# **RISE Control for 2DOF Human Lower Limb** with Antagonistic Bi-Articular Muscles

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*Abstract*— This paper considers RISE control for two-degreeof-freedom (2DOF) human lower limb with antagonistic biarticular muscles. The antagonistic bi-articular muscles straddle the waist joint and the knee joint in the lower limb. Because the nonlinear model of the lower limb of the human body is uncertain, a robust control method is developed yield semiglobal asymptotic tracking. Simulation results indicate that the torques in joint 2 of the 2DOF lower limb is lower than the previous method, because of antagonistic bi-articular muscles. It is verified that the 2DOF lower limb can move to the desired position in the presence of unmodeled bounded disturbances.

## I. INTRODUCTION

Rehabilitation robotics aims at developing novel solutions for assisted therapy and objective functional assessment of patients with reduced motor and/or cognitive abilities [1]. In particular, neuro-prostheses to replace motor function after disease or injury has been a major research area in rehabilitation engineering [2], [3]. Neuromuscular electrical stimulation (NMES), which is also called functional electrical stimulation (FES), is one technique employed to generate desired muscle contractions via electrical stimulus [4].

Models of the human limb can be used to design feedforward and feedback controllers. T. Schauer *et al.* [3] proposed estimated nonlinear models of the electrically stimulated quadriceps muscle group under nonisometric conditions. M. Ferrarin *et al.* [6] developed an adaptive feedforward controller for a nonlinear dynamic model of the lower limb. N. Sharma *et al.* [4], [5] developed a neural networkbased nonlinear NMES tracking controller for a human limb in the presence of a nonlinear uncertain muscle model with nonvanishing additive disturbances. In these works, the human limb is modeled as one degree of rotational freedom about the knee joint.

Kumamoto *et al.* model and examine the effect antagonistic bi-articular muscles [7]–[9]. Antagonistic bi-articular muscles act between the waist joint and the knee joint in the lower limb. If the lower limb is modeled with antagonistic bi-articular muscles, then lower torques at a each joint are expected. The motion of the lower limb produced by antagonistic bi-articular muscles approaches the human motion in NMES. Oh *et al.* considered two-degree-of-freedom(2DOF) control for robot manipulators with antagonistic bi-articular

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muscles [10], [11]. In these works, the stability analysis has not been discussed. Passivity-based control and openloop control are considered in [12], [13]. However, these proposed control laws are composed of parameters of the whole dynamical model of the human upper limb.

In this paper, RISE control for a 2DOF human lower limb with antagonistic bi-articular muscles is considered. The lower limb is modeled similar to a 2DOF robot manipulator, where the number of control inputs is three. The model is slightly different from previous works [12], [13] in the coefficient of velocities, however, the developed model is similar to the model of [10]. Because the model parameters of the lower limb of the human body cannot be measured exactly, the robust integral of the sign of the error (RISE) methods [4], [5], [14], [15] are applied. The proposed RISE based controller is composed of PID controller, the integral of the sign of the error, and one of the model parameters. In the stability analysis, the region of attraction is expanded by using one of the model parameters in proposed controller. It is expected that the 2DOF lower limb with antagonistic bi-articular muscles can be controlled with lower torque at a joint. The simulation results show that the 2DOF lower limb with antagonistic bi-articular muscles can move with lower torques in joint 2. Simulation results verify that the 2DOF lower limb can move to the desired position in the presence of unmodeled bounded disturbances.

The organization of this paper is as follows. The problem formulation and the model are shown in Section 2. In Section 3, the stability of the 2DOF robot manipulators with antagonistic bi-articular muscle is presented. In Section 4, simulation results are indicated. Finally, our conclusions are presented.

#### **II. PROBLEM FORMULATION**

Consider the dynamics of n-link rigid robot manipulators which can be represented as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + T_d = T,$$
(1)

where  $M(q) \in \mathbb{R}^{n \times n}$  is the positive definite inertia matrix, and q,  $\dot{q}$  and  $\ddot{q}$  are the joint angles, velocities, and accelerations, respectively. The vector  $C(q, \dot{q})\dot{q} \in \mathbb{R}^n$  represents the Coriolis and centrifugal torques,  $g(q) \in \mathbb{R}^n$  is the gravitational torques,  $T_d \in \mathbb{R}^n$  is a general disturbance (e.g., unmodeled effects), and  $T \in \mathbb{R}^n$  is the control input [14], [16]. From Fig. 1, when n = 2, terms in the dynamic

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equation (1) are

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \quad T_d = \begin{bmatrix} T_{d1} \\ T_{d2} \end{bmatrix},$$
$$M(q) = \begin{bmatrix} M_1 + 2M_2 + 2RC_2 & 2M_2 + RC_2 \\ 2M_2 + RC_2 & 2M_2 \end{bmatrix},$$
$$C(q, \dot{q}) = \begin{bmatrix} -RS_2\dot{q}_2 & -RS_2(\dot{q}_1 + \dot{q}_2) \\ RS_2\dot{q}_1 & 0 \end{bmatrix},$$
$$g(q) = \begin{bmatrix} g(m_1l_{g1} + m_2l_1)S_1 + g(m_2l_{g2})S_{12} \\ g(m_2l_{g2})S_{12} \end{bmatrix},$$

where  $M_1 = m_1 l_{g1}^2 + m_2 l_1^2 + \tilde{I}_1$ ,  $M_2 = \frac{1}{2}(m_2 l_{g2}^2 + \tilde{I}_2)$ and  $R = m_2 l_1 l_{g2}$ .  $S_i$ ,  $C_i$ ,  $S_{ij}$  and  $C_{ij}$  mean  $\sin q_i$ ,  $\cos q_i$ ,  $\sin(q_i + q_j)$  and  $\cos(q_i + q_j)$  (i, j = 1, 2), respectively.  $m_i$ ,  $l_i$ ,  $l_{gi}$  and  $\tilde{I}_i$  are the weight of the link i, the length of the link i, the distance from the center of the joint i to the center of the gravity point of the link i, and the moment of inertia about an axis through the center of mass of the link i.



(a) 2DOF robot leg (b) Human leg model

Fig. 1. (a)2DOF robot leg. (b)Human leg model. Two couples of the antagonistic mono-articular muscles of  $f_1$  and  $e_1$ , and of  $f_2$  and  $e_2$  are attached to the joints of  $J_1$  and  $J_2$ , respectively. A couple of the antagonistic bi-articular muscles  $f_3$  and  $e_3$  are attached to both joints of  $J_1$  and  $J_2$ .



Fig. 2. Visco-elastic muscle model [8].  $F_{ei/fi}$ : output force,  $u_{ei/fi}$ : contractile force,  $k_i$ : coefficient w.r.t. elastic,  $x_i$ : contracting length,  $l_p$ : radius of the joint pulley.

The human leg model can be represented as three pairs of antagonistic muscles as shown in Fig. 1. In the robot motion

control, the joint torque T directly works as the control input. A couple of bi-articular muscles is connected to both the attached waist joint and the knee joint in the human leg model as shown in Fig. 2. The joint torques are indicated as

$$T_i = (F_{ei} - F_{fi})l_p + (F_{e3} - F_{f3})l_p, \quad (i = 1, 2),$$
(2)

where  $F_{fi}$  and  $F_{ej}$  (i = 1, 2) indicate the forces by the flexor muscle and the extensor muscle, respectively in Fig. 2. By using the contractile force of the flexor muscle  $u_{fi}$  and the contractile force of the extensor muscle  $u_{ei}$ , Eq. (2) is derived as follows

$$T_{i} = (u_{ei} - u_{fi})l_{p} - (u_{ei} + u_{fi})k_{i}l_{p}^{2}q_{i} + (u_{e3} - u_{f3})l_{p} - (u_{e3} + u_{f3})k_{3}l_{p}^{2}(q_{1} + q_{2}), \quad (i = 1, 2),$$
(3)

where  $l_p$  and  $k_i$  are the radius of the joint, coefficient w.r.t. elastic [7]. Note that the joint torques (3) is close to the model of [10], although the coefficient  $b_j$  is utilized in [12], [13]. The  $u_{fi}$  and  $u_{ei}$  have the following relation [8]

$$u_{fi} + u_{ei} = 1, \quad (i = 1, 2, 3).$$
 (4)

Because only the contractile force of the extensor muscle  $u_{ei}$  can be controlled by an actuator,  $u_{fi} = 1 - u_{ei}$  is substituted into Eq. (3)

$$T_{i} = (2u_{ei} - 1)l_{p} - k_{i}l_{p}^{2}q_{i} + (2u_{e3} - 1)l_{p} - k_{3}l_{p}^{2}(q_{1} + q_{2}) = \tau_{i} + \tau_{3} - k_{i}l_{p}^{2}q_{i} - k_{3}l_{p}^{2}(q_{1} + q_{2}), \quad (i = 1, 2), (5)$$

where  $\tau_i$  is defined as  $\tau_i = (2u_{ei} - 1)l_p$ . We suggest the antagonistic bi-articular muscles torque as

$$\tau_3 = M_2(\ddot{q}_1 + \ddot{q}_2) + g(m_2 l_{g2})S_{12} + k_3 l_p^2(q_1 + q_2).$$
(6)

From Eqs. (1), (5), (6), the manipulator dynamics with antagonistic bi-articular muscles which is called the bi-articular manipulator dynamics [12] as shown in Fig. 1.

$$M_b(\theta)\ddot{\theta} + C_b(\theta,\dot{\theta})\dot{\theta} + g_b(\theta) + K_r\theta + \tau_d = \tau,$$
(7)

where the elements of  $M_b(\theta) \in R^{3\times 3}$ ,  $C_b(\theta, \dot{\theta}) \in R^{3\times 3}$  and  $g_b(\theta) \in R^3$ ,  $\tau_d \in R^3$ ,  $\tau \in R^3$ ,  $\theta \in R^3$  are derived as follows

$$\begin{split} M_b(\theta) &= \begin{bmatrix} M_1 + M_2 + 2RC_2 & M_2 + RC_2 & 0 \\ M_2 + RC_2 & M_2 & 0 \\ 0 & 0 & M_2 \end{bmatrix}, \\ C_b(\theta, \dot{\theta}) &= \begin{bmatrix} -RS_2 \dot{q}_2 & -RS_2 (\dot{q}_1 + \dot{q}_2) & 0 \\ RS_2 \dot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ g_b(\theta) &= \begin{bmatrix} g(m_1 l_{g1} + m_2 l_1)S_1 \\ 0 \\ g(m_2 l_{g2})S_{12} \end{bmatrix}, \quad K_r = l_p^2 \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}, \\ \tau_d &= \begin{bmatrix} \tau_{d1} \\ \tau_{d2} \\ \tau_{d3} \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}, \quad \theta = \begin{bmatrix} q_1 \\ q_2 \\ q_1 + q_2 \end{bmatrix}, \end{split}$$

 $K_r$  means the matrices w.r.t. elastic. It is assumed that  $\theta(t)$ ,  $\dot{\theta}(t)$  are measurable and  $K_r$  is known parameter.  $M_b(\theta)$ ,  $C_b(\theta, \dot{\theta})$ , and  $g_b(\theta)$  are unknown model,  $\tau_d$  is unmodeled

bounded disturbances. In addition, the bi-articular manipulator dynamics has following property.

Property 1: Under the condition  $M_1 + M_2 > 2R$  and  $M_1M_2 > R^2$ , the inertia matrix  $M_b(\theta)$  preserves the positive definiteness.

*Property 2:* The disturbance  $\tau_d$  term and its first two derivatives are bounded (i.e.,  $\tau_d$ ,  $\dot{\tau}_d$ ,  $\ddot{\tau}_d \in \mathcal{L}_{\infty}$ ) [4], [5].

For the bi-articular manipulator dynamics (7), the objective is to converge to the desired position in the presence of unmodeled bounded disturbances.

## **III. STABILITY OF BI-ARTICULAR MUSCLES**

The desired position is defined as  $\theta_d$ , the position tracking error is indicated as

$$e_1 = \theta_d - \theta. \tag{8}$$

The tracking errors about the velocity, acceleration are defined as  $e_2$ , r, respectively

$$e_2 = \dot{e}_1 + \alpha_1 e_1, \ r = \dot{e}_2 + \alpha_2 e_2. \tag{9}$$

From Eqs. (8)–(9), the following relationships are derived

$$\dot{\theta} = -e_2 + \dot{\theta}_d + \alpha_1 e_1, \tag{10}$$

$$\ddot{\theta} = -r + \ddot{\theta}_d + \alpha_1 \dot{e}_1 + \alpha_2 e_2. \tag{11}$$

Substituting Eqs. (10) and (11) into the bi-articular manipulator dynamics (7) yields

$$M_{b}(-r + \dot{\theta}_{d} + \alpha_{1}\dot{e}_{1} + \alpha_{2}e_{2}) + C_{b}(-e_{2} + \dot{\theta}_{d} + \alpha_{1}e_{1}) + g_{b} + K_{r}\theta + \tau_{d} = \tau.$$
(12)

Then, Eq. (12) becomes

$$M_{b}r = h + K_{r}\theta + M_{b}\alpha_{2}e_{2} - C_{b}e_{2} + \tau_{d} - \tau, \qquad (13)$$

where the nonlinear function h is defined as

$$h = M_b(\ddot{\theta}_d + \alpha_1 \dot{e}_1) + C_b(\dot{\theta}_d + \alpha_1 e_1) + g_b.$$
(14)

The auxiliary function  $f_d$  and  $\bar{h}$  are defined as

$$f_d = M_b \dot{\theta}_d + C_b \dot{\theta}_d + g_b, \tag{15}$$

$$\bar{h} = h - f_d = M_b \alpha_1 \dot{e}_1 + C_b \alpha_1 e_1.$$
 (16)

The nonlinear function (14) is rewritten as

$$h = f_d + \bar{h},\tag{17}$$

and when Eq. (17) is substituted into (13), the bi-articular manipulator dynamics become

$$M_b r = M_b \alpha_2 e_2 - C_b e_2 + \bar{h} + f_d + K_r \theta + \tau_d - \tau.$$
 (18)

The derivative of Eq. (18) is given by

$$M_b \dot{r} = -\frac{1}{2} \dot{M}_b r + \tilde{N} + N_d - e_2 + K_r \dot{\theta} - \dot{\tau}, \qquad (19)$$

where unmeasurable auxiliary terms  $\tilde{N}$  and  $N_d$  are defined as

$$\tilde{N} = -\frac{1}{2}\dot{M}_{b}r + \dot{M}_{b}\alpha_{2}e_{2} + M_{b}\alpha_{2}\dot{e}_{2} -\dot{C}_{b}e_{2} - C_{b}\dot{e}_{2} + \dot{\bar{h}} + e_{2} + \dot{\tau}_{d}, \qquad (20)$$

$$N_d = \dot{f}_d. \tag{21}$$

The mean value theorem is applied to upper bound  $\tilde{N}$  as

$$\|\tilde{N}\| \le \rho(\|y\|) \|y\|,$$
 (22)

where  $y(t) \in R^3$  is defined as

$$y = \begin{bmatrix} e_1^T & e_2^T & r^T \end{bmatrix}^T, \tag{23}$$

and the bounding function  $\rho(||y||) \in R$  is a positive, globally invertible, nondecreasing function. From  $q_d(t)$ ,  $\dot{q}_d(t)$ ,  $\ddot{q}_d(t)$ ,  $\ddot{q}_d(t) \in \mathcal{L}_{\infty}$  are bounded, the known positive constants  $\zeta_1, \zeta_2 \in R$  exist as follows

$$|N_d|| \le \zeta_1, \quad ||\dot{N}_d|| \le \zeta_2. \tag{24}$$

*Remark 1:* While  $N_d$  and  $\dot{N}_d$  are unmeasurable as shown Eq. (21), it is assumed that  $N_d$  and  $\dot{N}_d$  are bounded in order to design a controller's parameter.

The input torque based on RISE control law is proposed as follows

$$\tau(t) = k_s e_2(t) + K_r \theta_d(t) + \nu(t), \dot{\nu}(t) = k_s \alpha_2 e_2(t) + \beta \text{sgn}(e_2(t)) + K_r \alpha_1 e_1(t),$$
(25)

where  $k_s \in R, \ \beta \in R$  denote positive constant gains.

*Remark 2:* The controller (25) is different from a standard RISE controller used in [5] with respect to the terms  $K_r\theta_d$  and  $K_r\alpha_1e_1(t)$ . Though it is difficult to obtain the correct parameters of  $K_r$ , adding the terms  $K_r\theta_d$  and  $K_r\alpha_1e_1(t)$  is useful to enlarge the region of attraction.

The derivative of (25) is

$$\dot{\tau} = k_s r + \beta \operatorname{sgn}(e_2) + K_r \alpha_1 e_1 + K_r \dot{\theta}_d.$$
(26)

Substituting Eq. (26) into Eq. (19), the closed loop system is

$$M_b \dot{r} = -\frac{1}{2} \dot{M}_b r + \tilde{N} + N_d - e_2 -K_r e_2 - k_s r - \beta \operatorname{sgn}(e_2).$$
(27)

The domain  $\mathcal{D} \in \mathbb{R}^{3n+1}$  containing  $\Phi(t) = 0$  is defined, where  $\Phi(t)$  is defined as

$$\Phi(t) = \begin{bmatrix} y^T(t) & \sqrt{P(t)} \end{bmatrix}^T,$$
(28)

where P satisfies the following property as [5]

$$\dot{P} = -r^T \left( N_d - \beta \operatorname{sgn}(e_2) \right), \tag{29}$$

$$P(0) = \beta \sum_{i=1}^{T} |e_{2i}(0)| - e_2(0)^T N_d(0).$$
 (30)

Then, the main result of this paper is stated.

*Theorem 1:* Consider the system described by the biarticular manipulator dynamics (18) and control law (25). It is assumed that all system signals are bounded. The position tracking errors are regulated in the sense that

$$||e_1||, ||e_2||, ||r|| \to 0 \text{ as } t \to \infty,$$
 (31)

for the region of attraction  $\mathcal{D}$ 

$$\mathcal{D} = \left\{ \Phi \in R^{3n+1} | \| \Phi \| \le \rho^{-1} \sqrt{2\lambda_3 k_s} \right\},\tag{32}$$

where  $\rho$  is defined in Eq. (22)

$$\lambda_3 = \min \left\{ \begin{array}{l} 2\lambda_{min}(\alpha_1) - 1\\ \lambda_{min}(\alpha_2) + \lambda_{min}(K_r) - 1\\ \frac{1}{2}k_s \end{array} \right\}.$$
 (33)

The gain  $k_s$  is designed sufficiently large, and  $\beta$  is selected according to the following condition

$$\beta > \zeta_1 + \frac{1}{\lambda_{min}(\alpha_2)}\zeta_2,\tag{34}$$

and  $\alpha_1$ ,  $\alpha_2$  are selected as

$$\lambda_{\min}(\alpha_1) > \frac{1}{2}, \quad \lambda_{\min}(\alpha_2) > 1 - \lambda_{\min}(K_r), \tag{35}$$

where  $\lambda_{min}(\cdot)$  is minimum eigenvalue, and  $\rho > 0$  depends on unmeasurable uncertainties.

*Proof:* A continuously differentiable positive definite function for the system is proposed

$$V_L(\Phi, t) = e_1^T e_1 + \frac{1}{2} e_2^T \left( I + K_r \right) e_2 + \frac{1}{2} r^T M_b r + P.$$
 (36)

The time derivative of the function (36) along the trajectories (27) is obtained as follows

$$\dot{V}_{L}(\Phi, t) = 2e_{1}^{T}(e_{2} - \alpha_{1}e_{1}) + e_{2}^{T}(r - \alpha_{2}e_{2}) + r^{T} \left\{ -\frac{1}{2}\dot{M}_{b}r + \tilde{N} + N_{d} - e_{2} - K_{r}e_{2} - k_{s}r - \beta \text{sgn}(e_{2}) \right\} + \frac{1}{2}r^{T}\dot{M}_{b}r + \dot{P} + e_{2}^{T}K_{r}(r - \alpha_{2}e_{2}) = 2e_{1}^{T}e_{2} - 2e_{1}^{T}\alpha_{1}e_{1} - e_{2}^{T}\alpha_{2}e_{2} + r^{T}\tilde{N} + r^{T}N_{d} - r^{T}k_{s}r - r^{T}\beta \text{sgn}(e_{2}) + \dot{P} - e_{2}^{T}K_{r}\alpha_{2}e_{2}.$$
(37)

The dynamics (29) is substituted into (37)

$$\dot{V}_{L}(\Phi, t) = 2e_{1}^{T}e_{2} - 2e_{1}^{T}\alpha_{1}e_{1} - e_{2}^{T}(I + K_{r})\alpha_{2}e_{2} + r^{T}\tilde{N} - r^{T}k_{s}r.$$
(38)

By using  $k_s$  is scalar, the following relation is satisfied from (22)

$$2e_1^T e_2 \le ||e_1||^2 + ||e_2||^2, \tag{39}$$

the function (38) is given by

$$\dot{V}_{L}(\Phi, t) \leq \|e_{1}\|^{2} + \|e_{2}\|^{2} - 2e_{1}^{T}\alpha_{1}e_{1} \\
-e_{2}^{T}(I + K_{r})\alpha_{2}e_{2} + r^{T}\tilde{N} - k_{s}\|r\|^{2}.$$
(40)

The following properties are satisfied

$$e_{1}^{T}\alpha_{1}e_{1} \geq \lambda_{min}(\alpha_{1})\|e_{1}\|^{2}$$

$$e_{2}^{T}(I+K_{r})\alpha_{2}e_{2} \geq (\lambda_{min}(\alpha_{2})+\lambda_{min}(K_{r}))\|e_{2}\|^{2},$$
(41)
(41)

$$\|r^T \tilde{N}\| \le \rho(\|y\|) \|r\| \|y\|.$$
(43)

The function (40) is can be described as follows

$$\dot{V}_{L}(\Phi, t) \leq \|e_{1}\|^{2} + \|e_{2}\|^{2} - 2\lambda_{min}(\alpha_{1})\|e_{1}\|^{2} \\
- (\lambda_{min}(\alpha_{2}) + \lambda_{min}(K_{r}))\|e_{2}\|^{2} \\
+ \rho(\|y\|)\|r\|\|y\| - k_{s}\|r\|^{2} \\
\leq - (2\lambda_{min}(\alpha_{1}) - 1)\|e_{1}\|^{2} \\
- (\lambda_{min}(\alpha_{2}) + \lambda_{min}(K_{r}) - 1)\|e_{2}\|^{2} \\
- \frac{1}{2}k_{s}\|r\|^{2} - \left(\frac{1}{2}k_{s}\|r\|^{2} - \rho(\|y\|)\|r\|\|y\|\right).$$
(44)

By using the completing square, the following relation is obtained

$$\frac{1}{2}k_{s}\|r\|^{2} - \rho\left(\|y\|\right)\|r\|\|y\|$$
$$= \frac{1}{2}k_{s}\left(\|r\| - \frac{\rho}{k_{s}}\|y\|\right)^{2} - \frac{1}{2}\frac{\rho^{2}}{k_{s}}\|y\|^{2}.$$
(45)

From the relation (45), the function (44) is translated into

$$\dot{V}_{L}(\Phi, t) \leq -(2\lambda_{min}(\alpha_{1}) - 1) \|e_{1}\|^{2} \\
-(\lambda_{min}(\alpha_{2}) + \lambda_{min}(K_{r}) - 1) \|e_{2}\|^{2} \\
-\frac{1}{2}k_{s}\|r\|^{2} + \frac{1}{2}\frac{\rho^{2}}{k_{s}}\|y\|^{2} \\
\leq -\lambda_{3}\|y\|^{2} + \frac{\rho^{2}}{2k_{s}}\|y\|^{2} \\
\leq -U(\Phi),$$
(46)

where  $\lambda_3$  is defined as

$$\lambda_3 = \min \left\{ \begin{array}{l} 2\lambda_{min}(\alpha_1) - 1\\ \lambda_{min}(\alpha_2) + \lambda_{min}(K_r) - 1\\ \frac{1}{2}k_s \end{array} \right\}, \tag{47}$$

and  $U(\Phi)$  is a continuous semi-positive function, which is defined as follows for some positive constant c

$$U(\Phi) = c \|y\|^2.$$
 (48)

Note that  $U(\Phi)$  is valid for the region of attraction  $\mathcal{D}$ 

$$\mathcal{D} = \left\{ \Phi \in R^{3n+1} |||\Phi|| \le \rho^{-1} \sqrt{2\lambda_3 k_s} \right\}.$$
(49)

Then, the following relation is satisfied

$$c \|y(t)\|^2 \to 0, \quad t \to \infty, \quad \forall y(0) \in \mathcal{D}.$$
 (50)

Therefore,  $e_1$ ,  $e_2$ , r satisfy the following condition [17]

$$||e_1(t)||, ||e_2(t)||, ||r(t)|| \to 0, t \to \infty.$$
 (51)

*Remark 3:* The norms of  $N_d$  and  $\dot{N}_d$  are bounded and unmeasurable in Eq. (24). Thus, the gain  $\beta$  should be designed to be large along the relation Eq. (34).

*Remark 4:* In the nonlinear function Eq. (14), if  $K_r \theta$  is added as follows

$$h = M_b(\ddot{\theta}_d + \alpha_1 \dot{e}_1) + C_b(\dot{\theta}_d + \alpha_1 e_1) + g_b + K_r \theta, \quad (52)$$

then, Eq. (47) becomes

$$\lambda_3 = \min \left\{ \begin{array}{l} 2\lambda_{min}(\alpha_1) - 1\\ \lambda_{min}(\alpha_2) - 1\\ \frac{1}{2}k_s \end{array} \right\}$$
(53)

by using the previous work [5]. Therefore, the region of attraction  $\mathcal{D}$  by Eq. (47) obtained from our proposed method can be larger than one by Eq. (53) obtained from the previous work [5].

## IV. SIMULATION

In this section, the performance of the proposed control law in section 3 is verified. The model parameters are given as  $m_1 = 7.0$  [kg],  $m_2 = 4.0$  [kg],  $l_1 = 0.4$  [m],  $l_2 = 0.5$  [m],  $l_{g1} = l_1/2$  [m],  $l_{g2} = l_2/2$  [m],  $\tilde{I}_1 = 0.093$  [kg · m<sup>2</sup>],  $\tilde{I}_2 = 0.083$  [kg · m<sup>2</sup>],  $l_p = 0.08$  [m],  $k_1 = 4688$  [N/m],  $k_2 = 3125$  [N/m],  $k_3 = 6250$  [N/m]. The control parameters are designed

$$\alpha_1 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \ \alpha_2 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, 
k_s = 20, \ \beta = 1.$$
(54)

The initial conditions are given as  $q_1(0) = 0$  [rad],  $q_2(0) = 0$  [rad],  $\dot{q}_1(0) = 0$  [rad/s],  $\dot{q}_2(0) = 0$  [rad/s] and the constraint of  $q_2$  is  $-\pi \le q_2 \le 0$  [rad], because the limitation of the lower limb. The references are selected as  $q_{d1} = \frac{\pi}{4}$  [rad],  $q_{d2} = -\frac{\pi}{2}$  [rad]. The simulation results are obtained by using MATLAB.

Figs. 3–4 show the step responses without disturbance  $\tau_d$  where the solid line and dashed line represents the bi-articular manipulator dynamics (7) and the manipulator dynamics (1), respectively. From the results, it is derived that  $\theta = \theta_d$  ( $t \to \infty$ ) is achieved and errors  $e_1$ ,  $e_2$  and r converge to zero. Though the input torque  $\tau_1$  is larger than input torque  $T_1$ , the input torque  $\tau_2$  is smaller than input torque  $T_2$ . To flex the knee joint  $J_2$ , the input torque  $\tau_3$  by the bi-articular muscle  $f_3$  works instead of  $\tau_2$ . Fig. 5 indicates that the function  $V_L$  shown in Eq. (36) is positive definite function.



Fig. 3. Step responses of joint angles without disturbance  $(q_{d1} = \frac{\pi}{4}, q_{d2} = -\frac{\pi}{2})$ 



Fig. 4. Control inputs without disturbance  $(q_{d1} = \frac{\pi}{4}, q_{d2} = -\frac{\pi}{2})$ 



Fig. 5. Positive definite function (36) without disturbance

Second, the step responses with the unmodeled dynamics are verified in Figs. 6–7 where the solid line and dashed line show the response without disturbances and the response with disturbances, respectively. It is assumed that a disturbance  $\tau_d$  is muscle contraction model [7] as

$$\tau_d = B_r \dot{\theta},\tag{55}$$

where

$$B_r = 0.03 \times \begin{bmatrix} 300 & 0 & 0\\ 0 & 200 & 0\\ 0 & 0 & 400 \end{bmatrix}.$$
 (56)

Note that the muscle contraction model have already been shown in previous research [3], [7]. Because the purpose of simulations is disturbance attenuation by the proposed RISE controller, it is assumed that  $B_r\dot{\theta}$  is an unmodeled dynamics.

Fig. 6 shows the RISE control for the bi-articular manipulator dynamics (7), the responses of the passivity-based control [12] are indicated in Fig. 7. The transient property of RISE control is the same as no disturbance response. On the other hand, in the passivity-based control, the transient properties are different from no disturbance responses. Fig. 8 also indicates the trajectories with disturbances. The RISE control can raise a lower limb without swinging of the ankle.



Fig. 6. Step responses of joint angles with disturbances by using proposed controller



Fig. 7. Step responses with disturbances by using passivity-based controller

#### V. CONCLUSIONS

This paper considered RISE based 2DOF robot manipulators with antagonistic bi-articular muscles. The lower limb which is modeled by using the model of 2DOF robot manipulators was considered where the number of control inputs is three. The RISE methods was applied, because the unmodeled disturbances of the lower limb of the human body exist. The stability analysis was indicated by using the proposed controller. It was shown that the errors converge to zero. The simulation result indicated that the errors converge to zero, the high torque at the joint 2 is not needed by using antagonistic bi-articular muscles, and the RISE control can move to the desired position in the presence of additive disturbances. In the future works, we would like to verify the fatigue by using the antagonistic bi-articular muscles in experiments.

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