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An Interpretation of Vision-based Control for Rigid Body Motion: A Geometric Framework

1 Introduction

This paper investigates the rigid body motion (involving both translation and rotation) control problem of vision-based robotic systems. Visual feedback systems of the eye-in-hand configuration typically use three coordinate frames which consist of a world frame Σ_w, a target object frame Σ_o and a camera (end-effector) frame Σ_c. In this control strategy, one of the control objectives is to track the moving target object in a three-dimensional workspace by image information. Hence the model of the relative rigid body transformation, which represents the position p and the rotation $e^{\hat{\xi}\theta}$ of the target object frame Σ_o relative to the camera frame Σ_c, can be described by the nonlinear systems on the group of rigid motions, which shall be denoted as $SE(3)$. In order to regard the rotation as one of the state for visual feedback system, we derive the model of the relative rigid body motion using the homogeneous representation and the adjoint transformation. The main contribution of this paper is that the interpretation of our proposed strategy [1] has been given based on a geometric framework.

2 Visual Feedback Control

The homogeneous representation and the adjoint transformation [2] will give us the model of the relative rigid body motion(RRBM) as

$$V^b = -\mathrm{Ad}_{(g^{-1})} V^b_{wc} + V^b_{wo}, \tag{1}$$

where V^b, V^b_{wc} and V^b_{wo} represent the body velocity of the relative rigid body motion, the body velocity of the camera relative to Σ_w and the body velocity of the target object relative to Σ_w, respectably. The notation $g = (p, e^{\hat{\xi}\theta})$ expresses the homogeneous representation of the relative rigid body motion. The adjoint transformation associated with g is given as $\mathrm{Ad}_{(g)}$. We will consider the model of the estimated relative rigid body motion as follows,

$$\bar{V}^b = -\mathrm{Ad}_{(\bar{g}^{-1})} V^b_{wc} + u_e \tag{2}$$

where $\bar{g} = (\bar{p}, e^{\hat{\bar{\xi}}\bar{\theta}})$ and $\bar{V}^b$ are the estimated value of the relative rigid body motion and the estimated body velocity. The new input u_e is to be determined in order to converge the estimated value to the actual relative rigid body motion. The reference $g_d = (p_d, e^{\hat{\xi}\theta_d})$ for the relative rigid motion g is assumed to be constant in this paper. Then, we define the control error and the estimation error as $g_{ec} := g_d^{-1}\bar{g} = (p_{ec}, e^{\hat{\xi}\theta_{ec}})$ and $g_{ee} := \bar{g}^{-1}g = (p_{ee}, e^{\hat{\xi}\theta_{ee}})$, respectively. From (1) and (2), the visual feedback system can be derived as

$$\begin{bmatrix} V^b_{ec} \\ V^b_{ee} \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} \begin{bmatrix} u_c \\ u_e \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} V^b_{wo} \tag{3}$$

where V^b_{ec} and V^b_{ee} represent the body velocity of the control error and the estimation error, respectably. Since the camera velocity V^b_{wc} is considered as an input, the notation u_c will be used instead of V^b_{wc}. The block diagram of the visual feedback system is shown in Fig. 1.

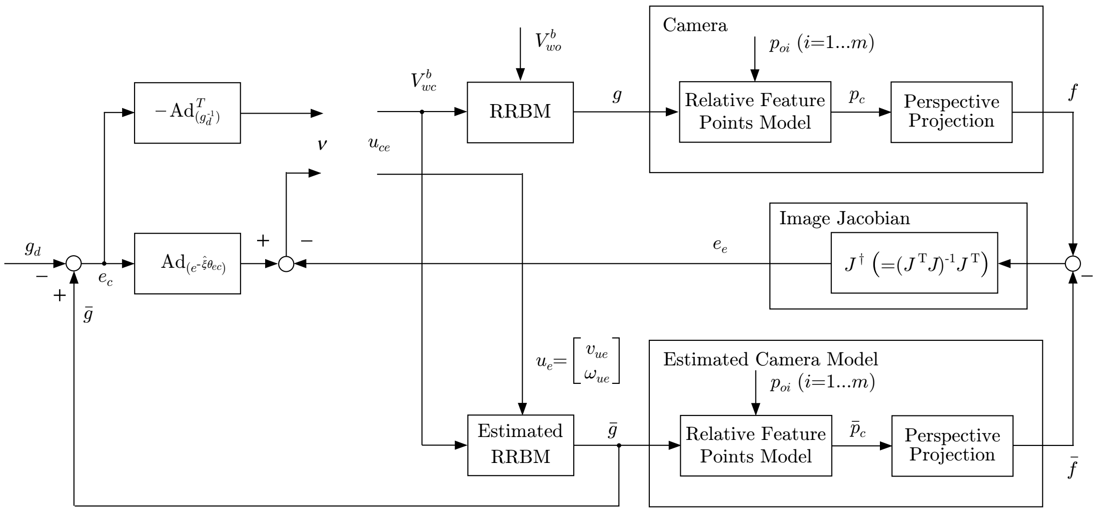

Figure 1: Block diagram of the visual feedback system

If $V_{wo}^b = 0$, then the visual feedback system (3) with the energy function $V = \frac{1}{2}\|p_{ec}\|^2 + \phi(e^{\hat{\xi}\theta_{ec}}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(e^{\hat{\xi}\theta_{ee}})$ satisfies

$$\int_0^T u_{ce}^T \nu d\tau \geq -\gamma_{ce}, \quad \nu := \begin{bmatrix} -\mathrm{Ad}^T_{(g_d^{-1})} & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{bmatrix} e, \tag{4}$$

where ϕ is the error function of the rotation matrix. This iniquity would suggest that the system (3) is *passive* from the input u_{ce} to the output ν. Based on this property, stability and L_2-gain performance analysis can be discussed with the energy function V.

3 Conclusions

The main contribution of this paper is that the interpretation of our proposed strategy has been given based on a geometric framework. By using the homogeneous representation and the adjoint transformation, we have derived the relative rigid body motion dynamics between the moving target object and the camera. Stability and L_2-gain performance analysis for the visual feedback system can be discussed based on passivity with the energy function.

References

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