta) (\dot{e} + k_p \operatorname{Sin}(e)) + \frac{1}{2} (\lambda_{\min} \{K_b\} - k_p^2 \lambda_{\max} \{M_b(\theta)\}) ||e||^2.$$
(20)

Therefore, the function (19) satisfies positive definiteness through the condition (17).

Using Property 2, the time derivative of the function (19) along the trajectories of Eq. (14) yields

$$\dot{V}(e, \dot{e}) = -\dot{e}^T B_b \dot{e} + k_p \dot{e}^T \operatorname{Cos}(e)^T M_b(\theta) \dot{e} - k_p \operatorname{Sin}(e)^T K_b e -k_p \operatorname{Sin}(e)^T B_b \dot{e} + k_p \operatorname{Sin}(e)^T C_b(\theta, \dot{\theta})^T \dot{e} - \dot{e}^T h(e, \dot{e}) -k_p \operatorname{Sin}(e)^T h(e, \dot{e}) - (\dot{e} + k_p \operatorname{Sin}(e))^T (K_y y - \tau_e).$$
(21)

From the property of the saturated function and Properties 3 and 4, we have the following upper-bounds of each term of Eq. (21):

$$-\dot{e}^T B_b \dot{e} \le -\lambda_{\min} \left\{ B_b \right\} \left\| \dot{e} \right\|^2, \tag{22}$$

$$k_p \dot{e} \operatorname{Cos}(e)^T M_b(\theta) \dot{e} \le k_p \lambda_{\operatorname{Max}} \left\{ M_b(\theta) \right\} \left\| \dot{e} \right\|^2,$$
(23)

$$-k_p \text{Sin}(e)^T K_b e \le -k_p \lambda_{\min} \{K_b\} \|\text{Sin}(e)\|^2, \qquad (24)$$

$$-k_p \operatorname{Sin}(e)^T B_b \dot{e} \leq -k_p \lambda_{\min} \left\{ B_b \right\} \left\| \dot{e} \right\| \left\| \operatorname{Sin}(e) \right\|, \qquad (25)$$
$$k_p \operatorname{Sin}(e)^T C_b(\theta, \dot{\theta})^T \dot{e}$$

$$\leq k_p \sqrt{3} \alpha_{C1} \|\dot{e}\|^2 + k_p \alpha_{C1} \sup \left\| \dot{\theta}_d \right\| \|\dot{e}\| \|\operatorname{Sin}(e)\|, (26)$$

$$-\dot{e}^{T}h(e,\dot{e}) \leq \alpha_{h1} \|\dot{e}\|^{2} + \alpha_{h2} \|\dot{e}\| \|\operatorname{Sin}(e)\|, \qquad (27)$$
$$-k_{p}\operatorname{Sin}(e)^{T}h(e,\dot{e})$$

$$\leq k_p \alpha_{h1} \|\dot{e}\| \|\operatorname{Sin}(e)\| + k_p \alpha_{h2} \|\operatorname{Sin}(e)\|^2.$$
(28)

By using Eqs. (22)-(28), Eq. (21) satisfies

$$\dot{V}(e,\dot{e}) \leq -k_p \begin{bmatrix} \|\operatorname{Sin}(e)\| \\ \|\dot{e}\| \\ +y^T \tau_e - y^T K_y y, \end{bmatrix}^T G(k_p) \begin{bmatrix} \|\operatorname{Sin}(e)\| \\ \|\dot{e}\| \end{bmatrix}$$
(29)

where

$$G(k_p) = \begin{bmatrix} \lambda_{\min} \{K_b\} - \alpha_{h2} & -a - \frac{1}{k_p} \frac{\alpha_{h2}}{2} \\ -a - \frac{1}{k_p} \frac{\alpha_{h2}}{2} & \frac{1}{k_p} \{\lambda_{\min} \{B_b\} - \alpha_{h1}\} - b \end{bmatrix}.$$

From the theorem of Sylvester, it can be shown that  $G(k_p)$  with the conditions (16) and (17) is a positive definite matrix. Integrating Eq. (29) from 0 to T yields

$$\int_{0}^{T} \tau_{e}^{T} y dt \geq V(T) - V(0) + \int_{0}^{T} y^{T} K_{y} y dt + k_{p} \left[ \begin{array}{c} \|\operatorname{Sin}(e)\| \\ \|\dot{e}\| \end{array} \right]^{T} G(k_{p}) \left[ \begin{array}{c} \|\operatorname{Sin}(e)\| \\ \|\dot{e}\| \end{array} \right] \geq -V(0) + \int_{0}^{T} y^{T} K_{y} y := -\beta + \int_{0}^{T} y^{T} K_{y} y,$$
(30)

where  $\beta$  is the positive scalar which only depends on the initial state of e and  $\dot{e}$ .

Lemma 1 implies that the error dynamics of the biarticular manipulator for iterative learning control (14) is *output strictly passive* from the input  $\tau_e$  to the output y as in the definition in [18]. One of the main contributions of this paper is that we construct the error dynamics (14) to satisfy an output strictly passivity.

# IV. ITERATIVE LEARNING CONTROL FOR BI-ARTICULAR MANIPULATOR

In this section, we present an iterative learning control law for the bi-articular manipulator. Generally, the problem of iterative learning control is to find a recursive form of a learning control law  $\tau_l^{k+1} = F(\tau_l^k, y^k)$  in trial number k that eventually realizes the convergence  $y \to 0$  as  $k \to \infty$  [16]. In this paper, for the bi-articular manipulator, we tackle the same problem. We now propose the following learning control update law for the the bi-articular manipulator:

$$\tau_l^{k+1} = \tau_l^k - K_l y^k, \tag{31}$$

where  $K_l := \text{diag}\{k_{l1}, k_{l2}, k_{l3}\}$  is the positive gain matrix for iterative learning. From Eqs. (13) and (31), we can easily derive the following relationship:

$$\tau_e^{k+1} = \tau_e^k - K_l y^k. \tag{32}$$

Suppose that the conditions (16) and (17) of Lemma 1 are satisfied, the following theorem concerning the convergence of the iterative learning control for the bi-articular manipulator holds.

Theorem 1: Suppose that  $B_b$  and  $K_b$  satisfy Eq. (16) and (17) and  $0 < K_l < 2K_y$ , then the iterative learning control law (31) for the error dynamics of the bi-articular manipulator guarantees the convergence of  $e^k = 0$  in the sense of a  $L_2[0,T]$  norm.

*Proof:* Multiplying both sides by the positive definite symmetric matrix  $K_l^{-1}$  for Eq. (32) gives us

$$K_l^{-1}\tau_e^{k+1} = K_l^{-1}\tau_e^k - y^k.$$
 (33)

Calculating inner products of both sides of Eqs. (32) and (33), we have

$$\begin{aligned} (\tau_e^{k+1})^T K_l^{-1} \tau_e^{k+1} \\ &= (\tau_e^k)^T K_l^{-1} \tau_e^k + (y^k)^T K_l y^k - 2(\tau_e^k)^T y^k. \end{aligned}$$
(34)

Integrating of both sides Eq. (34) from 0 to T yields

$$\int_{0}^{T} (\tau_{e}^{k+1})^{T} K_{l}^{-1} \tau_{e}^{k+1} dt$$

$$= \int_{0}^{T} (\tau_{e}^{k})^{T} K_{l}^{-1} \tau_{e}^{k} dt + \int_{0}^{T} (y^{k})^{T} K_{l} y^{k} dt$$

$$-2 \int_{0}^{T} (\tau_{e}^{k})^{T} y^{k} dt.$$
(35)

Using the definition  $||x||_A^2 := \int_0^T x^T A x dt$ , Eq. (35) can be represented in terms of a  $L_2$  norm as

$$\|\tau_e^{k+1}\|_{K_l^{-1}}^2 = \|\tau_e^k\|_{K_l^{-1}}^2 + \|y^k\|_{K_l}^2 - 2\int_0^1 (\tau_e^k)^T y^k dt.$$
(36)



Fig. 4. Block diagram of iterative learning controller.

In the proof of Lemma 1, the following relationship has been shown:

$$\int_0^T (\tau_e^k)^T y^k dt = V^{k+1} - V^k + \|y^k\|_{K_y}^2, \qquad (37)$$

where we have regarded V(T) and V(0) as  $V^{k+1}$  and  $V^k$ , respectively. From Eqs. (36) and (37), it can be easily shown that

$$\|\tau_e^{k+1}\|_{K_l^{-1}}^2 + 2V^{k+1} = \|\tau_e^k\|_{K_l^{-1}}^2 + 2V^k - \|y^k\|_{(2K_y - K_l)}^2.$$
(38)

From Eq. (38) and  $0 < K_l < 2K_y$ , we find that the function  $\{\|\tau_e^k\|_{K_l^{-1}}^2 + 2V^k\}$  is a monotonically non-increasing function and bounded below. Hence, it is obvious that it converges to a non-negative value when  $k \to \infty$ . Then, the output  $\|y^k\|_{(2K_y - K_l)}^2$  tends to zero as  $k \to \infty$ , i.e.,  $y^k$  converges to zero in the sense of a  $L_2[0,T]$  norm. Since it is clear that  $e \to 0$  when  $y \to 0$ , it can be concluded that the joint error e in trial number k converges to zero.

Theorem 1 guarantees the convergence of the joint angle  $\theta$ in trial number k to the desired one  $\theta_d$  using the property that the error dynamics of the bi-articular manipulator (14) is output strictly passive. From Theorem 1, it can be also shown that the input torque error  $\|\tau_e^k\|_{K_l^{-1}}^2$  tends to zero as  $k \to \infty$ . Therefore, the input torque  $\tau$  converges to the ideal input  $\tau_d$  in the sense of a  $L_2[0,T]$  norm. Fig. 4 shows a block diagram of the iterative learning controller. It should be noted that the ideal input torque  $\tau_d$  does not need in practice as shown in Fig. 4 even though it is assumed its existence theoretically.

Since the iterative learning control approach is generally to improve the transient response and tracking performance for the repeatability, the control performance of the proposed control law for the bi-articular manipulator should be improved with increasing the trial number. The control laws proposed in [11] and [12] depend on the parameters for the accurate models. On the contrary, the proposed law (10) and (31) does not need the desired dynamics compensation term, i.e.,  $M_b(\theta_d)\ddot{\theta}_d + C_b(\theta_d, \dot{\theta}_d)\dot{\theta}_d + g_b(\theta_d)$ . Therefore, thanks to learning, it can be found that the proposed controller in this paper is more robust than that in the previous ones. This is one of the main advantages of this work.

#### V. SIMULATION RESULTS

In this section, we present simulation results for the iterative learning control with a 2DOF bi-articular planar manipulator. The parameters of the bi-articular manipulator are  $l_1 = 0.26$  m,  $l_2 = 0.26$  m,  $l_{g1} = 0.0983$  m,  $l_{g2} = 0.0229$ 



Fig. 5. Joint error e: Dotted (Blue); with k = 1: Dashed (Purple); with k = 5: Solid (Red); with k = 12.



Fig. 6. Trajectory of end-effector of bi-articular manipulator with k = 1.

m,  $m_1 = 6.5225$  kg,  $m_2 = 2.0458$  kg,  $\tilde{I}_1 = 0.1213$  kg·m<sup>2</sup>,  $\tilde{I}_2 = 0.0116$  kg·m<sup>2</sup>, g = 0 m/s<sup>2</sup> and  $r_p = 0.05$  m. The gains are selected as  $k_p = 2$ ,  $K_y = 0.75I$  and  $K_l = I$ . The coefficients w.r.t. viscosity and elastic are  $b_1 = b_2 = b_3 = 800$  Ns/m and  $k_1 = k_2 = k_3 = 1400$  N/m, respectively, which are set to satisfy the Eqs. (16) and (17). The simulation is carried out with the initial condition  $\theta = [0.5844 - 0.7522 - 0.1678]^T$  rad,  $\dot{\theta} = [0 \ 0 \ 0]^T$  rad/s. We give a reference trajectory so that the end-effector of the bi-articular manipulator moves along a "Figure 8" motion.

The simulation results are presented in Figs. 5–8. Fig. 5 shows the joint error e. The dotted, dashed and solid lines denote the errors applying the update control law in trial number k = 1, k = 5 and k = 12, respectively. Figs. 6–8 depict the trajectory of the end-effector of the bi-articular manipulator in trial number k = 1, k = 5 and k = 12, respectively. The dashed lines in Figs. 6–8 show the reference trajectory. From Figs. 5–8, it is clear that the joint error decreases with increasing repetition. The trajectory of the end-effector of the bi-articular manipulator in trial number k = 12 almost coincides with the reference trajectory. Therefore, the effectiveness of the iterative learning control law can be verified through the simulation results.

#### VI. CONCLUSIONS

This paper proposes iterative learning control based on passivity for 2DOF robot manipulators with antagonistic bi-articular muscles. The main contribution of this paper is to show that the iterative learning control law, which can improve tracking performance for the repeatability, is designed for the bi-articular manipulator without the pa-



Fig. 7. Trajectory of end-effector of bi-articular manipulator with k = 5.



Fig. 8. Trajectory of end-effector of bi-articular manipulator with k = 12.

rameters for the accurate models. Based on output strictly passivity, convergence analysis of the closed-loop system is discussed. Simulation results are presented to verify the control performance of the proposed control scheme.

#### APPENDIX

#### A. Saturated Function

Sin(e) represents the saturated function as follows [19]:

$$Sin(e) = [Sin(e_1), Sin(e_2), Sin(e_3)]^T$$
, (39)

$$\operatorname{Sin}(e_i) = \begin{cases} 1 & e_i \ge \frac{\pi}{2} \\ \sin(e_i) & |e_i| < \frac{\pi}{2} \\ -1 & e_i \le -\frac{\pi}{2}, \end{cases}$$
(40)

$$\frac{\partial \operatorname{Sin}(e)}{\partial e} = \operatorname{Cos}(e)$$
  
= diag{Cos(e\_1), Cos(e\_2), Cos(e\_3)}. (41)

In this paper, the saturated function Sin(e) satisfies the following properties:

$$\|\operatorname{Sin}(e)\| \le \|e\|,$$
 (42a)

$$\|\operatorname{Sin}(e)\| \le \sqrt{3},\tag{42b}$$

$$\|\operatorname{Sin}(e)\|^2 \le \operatorname{Sin}(e)^T e, \tag{42c}$$

$$\|\cos(e)\dot{e}\| < \|\dot{e}\|,$$
 (42d)

for all  $e, \dot{e} \in \mathcal{R}^3$ .

#### B. Properties of Error Dynamics

The error dynamics of the bi-articular manipulator satisfies the following properties as well as standard robot manipulators [17]:

*Property 3:* There exist numbers  $\alpha_{C1} > 0$ ,  $\alpha_{C2} > 0$  such that

$$\|C_b(x,z)w - C_b(y,v)w\| \le \alpha_{C1} \|z - v\| \|w\| + \alpha_{C2} \|x - y\| \|w\| \|z\|, (43)$$

for all vectors  $v, w, x, y, z \in \mathbb{R}^3$ .

*Property 4:* There exist constants  $\alpha_{h1}$ ,  $\alpha_{h2} \ge 0$  such that the norm of the residual dynamics satisfies

$$\|h(e, \dot{e})\| \le \alpha_{h1} \|\dot{e}\| + \alpha_{h2} \|\operatorname{Sin}(e)\|,$$
 (44)

for all  $e, \dot{e} \in \mathcal{R}^3$ .

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