Passivity-based Synchronized Visual Feedback Control for Eye-to-Hand Systems

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Abstract—This paper investigates passivity based synchronized visual feedback control for eye-to-hand systems. Firstly the brief summary of the estimation and the control error systems is given with the basic representation of a relative rigid body motion. Secondly we construct the synchronized visual feedback system with an eye-to-hand configuration by combining a synchronization error system. Next, we derive the passivity of the synchronized visual feedback system. Based on passivity and dissipative systems theory, stability and $L_2$-gain performance analysis are discussed. Finally the validity of the proposed control law can be confirmed by the simulation results.

I. INTRODUCTION

Robotics and intelligent machines need sensory information to behave autonomously in dynamical environments. Visual information is particularly suited to recognize unknown surroundings. In this sense, vision is one of the highest sensing modalities that currently exist. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control is important when we consider a mechanical system working in dynamical environments [1][2].

In classical visual servoing, many practical methods are reported by two well known approaches with two camera configurations, i.e., position-based visual feedback control and image-based one with an eye-in-hand configuration or an eye-to-hand (fixed camera) one [1]. Recently, new camera configurations combined each classical one have been proposed. Flandin et al. [3] addressed an eye-in-hand and an eye-to-hand cooperation approach that each camera information is partitioned into the positioning task and the orientation one, respectively. In [4], the occlusion problem has been tackled by using multi eye-in-hand and eye-to-hand cameras. The authors discussed passivity based visual feedback control in the 3D (three dimensional) workspace with an eye-in/to-hand configuration which consists of a robot manipulator (a work manipulator) and a camera that is attached to the end-effector of another robot manipulator (a camera manipulator) [5][6].

A common feature among these visual feedback systems with a new camera configuration [3]–[6] is to consist of multiple robot manipulators.

On the other hand, some efforts have recently been made to design multicomposed system which is shared information among robot manipulators. Especially, the mutual synchronized control problem can be formulated as to design interconnections and controllers for the robot manipulators in the system, such that the motion of the manipulator has to be synchronized with respect to not only the desired trajectory but also the motion of other manipulators. In production process tasks that cannot be carried out by a single robot, either because of the complexity of the task or limitations of the robot, the use of multirobot systems working in mutual synchronization, e.g., cooperative schemes, has proved to be a good alternative [7]. One of the advantages of synchronized control is that the controller can reduce the magnitude of the error for the whole system in the transient stage because of synchronous manner, in adding disturbance to some manipulators.

One recent representative work on synchronization of robot manipulators is in [7]. In this work, a scheme which needs only position measurements was proposed to solve the problem of position synchronization of multiple cooperative robot systems, and the controller was shown to be semi-globally exponentially stable. In [8], a leader-follower synchronization output feedback control scheme for the ship replenishment problem was proposed. Chung et al. [9] provided a method that eliminates both the all-to-all coupling and the feedback of the acceleration terms. Sun et al. [9] designed an adaptive cross-coupled controller [10] and a model-free one [11] to stabilize multi-axis motions for synchronizing mechanical systems. Visual feedback, however, is not considered here. The scheme, which applies to mutual synchronized control for the aforementioned visual feedback system with multiple robot manipulators, should allow us to extend the technological application area.

This paper deals with synchronized visual feedback control for eye-to-hand systems as depicted in Fig. 1. In this paper, for simplicity, the synchronized visual feedback system consists of only two manipulators and a fixed camera, while it can...
be easily extended the system which consists of more mani-
ippers and/or a movable camera [5][6]. In our proposed
system, not only the convergence of the relative rigid body
motion from the hand to the target object to the desired one,
but also that of between manipulators are guaranteed, since
the synchronization error system is constructed. Based on
passivity and dissipative systems theory, stability and \(L_2\) gain
performance analysis for the synchronized visual feedback
system are discussed. Finally simulation results are shown
to verify the stability and the performance of the proposed
method.

II. ESTIMATION AND CONTROL ERROR SYSTEMS

Throughout this paper, we use the notation \(e^{\hat{\theta}_{ab}} \in \mathbb{R}^{3 \times 3}\)
to represent the change of the principle axes of a frame \(\Sigma_b\)
relative to a frame \(\Sigma_a\). \(\xi_{ab} \in \mathbb{R}^3\) specifies the direction
of rotation and \(\theta_{ab} \in \mathbb{R}\) is the angle of rotation. For simplicity
we use \(\xi_{ab} \theta_{ab}\) to denote \(\xi_{ab} \theta_{ab}\). The notation \(\wedge\) (wedge)
is the skew-symmetric operator such that \(\xi \theta = [\xi \times \theta\) for the vector
product \(\times\) and any vector \(\theta \in \mathbb{R}^3\). The notation \(\vee\) (vee) denotes the inverse
operator to \(\wedge\), i.e., \(so(3) \rightarrow \mathbb{R}^3\). Recall that a skew-symmetric
matrix corresponds to an axis of rotation (via the mapping \(a \mapsto \hat{a}\)). We use the \(4 \times 4\) matrix
\[
\begin{bmatrix}
e^{\xi_{ab}} & p_{ab} \\
0 & 1
\end{bmatrix}
\]
as the homogeneous representation of \(g_{ab} = (p_{ab}, e^{\xi_{ab}}) \in SE(3)\) describing the configuration of a frame \(\Sigma_b\) relative
to a frame \(\Sigma_a\). The adjoint transformation associated with \(g_{ab}\)
is denoted by \(A_d(g_{ab})\) [12].

A. Generation of Desired Relative Rigid Body Motion

The visual feedback system considered in this paper has
two robot manipulators and a fixed camera as depicted in
Fig. 1, where the coordinate frames \(\Sigma_w\), \(\Sigma_c\), \(\Sigma_a\), \(\Sigma_w\), and
\(\Sigma_h\) represent the world frame, the camera frame, the object
frame, the base frame of the \(i\)-th manipulator, and the \(i\)-th
hand (end-effector of the \(i\)-th manipulator) frame, respectively.
Here, the subscript \(i\) denotes the index of the manipulators,
i.e., \(i = 1, 2\) in this paper. Then, the relative rigid body motion
from \(\Sigma_c\) to \(\Sigma_o\) can be represented by \(g_{co}\). Similarly, the rigid
body motions \(g_{wco}, g_{wco}, g_{wco}, g_{wco}, g_{wco}, g_{wco}, g_{wco}, g_{wco}, g_{wco}\),
and \(g_{wco}\) are represented, respectively, as shown in Fig. 1.

The objective of synchronized visual feedback control is
that both manipulators track the target object while the ma-
nipulators should mutually synchronize. In other words, it
is to bring the actual relative rigid body motions \(g_{h,1,2,0}\)
and \(g_{h,2,0}\) to given constant desired ones \(g_{h,1,2,0}\) and \(g_{h,2,0}\), respectively.
It should be noted that the desired motions \(g_{h,1,2,0}\) and \(g_{h,2,0}\) are
related each other because of the mutual synchronous behavior.
Therefore, we state how to set the desired rigid body motions
for synchronized visual feedback control.

Firstly, we consider the reference manipulator as depicted
in Fig. 2, which is the fundamental manipulator to track the
target object. The desired value of the synchronized visual
feedback system has only one degree of freedom. We define
it as \(g_{bd}\), which is the relative rigid body motion from the
reference hand frame \(\Sigma_h\) to the object frame \(\Sigma_o\). Next, we
set the reference target objects which have the coordinate
frames \(\Sigma_o\), as depicted in Fig. 2, to fulfill \(g_{wco} = g_{wco}\) (This
means \(g_{wco} = g_{wco}\)). Meanwhile, if the control objective
is achieved, the motion of all manipulators is corresponded
to the reference manipulator i.e. \(g_{h,1,2,0} = g_{wco}\) because of
the mutual synchronous behavior. This means to satisfy the
relationship \(g_{h,1,2,0} = g_{wco}\), too. Hence, we can set \(g_{h,1,2,0}\), using
\(g_{bd}\) as follows:
\[
g_{h,1,2,0} = g_{bd}^{-1} g_{wco} g_{bd} g_{wco}.
\] (2)

We define the control error \(e_{h,1,2,0}\) between the actual relative rigid
body motion \(g_{h,1,2,0}\) and the desired one \(g_{h,1,2,0}\), as
\[
e_{h,1,2,0} = g_{h,1,2,0}^{-1} g_{h,1,2,0}\] (3)
in other words, \(p_{eh,1,2,0} = \hat{e}_{h,1,2,0} (p_{h,1,2,0} - p_{h,1,2,0})\) and \(\hat{e}_{h,1,2,0}\).
Note that \(p_{h,1,2,0} = p_{h,1,2,0}\) and \(e_{h,1,2,0} = e_{h,1,2,0}\).
If \(g_{h,1,2,0} = I_3\), i.e., \(p_{h,1,2,0} = 0\) and \(e_{h,1,2,0} = I_3\). Using the
notation \(e_R(\hat{e}_{h,1,2,0}) := \frac{1}{2} (e^{\hat{e}_{h,1,2,0}} - e^{\hat{e}_{h,1,2,0}})\) which denotes
the error vector of the rotation matrix \(e^{\hat{e}_{h,1,2,0}}\), the vector of
the control error is given by
\[
e_{h,1,2,0} := \begin{bmatrix}
p_{eh,1,2,0}' \\
\hat{e}_{h,1,2,0}'
\end{bmatrix}^T.
\] (4)

Note that \(e_{h,1,2,0} = 0\) iff \(p_{h,1,2,0} = 0\) and \(e^{\hat{e}_{h,1,2,0}} = I_3\). Therefore, if
the vector of the control error is equal to zero, then the actual
relative rigid body motion \(g_{h,1,2,0}\) equals the desired relative rigid
body motion \(g_{h,1,2,0}\).

B. Basic Representation for Visual Feedback System and Es-
timation Error System

This subsection mainly reviews our previous works [13][14]
via the passivity based visual feedback control.
In order to achieve the control objective, the relative rigid body motion $g_{i,o}$ is needed. $g_{i,o}$ can be represented as follows:

$$g_{i,o} = g_{ch_i}^{-1} g_{co} g_{uw_i}.$$  

(5)

In (5), since $g_{ch_i}$ and $g_{uw_i}$ can be given by the measured information, we consider to get the relative rigid body motion $g_{co}$. The relative rigid body motion from $\Sigma_i$ to $\Sigma_o$ can be led by using the composition rule for rigid body transformations ([12], Chap. 2, pp. 37, eq. (2.24)) as follows:

$$g_{co} = g_{wc}^{-1} g_{wo}.$$  

(6)

The relative rigid body motion involves the velocity of each rigid body. To this end, let us consider the velocity of a rigid body as described in [12]. We define the body velocity of the camera relative to the world frame $\Sigma_w$ as $V^b_{wc} = [V^T_{wc} \omega^T_{wc}]^T$, where $V_{wc}$ and $\omega_{wc}$ represent the velocity of the origin and the angular velocity from $\Sigma_w$ to $\Sigma_c$, respectively ([12] Chap. 2, eq. (2.55)).

Differentiating (6) with respect to time, the body velocity of the relative rigid body motion $g_{co}$ can be written as follows (See [13]):

$$V^b_{co} = -Ad^1_{(g_{co})} V^b_{wc} + V^b_{wo}.$$  

(7)

where $V^b_{wo}$ is the body velocity of the target object relative to $\Sigma_w$. Eq. (7) is the basic representation for the three coordinate frames of the visual feedback system [13]. In the case of the fixed camera configuration, i.e. $V^b_{wc} = 0$, the body velocity of the relative rigid body motion $g_{co}$ can be rewritten as

$$V^b_{co} = V^b_{wo}.$$  

(8)

Roughly speaking, if both the camera and the target object move, then the relative rigid body motion $g_{co}$ will be derived from the difference between the camera velocity $V^b_{wc}$ and the target object velocity $V^b_{wo}$. Hence, the model of the relative rigid body motion from $\Sigma_c$ to $\Sigma_o$ equals the target object velocity $V^b_{wo}$.

The visual feedback control task requires information of the relative rigid body motion $g_{co}$. Since the measurable information is only the image information $f(g_{co})$ in the visual feedback system, we consider a nonlinear observer in order to estimate the relative rigid body motion $g_{co}$ from the image information $f(g_{co})$.

Firstly, using (8), we choose estimates $\hat{g}_{co}$ and $\hat{V}^b_{co}$ of the relative rigid body motion and velocity, respectively as

$$\hat{V}^b_{co} = u_e.$$  

(9)

The new input $u_e$ is to be determined in order to drive the estimated values $\hat{g}_{co}$ and $\hat{V}^b_{co}$ to their actual values.

In order to establish the estimation error system, we define the estimation error between the estimated value $\hat{g}_{co}$ and the actual relative rigid body motion $g_{co}$ as

$$g_{ee} = \hat{g}_{co}^{-1} g_{co}.$$  

(10)

Using the notation $e_R(e^\theta_{ae})$, the vector of the estimation error is defined as

$$e_e := \begin{bmatrix} p^T_{ee} \ e^T_R(e^{\theta_{ae}}) \end{bmatrix}^T.$$  

(11)

Therefore, if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\hat{g}_{co}$ equals the actual relative rigid body motion $g_{co}$.

Suppose the attitude estimation error $\theta_{ae}$ is small enough that we can let $e^{\theta_{ae}} \approx I + sk(e^{\theta_{ae}})$. Therefore, using a first-order Taylor expansion approximation, the estimation error vector $e_e$ can be obtained from image information $f(g_{co})$ and the estimated value of the relative rigid body motion $\hat{g}_{co}$ as follows ([13]):

$$e_e = J^1(\hat{g}_{co})(f - \hat{f}),$$  

(12)

where $\hat{f}$ is the estimated value of image information. In the same way as the basic representation (7), the estimation error system can be represented by

$$V^b_{ee} = -Ad^1_{(g_{we})} u_e + V^b_{wo}.$$  

(13)

It should be noted that if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\hat{g}_{co}$ equals the actual one $g_{co}$.
C. Control Error System

In this subsection which mainly reviews our previous work [6], let us consider the dual of the estimation error system, which we call the control error system.

In order to achieve the control objective, we have to derive \(g_{ehi}\) defined as (3). Using \(g_{ee}\), the control error can be transformed as

\[
g_{ehi} = g_{hi}^{-1}g_{hi}^{-1}g_{co}g_{wwi} = g_{hi}^{-1}g_{co}g_{wwi}.
\]  

(14)

In (14), \(g_{hi}, g_{co}, g_{wwi}\) are known information. While the estimation error matrix \(g_{ee}\) cannot be directly obtained, because \(g_{ee}\) is defined using non-measurable value \(g_{co}\) as (10). Therefore, we derive \(g_{ee}\) from \(g_{e}\).

Because of the definition of the estimation error vector \(g_{e}\), i.e., \(g_{e} := [p_{ee}, e^T(e^Tg_{ee})]_T\), the position estimation error \(p_{ee}\) can be derived directly from \(g_{e}\). Under the condition \(-\frac{T}{2} \leq \theta_{ee} \leq \frac{T}{2}\), \(\theta_{ee}\) can be derived as follows [6]:

\[
\theta_{ee} = \frac{\sin^{-1}(\|e^T(e^Tg_{ee})\|)}{\|e^T(e^Tg_{ee})\|} e^T(e^Tg_{ee}).
\]

(15)

Hence, \(g_{ee}\) can be derived from \(g_{e}\) through \(\theta_{ee}\).

The reference of the relative rigid body motion \(g_{hi,d}\) is constant in this paper, i.e., \(g_{hi,d} = 0\), hence, \(V_{ehi} = V_{hi,o}^b\).

Thus, differentiating (3) with respect to time, the control error system can be represented as

\[
V_{ehi}^b = -Ad_{(g_{hi},b)} Ad_{(g_{hi},a)} V_{whi}^b + V_{wo}^b.
\]

(16)

This is dual to the estimation error system.

III. PASSIVITY-BASED SYNCHRONIZED VISUAL FEEDBACK CONTROL

For assembly tasks in modern manufacturing and space applications, a scheme reduces not only the error concerning the control objective but also that of synchronous manner must have the great potential [10]. For the visual feedback system which consists of two manipulators and a fixed camera shown as Fig. 1, the straightforward extension of the controller using only the estimation and the control error systems in [14] can be also proposed without synchronous manner. However, although it can be treated the tracking problem, it can not be ensured the synchronous behavior. This section considers synchronized visual feedback control based on passivity which is a main contribution in this paper.

A. Synchronization Error System

In this subsection, we derive the synchronization error system in order to impose the synchronous manner. Firstly, we define the error \(g_{ehi} = (p_{ehi}, e^{\theta_{ehi}})\) between the relative rigid body motion of the one manipulator \(g_{hi,a}\), and that of another manipulator \(g_{hi,o}\) as

\[
g_{ehi} = g_{hi,a}^{-1}g_{hi,o}, \quad (i, j) = (1, 2), (2, 1).
\]

Moreover, the synchronization error is defined using the element of the predefined error (17) and the coupling gains which denote the interactions between the manipulators in the system as follows:

\[
g_{esij} := 
\begin{bmatrix}
K_{Rij} e^{(g_{esij})} & k_{pij} p_{ehij} \\
0 & 1
\end{bmatrix}
\]

(18)

where \(k_{pij}\) is the coupling gain for the position and \(k_{Rij}\) is that for the rotation. Using the notation \(e^R(e^{\theta_{ee}})\), the synchronization error vector is defined as

\[
e_{esij} := 
\begin{bmatrix}
p_{esij}^T & e^R(e^{\theta_{esij}})
\end{bmatrix}_T.
\]

(19)

Differeniating (18) with respect to time, the synchronization error system can be represented as

\[
V_{esij}^b = -K_{sij} Ad_{(g_{hi,a})} Ad_{(g_{hi,o})} V_{whi}^b + K_{sij} Ad_{(g_{hi,a})} Ad_{(g_{hi,o})} V_{whj}^b + K_{sij} Ad_{(g_{hi,o})} (1 - Ad_{(g_{hi,a})}) V_{wo}^b.
\]

(20)

where \(K_{sij} := diag\{k_{pij}, k_{Rij}, I, k_{Rij}^2, I\}\) for all \(i, j = 1, 2, i \neq j\).

B. Synchronized Visual Feedback System

The dynamics of the rigid body manipulators can be written as

\[
M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = \tau_i + \tau_{id}
\]

(21)

where \(M_i \in \mathbb{R}^{n \times n}\) is the inertia matrix, \(C_i \in \mathbb{R}^{n \times n}\) is the Coriolis matrix, \(g_i \in \mathbb{R}^{n}\) is the gravity vector, and \(q_i, \dot{q}_i, \ddot{q}_i\) are the joint angle, velocity and acceleration, respectively. \(\tau_i\) is the vector of the input torque, and \(\tau_{id}\) represents a disturbance input. Since the manipulator dynamics is considered, the hand body velocity is given by \(V_{whi}^b = J_i(q_i) \dot{q}_i\), where \(J_i(q_i)\) is the body manipulator Jacobian [12].

Next, we propose the control law for the manipulator as

\[
\tau_i = M_i(q_i) \ddot{q}_{id} + C_i(q_i, \dot{q}_i) \dot{q}_{id} + g_i(q_i) + J_i(q_i)Ad_{(g_{hi,a})} \dot{q}_{id} + J_i(q_i)Ad_{(g_{hi,o})} \dot{q}_{id},
\]

\[
K_{sij} e_{sij}
\]

\[
- J_i^T(q_i)Ad_{(g_{hi,a})}^T K_{sij} e_{sij} + u_{\xi},
\]

(22)

where \(\ddot{q}_{id}\) and \(\dot{q}_{id}\) represent the desired joint velocity and acceleration, respectively. The new input \(u_{\xi}\) is to be determined in order to achieve the control objective.

Let us define the error vector with respect to the joint velocity of the manipulator as

\[
\xi_i := \dot{q}_i - \dot{q}_{id}.
\]

(23)

Moreover, we design the reference of the joint velocity as \(\dot{q}_{id} := J_i(q_i)u_{wh,d}\) where \(u_{wh,d}\) is the desired body velocity which will be obtained from the aforementioned error systems. Thus, \(V_{whi}^b\) should be replaced by \(u_{wh,d}\).

Using (13), (16) and (20)–(22), the synchronized visual feedback system can be derived as the equation (24) at the bottom of the next page, where the input \(u := \bar{u}_{\xi} u_{\xi}^T (Ad_{(g_{hi,a})} u_{wh,d})^T (Ad_{(g_{hi,o})} u_{wh,d}) u_{\xi}^T u_{\xi}^T\). We define the state and the disturbance of the synchronized visual
feedback system as $x := [\xi_1^T \xi_2^T e_{h1} e_{h2} e_{s12} e_{s21} e_{e}^T]^T$ and $w := [\tau_{id} \tau_{id} (V_{by}^b)^T]^T$, respectively.

**Remark 1**: If the coupling gains $k_{pi} = k_{qi} = 0$ and the synchronization error $e_{sij}$ is not considered, then the synchronized visual feedback system (24) can be regarded as the non-synchronized one which consists of two manipulators and a fixed camera. Non-synchronized system is a straightforward extension of the eye-to-hand visual feedback system [14]. Thus, the visual feedback system with the eye-to-hand configuration [14] is included as the special case of the system (24).

**C. Passivity of Synchronized Visual Feedback System**

Before proposing a synchronized visual feedback control law, we derive an important lemma.

**Lemma 1**: If $w = 0$, then the synchronized visual feedback system (24) satisfies

$$\int_0^T u^T \nu \geq -\beta, \quad \forall T > 0$$

where

$$\nu := NK_s x$$

$$N = \begin{bmatrix}
I & 0 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & -I & 0 & -Ad^T_{(g_{h1})_{e}} g_{h1} & Ad^T_{(g_{h1})_{e}} \\
0 & 0 & 0 & -I & -Ad^T_{(g_{h2})_{e}} g_{h2} & Ad^T_{(g_{h2})_{e}} g_{h2} \\
0 & 0 & 0 & 0 & 0 & K_s
\end{bmatrix}$$

and $\beta$ is a positive scalar.

**Proof**: Consider the following positive definite function

$$V = \frac{1}{2} \xi_1^T M_1 \xi_1 + \frac{1}{2} \xi_2^T M_2 \xi_2 + E(g_{eh1}) + E(g_{eh2})$$

$$+ \frac{1}{k_{R_{12}}} E(g_{se12}) + \frac{1}{k_{R_{21}}} E(g_{se21}) + E(g_{ee}), \quad (26)$$

where $E(g_{ab}) := \frac{1}{2} ||p_{ab}||^2 + \phi(e_{\theta_{ab}})$ and $\phi(e_{\theta_{ab}}) := \frac{1}{2} \text{tr}(I - e^{i\theta_{ab}}) \text{ is the error function of the rotation matrix (see, e.g., [15])}$. Differentiating (26) with respect to time yields

$$\dot{V} = \frac{1}{2} \xi_1^T M_1 \xi_1 + \frac{1}{2} \xi_2^T M_2 \xi_2$$

$$+ x^T \begin{bmatrix}
M_1 & 0 & 0 & 0 & 0 & 0 \\
0 & M_2 & 0 & 0 & 0 & 0 \\
0 & 0 & Ad_{(e_{h1})_{e}} & 0 & 0 & 0 \\
0 & 0 & 0 & Ad_{(e_{h2})_{e}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Ad_{(e_{s12})_{e}} \\
0 & 0 & 0 & 0 & 0 & Ad_{(e_{s21})_{e}}
\end{bmatrix} x$$

$$= x^T K_s N^T u,$$

(27)

using the skew-symmetry of the matrices $\hat{p}_{eh1}, \hat{p}_{eh2}, \hat{p}_{eh12}, \hat{p}_{eh21}, \hat{p}_{ee}, M_1 - 2C_1$ and $M_2 - 2C_2$. Note that we have utilized the fact that $c_R(k_{R_{12}}e^{i\theta_{ab}}) = k_{R_{12}}c_R(e^{i\theta_{ab}})$. Integrating (27) from 0 to $T$, we obtain

$$\int_0^T u^T v dt = V(T) - V(0) \geq -\beta$$

(28)

where $\beta$ is a positive scalar that only depends on the initial states of $\xi_1, \xi_2, e_{h1}, e_{h2}, e_{s12}, e_{s21},$ and $e_{ee}$.

**Remark 2**: Lemma 1 would suggest that the synchronized visual feedback system is passive from the input $u$ to the output $v$ as in the definition in [16]. Note that Lemma 1 can be shown to be equivalent with the passivity property of the manipulator dynamics, the estimation error system, the control error system and the synchronization error system.

**D. Stability Analysis for Synchronized Visual Feedback System**

We now propose the following control input for the interconnected system:

$$u = -K \nu = -K NK_s x$$

$$K := \text{diag}\{K_{\xi_1}, K_{\xi_2}, K_{h1}, K_{h2}, K_{e}\}$$

(29)
where $K_{\xi} := \text{diag}\{k_{\xi,1}, \ldots, k_{\xi,n}\}$, $K_h := \text{diag}\{k_{h,1}, \ldots, k_{h,6}\}$ and $K_e := \text{diag}\{k_{e,1}, \ldots, k_{e,6}\}$ denote the positive gain matrices.

**Theorem 1:** If $w = 0$, then the equilibrium point $x = 0$ for the closed-loop system (24) and (29) is asymptotic stable.

**Proof:** In the proof of Lemma 1, we have already derived the time derivative of $V$ along the trajectory of the system (24) is formulated as (27). Using the control input (29), (27) can be transformed into

$$
\dot{V} = -x^T K_{\xi} N^T KN K_{\xi} x.
$$

(30)

This completes the proof.

Theorem 1 guarantees the stability of synchronized control using a Lyapunov function for the eye-to-hand synchronized visual feedback system (24) which is a highly nonlinear system. It is interesting to note that stability analysis is based on passivity as described in (25). For the tasks which need the synchronous manner, the control performance should be improved compared to the eye-to-hand non-synchronized visual feedback control law, because the synchronized control design considers the mutual synchronization error. For the visual feedback system, it is difficult that the desired trajectory in joint space is given explicitly, since the motion of the target object is unknown. Although the synchronization error is defined in the joint space in the previous works [7]–[11], we define it in the work space by making the reference manipulator and the reference target objects as shown in Fig 2.

**E. $L_2$-gain Performance Analysis for Synchronized Visual Feedback System**

Based on the dissipative systems theory, we consider $L_2$-gain performance analysis for the synchronized visual feedback system (24) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

$$
P := K_{\xi} N^T KN K_{\xi} - \frac{1}{2\gamma^2} ||W||^2 - \frac{1}{2} I
$$

(31)

where $\gamma \in \mathbb{R}$ is positive and $W$ is defined at the bottom of the next page. Then we have the following theorem.

**Theorem 2:** Given a positive scalar $\gamma$ and consider the control input (29) with the gains $K_{\xi}, K_{\xi}, K_h, K_h, K_e$ and $K_e$ such that the matrix $P$ is positive semi-definite, then the closed-loop system (24) and (29) has $L_2$-gain $\leq \gamma$.

The proof is omitted due to space limitations. Theorem 2 can be proved using the energy function (26) as a storage function for $L_2$-gain performance analysis. The $L_2$-gain performance analysis of the synchronized visual feedback system is discussed via the dissipative systems theory. In $H_\infty$-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed $L_2$-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other-type of generalized plants of the synchronized visual feedback systems.

**IV. SIMULATIONS**

In this section, the validity of the proposed control law can be confirmed by comparing the simulation results with a static target object. Although we only discuss simulation results, it should be noted that the model parameters were developed using actual synchronized visual feedback system.

The simulation results on two 2DOF manipulators as depicted in Fig. 3 are shown in order to understand our proposed method simply, though it is valid for 3D synchronized visual feedback systems. We defined the seven coordinates which were described in Fig. 3. The target object has four feature points. The control objective is to bring the actual relative rigid body motions $g_{h,1}$ and $g_{h,2}$ to a given references $g_{w,1}$ and $g_{w,2}$ with mutual synchronization of robotic manipulation. In other words, it can be stated as the desire to regulate not only the both hand control errors $e_{h,1}$ and $e_{h,2}$ but also the synchronization errors $e_{w,1}$ and $e_{w,2}$ to zero.

In this paper, we present simulation results for the synchronized visual feedback control, compared with the non-synchronized one which is the straightforward extension of the controller in [14]. The simulation is carried out with the initial condition $q_{11}(0) = \pi/4$ [rad], $q_{12}(0) = \pi/12$ [rad], $q_{21}(0) = \pi/6$ [rad], $q_{22}(0) = -\pi/6$ [rad], $p_{w,1} = \begin{bmatrix}0 0 -1\end{bmatrix}^T$ [m], $\xi_{\theta,1}(0) = \begin{bmatrix}0 0 0\end{bmatrix}^T$ [rad], $p_{w,2} = \begin{bmatrix}0 0 -2\end{bmatrix}^T$ [m], $\xi_{\theta,1}(0) = \begin{bmatrix}0 0 0\end{bmatrix}^T$ [rad], $p_{w,2} = \begin{bmatrix}0.4732 0.1\end{bmatrix}^T$ [m], $\xi_{\theta,1}(0) = \begin{bmatrix}0 0 0\end{bmatrix}^T$ [rad], $p_{w,2} = \begin{bmatrix}0.3086 0 -3\end{bmatrix}^T$ [m], $\xi_{\theta,1}(0) = \begin{bmatrix}0 0 -0.5087\end{bmatrix}^T$ [rad]. The desired relative rigid body motion is $p_{h,1d} = [0 0 -2]^T$ [m], $\xi_{\theta,1d} = [0 0 0]^T$ [rad], $p_{h,2d} = [0 0 -1]^T$ [m], $\xi_{\theta,2d} = [0 0 0]^T$ [rad], and the initial errors are calculated as $\xi_1(0) = \xi_2(0) = [0 0 0]^T$ [rad/s], $p_{h,1}(0) = [-0.294 -0.293 0]^T$ [m], $\xi_{\theta,1}(0) = [0 0 -1]^T$ [rad], $p_{h,2}(0) = [-0.075 -0.1 0]^T$ [m], $\xi_{\theta,2}(0) = [0 0 -0.487]^T$ [rad], $p_{w,12}(0) = [-0.063 -0.179 0]^T$ [m], $\xi_{\theta,12}(0) = [0 0 -0.563]^T$ [rad], $p_{w,21}(0) = [-0.124 -0.144 0]^T$ [m], $p_{w,21}(0) = [-0.124 -0.144 0]^T$ [m].
for mutual synchronization were chosen as $k$ were empirically selected as gains
compare the both control laws simply, except for the coupling
synchronized control law, the same gains are set in order to
important not to select too large values. In the case of the non-
of the translations of (Solid), respectively. In these figures, we focus on the errors
Fig. 4. Figs. 5 and 6 illustrate the hand control errors
In Figs. 4 and 5, the asymptotic stability applying the
control case in Fig. 5. This proves the mutual synchronization
behavior. Meanwhile, the errors $e_{h1}$, and $e_{h2}$ converge faster
than $e_{x21}$ for the non-synchronized control case in Fig. 6.
Moreover, we tested the system by adding a force disturbance $T_{dd} = [100 50]^{T}$ [Nm] to only the manipulator 1 from

$$W := \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & \text{Ad}_{\epsilon(\theta_{h1})} & 0 \\
0 & 0 & \text{Ad}_{\epsilon(\theta_{h2})} & 0 \\
\text{Ad}_{\epsilon(\theta_{h1})} & \text{Ad}_{\epsilon(\theta_{h2})} & 0 & 0 \\
\text{Ad}_{\epsilon(\theta_{h1})} & 0 & 0 & 0 \\
\text{Ad}_{\epsilon(\theta_{h2})} & 0 & 0 & 0 \\
\text{Ad}_{\epsilon(\theta_{h1})} & 0 & 0 & 0 \\
\end{bmatrix}$$

(32)

V. CONCLUSIONS

This paper considers synchronized visual feedback control for eye-to-hand systems. In our proposed system, we can treat not only the tracking problem but also the synchronized one, since it consists of the synchronization error system. Based on passivity and dissipative systems theory, stability and $L_2$-gain performance analysis for the synchronized visual feedback system are discussed. Finally, simulation results are shown to verify the stability and the performance of the proposed method.

REFERENCES