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Passivity-based Control of Visual Feedback Systems with a Movable Camera

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Abstract. This paper deals with the visual feedback control with a movable camera instead of a fixed camera in the fixed camera configuration. Firstly the brief summary of the visual feedback system with a fixed camera is given with the fundamental representation of a relative rigid body motion. Secondly we construct the new error system in order to enlarge the field of view. Next, we derive the passivity of the visual feedback system. Finally, stability and L_2-gain performance analysis are discussed based on the passivity and the dissipative systems theory.

1 Introduction

Robotics and intelligent machines need many information to behave autonomously under dynamical environments. Specifically, the combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments [1].

In classical visual servoing, many practical methods are reported by two well known approaches with two camera configurations, i.e., position-based visual feedback control and image-based one with an eye-in-hand configuration or a fixed camera one (see, e.g. [1]). Kelly *et al*. [2] considered a simple image-based controller for visual feedback system in the three dimensional(3D) workspace under the assumption that the objects' depths are

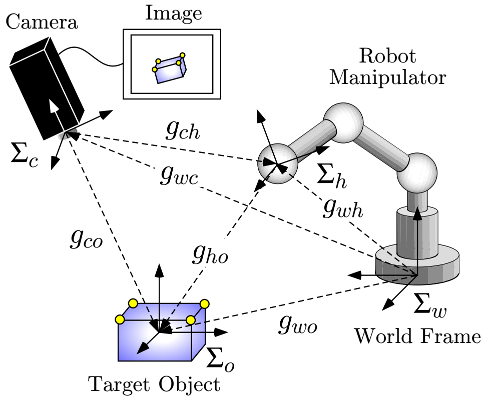

Figure 1: Visual feedback system in the fixed camera configuration.

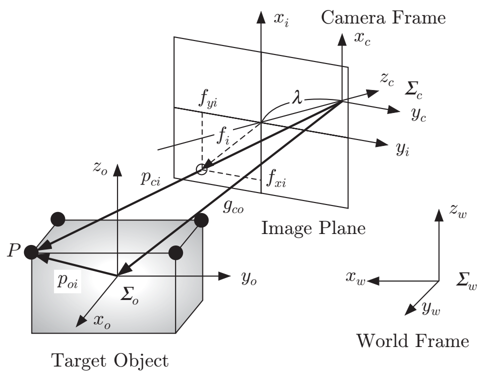

Figure 2: Pinhole camera

known. Zergeroglu *et al.* developed an adaptive control law for the position tracking and the camera calibration problems of the visual feedback system with parametric uncertainties in [3]. Cowan *et al.* [4] addressed the problems of the field of view for the visual feedback system by using the navigation functions. Although the good solutions to the set-point problems are reported in those papers, few results have been obtained for the tracking problems of the moving target object in the visual feedback system. Additionally, most of the previous works are discussed for the camera configurations separately, while the position-based visual feedback control and the image-based one are combined in some issues [5][6].

In this paper, we discuss the visual feedback control for the target tracking problem with a movable camera instead of a fixed camera in the fixed camera configuration as in Fig. 1. While the objective of this system is obviously to control the end-effector of the manipulator, we also control the camera in order to enlarge the field of view. Moreover, we can derive that the visual feedback system with a movable camera preserves the passivity of the visual feedback system which is obtained in our previous works [7][8]. Stability and L_2-gain performance analysis are discussed based on the passivity and the dissipative systems theory with the energy function.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times 3}$ to represent the change of the principle axes of a frame Σ_b relative to a frame Σ_a. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathcal{R}^3$. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. Here $\hat{\xi}\theta_{ab}$ denotes $\hat{\xi}_{ab}\theta_{ab}$ for the simplicity of notation. We use the 4×4 matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \tag{1}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$ which is the description of the configuration of a frame Σ_b relative to a frame Σ_a. The adjoint transformation associated with g_{ab} is denoted by $\mathrm{Ad}_{(g_{ab})}$ [9]. Let us define the vector form of the rotation matrix as $e_R(e^{\hat{\xi}\theta_{ab}}) := \mathrm{sk}(e^{\hat{\xi}\theta_{ab}})^{\vee}$ where $\mathrm{sk}(e^{\hat{\xi}\theta_{ab}})$ denotes $\frac{1}{2}(e^{\hat{\xi}\theta_{ab}} - e^{-\hat{\xi}\theta_{ab}})$.

2 Passivity-based Visual Feedback System in the Fixed Camera Configuration

2.1 Fundamental Representation for Visual Feedback System

Visual feedback systems typically use four coordinate frames which consist of a world frame Σ_w, a target object frame Σ_o, a camera frame Σ_c and a hand (end-effector) frame Σ_h as in Fig. 1. Then, g_{wh}, g_{wc} and g_{wo} denote the rigid body motions from Σ_w to Σ_h, from Σ_w to Σ_c and from Σ_w to Σ_o, respectively. Similarly, the relative rigid body motions from Σ_c to Σ_h, from Σ_c to Σ_o and from Σ_h to Σ_o can be represented by g_{ch}, g_{co} and g_{ho}, respectively, as shown in Fig. 1. Here, it is supposed that the relative rigid body motion from Σ_w to Σ_c, i.e. g_{wc} can be measured exactly. Since g_{wh} is known by the angle of manipulator, g_{ch} can be also available from using the composition rule for rigid body transformations ([9], Chap. 2, pp. 37, eq. (2.24)) as $g_{ch} = g_{wc}^{-1} g_{wh}$. Thus, g_{wc}, g_{wh} and g_{ch} are known information in the visual feedback system as in Fig. 1.

The relative rigid body motion from Σ_c to Σ_o can be led as $g_{co} = g_{wc}^{-1} g_{wo}$. Then, the fundamental representation of the relative rigid body motion g_{co} is described as follows [7].

$$V_{co}^b = -\mathrm{Ad}_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b \tag{2}$$

where V_{wc}^b and V_{wo}^b are the body velocity of the camera and the target object relative to Σ_w, respectively. Roughly speaking, the relative rigid body motion g_{co} will be derived from the difference between the camera velocity V_{wc}^b and the target object velocity V_{wo}^b.

2.2 Camera Model and Estimation Error System

Next, we derive the model of a pinhole camera with a perspective projection as shown in Fig. 2. Let λ be a focal length, $p_{oi} \in \mathcal{R}^3$ and $p_{ci} \in \mathcal{R}^3$ be coordinates of the target object's i-th feature point relative to Σ_o and Σ_c, respectively. Using a transformation of the coordinates, we have $p_{ci} = g_{co} p_{oi}$ where p_{ci} and p_{oi} should be regarded as $[p_{ci}^T \ 1]^T$ and $[p_{oi}^T \ 1]^T$ via the well-known representation in robotics, respectively (see, e.g., [9]). The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate $f_i := [f_{xi} \ f_{yi}]^T \in \mathcal{R}^2$ as follows

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{3}$$

where $p_{ci} := [x_{ci} \ y_{ci} \ z_{ci}]^T$. It is straightforward to extend this model to the m image points case by simply stacking the vectors of the image plane coordinate, i.e. $f(g_{co}) := [f_1^T \ \cdots \ f_m^T]^T \in \mathcal{R}^{2m}$. We assume that multiple point features on a known object can be used.

The visual information $f(g_{co})$ which includes the relative rigid body motion can be exploited, while the relative rigid body motion g_{co} can not be obtained directly in the visual feedback system. In order to bring the actual relative rigid body motion g_{ho} to a given reference g_d in Fig. 1, in addition to the control problem, we consider the estimation one in the visual feedback system. Firstly, we shall consider the following model which just comes from the fundamental representation (2).

$$\bar{V}_{co}^b = -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e \tag{4}$$

where $\bar{g}_{co} = (\bar{p}_{co}, e^{\hat{\bar{\xi}}\bar{\theta}_{co}})$ and $\bar{V}_{co}^b$ are the estimated value of the relative rigid body motion and the estimated body velocity from Σ_c to Σ_o, respectively. u_e is the input in order to converge the estimated value to the actual relative rigid body motion. Because the design of u_e needs a property of the whole visual feedback system, we will propose u_e in Section 3.

The estimated image feature point $\bar{f}_i$ $(i = 1, \cdots, m)$ should have the same form as (3). In order to establish the estimation error system, we define the estimation error between the estimated value $\bar{g}_{co}$ and the actual relative rigid body motion g_{co} as $g_{ee} = \bar{g}_{co}^{-1} g_{co}$. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the estimation error is given by $e_e := [p_{ee}^T \; e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$. The vector of the estimation error e_e can be derived from the following relation

$$e_e = J^{\dagger}(\bar{g}_{co})(f - \bar{f}) \tag{5}$$

where $J(\bar{g}_{co})$ is defined in [7] and $\dagger$ denotes the pseudo-inverse. Therefore the estimation error e_e can be exploited in the 3D visual feedback control law using image information f obtained from the camera. Hence, the nonlinear observer is constructed by (4) and the estimation input u_e which can be determined from e_e in (5) with an estimation gain in Section 3.3. Then, the estimation error system is represented by

$$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \tag{6}$$

It should be noted that if the vector of the estimation error is equal to zero, then the estimated relative rigid body motion $\bar{g}_{co}$ equals the actual one g_{co}.

2.3 Visual Feedback System in the Fixed Camera Configuration

Similar to the estimation error system, we consider the control error system. Because g_{co} can not be obtained directly, we represent the relative rigid body motion from Σ_h to Σ_o with the estimated one $\bar{g}_{co}$ as $\bar{g}_{ho} = g_{ch}^{-1} \bar{g}_{co}$. Differentiating $\bar{g}_{ho}$ with respect to time, the estimated body velocity from Σ_h to Σ_o can be obtained as follows.

$$\bar{V}_{ho}^b = -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{wh}^b + u_e. \tag{7}$$

where V_{wh}^b is the body velocity of the hand relative to Σ_w. Similarly, we define the error between g_d and $\bar{g}_{ho}$, which is called the control error, as $g_{ec} = g_d^{-1} \bar{g}_{ho}$. The vector of the control error is defined as $e_c := [p_{ec}^T \; e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$. The control error system is described by

$$V_{ec}^b = -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{wh}^b + u_e - \mathrm{Ad}_{(g_{ec}^{-1})} V_d^b \tag{8}$$

where V_d^b is the desired body velocity of the relative rigid body motion g_{ho}.

Combining (6) and (8), the visual feedback system in the fixed camera configuration is constructed as follows

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b, \quad u_{ce} := \begin{bmatrix} V_{wh}^b + \mathrm{Ad}_{(g_d)} V_d^b \\ u_e \end{bmatrix}. \tag{9}$$

Let us define the error vector of the visual feedback system as $e_{ce} := \left[e_c^T \ e_e^T\right]^T$ which contains of the control error vector e_c and the estimation error vector e_e. Here, we define the output of the visual feedback system (9) as follows

$$\nu_{ce} := \begin{bmatrix} -\mathrm{Ad}^T_{(g_d^{-1})} & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{bmatrix} e_{ce}.$$

Then the visual feedback system (9) satisfies $\int_0^T u_{ce}^T \nu_{ce} d\tau \geq -\beta_{ce}$ where β_{ce} is a positive scalar [8]. This would suggest that the visual feedback system (9) is *passive* from the input u_{ce} to the output ν_{ce} just formally as in the definition in [10].

3 Visual Feedback System with a Movable Camera

3.1 Camera Field Error System

In this section, we construct the error system of the movable camera in the fixed camera configuration, we call the camera field error system, in order to increase the available workspace for the robot hand. Here we define the camera field error between the estimated value $\bar{g}_{co}$ and a given reference g_{cd} for the camera motion as $g_{ev} = g_{cd}^{-1}\bar{g}_{co}$. If $\bar{g}_{co}$ is equal to g_{cd}, then the target object can be kept in the center of the camera field of view. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the camera field error is defined as $e_v := [p_{ev}^T \ e_R^T(e^{\hat{\xi}\theta_{ev}})]^T$. Note that $e_v = 0$ iff $p_{ev} = 0$ and $e^{\hat{\xi}\theta_{ev}} = I_3$. Similarly to (6) and (8), the camera field error system can be obtained as

$$V_{ev}^b = u_e - \mathrm{Ad}_{(\bar{g}_{co}^{-1})} V_{wc}^b - \mathrm{Ad}_{(g_{ev}^{-1})} V_{cd}^b \tag{10}$$

where V_{cd}^b is the desired body velocity of the relative rigid body motion g_{co}.

3.2 Property of Visual Feedback System

Combining (6), (8) and (10), we construct the visual feedback system with a movable camera in the fixed camera configuration as follows

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \\ V_{ev}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I & 0 \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} & 0 \\ 0 & I & -\mathrm{Ad}_{(\bar{g}_{co}^{-1})} \end{bmatrix} u_{cev} + \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} V_{wo}^b, \quad u_{cev} := \begin{bmatrix} V_{wh}^b + \mathrm{Ad}_{(g_d)} V_d^b \\ u_e \\ V_{wc}^b + \mathrm{Ad}_{(g_{cd})} V_{cd}^b \end{bmatrix}. \tag{11}$$

Let us define the error vector of the visual feedback system (11) as $e := \left[e_c^T \ e_e^T \ e_v^T\right]^T$. It should be noted that if the vector of the estimation error is equal to zero, not only $\bar{g}_{co}$ equals g_{co} but also $\bar{g}_{ho}$ equals g_{ho}. Moreover, if the vectors of the control error and the camera field error are equal to zero, then $\bar{g}_{ho}$ and $\bar{g}_{co}$ equal g_d and g_{cd}, respectively. Thus, when $e \to 0$, g_{ho} and g_{co} tend to g_d and g_{cd}, respectively. This states that the control objective can be achieved, in addition, the available workspace for the robot hand will be increased by moving of the camera.

Lemma 1 If $V_{wo}^b = 0$, then the visual feedback system (11) satisfies

$$\int_0^T u_{cev}^T \nu_{cev} d\tau \geq -\beta_{cev}, \quad \forall T > 0, \quad \nu_{cev} := N_{cev} e = \begin{bmatrix} -\mathrm{Ad}_{(g_d^{-1})}^T & 0 & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ev}})} \\ 0 & 0 & -\mathrm{Ad}_{(g_{cd}^{-1})}^T \end{bmatrix} e \quad (12)$$

where β_{cev} is a positive scalar.

Proof Consider the following positive definite function

$$V_{cev} = E(g_{ec}) + E(g_{ee}) + E(g_{ev}) \quad (13)$$

where $E(g) := \frac{1}{2}\|p\|^2 + \phi(e^{\hat{\xi}\theta})$ and $\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta})$ which is the error function of the rotation matrix. Differentiating (13) with respect to time yields

$$\dot{V}_{cev} = e^T \begin{bmatrix} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & 0 & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} & 0 \\ 0 & 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ev}})} \end{bmatrix} \begin{bmatrix} V_{ec}^b \\ V_{ee}^b \\ V_{ev}^b \end{bmatrix} \quad (14)$$

where we use the property $\dot{\phi}(e^{\hat{\xi}\theta}) := e^{\hat{\xi}\theta}\omega$. Observing the skew-symmetry of the matrices $\hat{p}_{ec}$, $\hat{p}_{ee}$ and $\hat{p}_{ev}$, the above equation along the trajectories of the system (11) can be transformed into

$$\dot{V}_{cev} = u_{cev}^T \nu_{cev}. \quad (15)$$

Integrating (15) from 0 to T, we can obtain

$$\int_0^T u_{cev}^T \nu_{cev} d\tau \geq -V_{cev}(0) := -\beta_{cev} \quad (16)$$

where β_{cev} is the positive scalar which only depends on the initial states of g_{ec}, g_{ee} and g_{ev}. □

Remark 1 In the visual feedback system, $p_{ec}^T(e^{-\hat{\xi}\theta_d}\ \omega_{ec})^\wedge p_{ec} = 0$, $p_{ee}^T \hat{\omega}_{ee} p_{ee} = 0$, $p_{ev}^T(e^{-\hat{\xi}\theta_{cd}}\omega_{ev})^\wedge p_{ev} = 0$ hold. Let us take u_{cev} as the input and ν_{cev} as its output in Fig. 3. OMFC and HMFC represent the object motion relative to the camera frame Σ_c and the hand motion relative to the camera frame Σ_c, respectivery. Lemma 1 suggests that the visual feedback system (11) is *passive* from the input u_{cev} to the output ν_{cev} as in the definition in [10].

3.3 Stability Analysis for Visual Feedback System

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

$$u_{cev} = -K_{cev}\nu_{cev} = -K_{cev}N_{cev}e, \ K_{cev} := \begin{bmatrix} K_c & 0 & 0 \\ 0 & K_e & 0 \\ 0 & 0 & K_v \end{bmatrix} \quad (17)$$

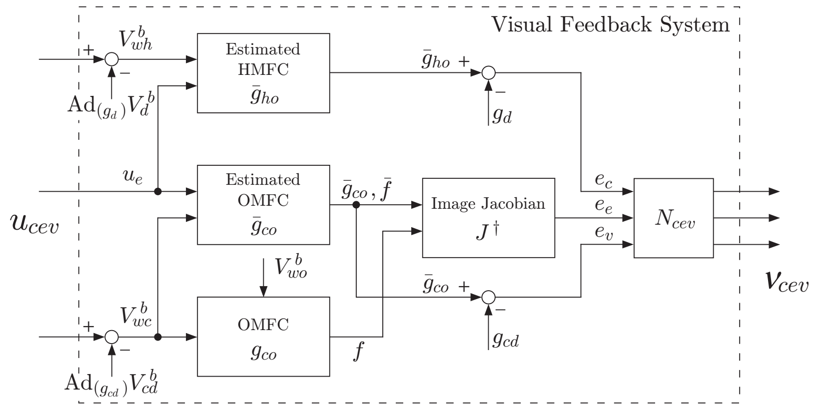

Figure 3: Block diagram of the visual feedback system

where $K_c := \mathrm{diag}\{k_{c1}, \cdots, k_{c6}\}$, $K_e := \mathrm{diag}\{k_{e1}, \cdots, k_{e6}\}$ and $K_v := \mathrm{diag}\{k_{v1}, \cdots, k_{v6}\}$ are the positive gain matrices of x, y and z axes of the translation and the rotation for the control error, the estimation one and the camera field one, respectively. The result with respect to asymptotic stability of the proposed control input (17) can be established as follows.

Theorem 1 If $V_{wo}^b = 0$, then the equilibrium point $e = 0$ for the closed-loop system (11) and (17) is asymptotic stable.

Theorem 1 can be proved using the energy function (13) as a Lyapunov function. It is interesting to note that stability analysis is based on the passivity as described in (12).

3.4 L_2-gain Performance Analysis for Visual Feedback System

Based on the dissipative systems theory, we consider L_2-gain performance analysis for the visual feedback system (11) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

$$P := N_{cev}^T K_{cev} N_{cev} - \frac{1}{2\gamma^2}W - \frac{1}{2}I \tag{18}$$

where $\gamma \in \mathcal{R}$ is positive and $W := \mathrm{diag}\{0, I, 0\}$. Then we have the following theorem.

Theorem 2 Given a positive scalar γ and consider the control input (17) with the gains K_c, K_e and K_v such that the matrix P is positive semi-definite, then the closed-loop system (11) and (17) has L_2-gain $\leq \gamma$.

Theorem 2 can be proved using the energy function (13) as a storage function for L_2-gain performance analysis. The L_2-gain performance analysis of the visual feedback system is discussed via the dissipative systems theory. In H_∞-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed L_2-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other-type of generalized plants of the visual feedback systems.

4 Conclusions

This paper dealt with the visual feedback control with a movable camera instead of a fixed camera in the fixed camera configuration in order to increase the available workspace for the robot hand. Moreover, we derived that the visual feedback system preserved the passivity of the visual feedback system by the same strategy in our previous works, [7], [8]. Stability and L_2-gain performance analysis for the visual feedback system have been discussed based on passivity with the energy function.

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