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An Interpretation of Vision-based Control for Rigid Body Motion: A Geometric Framework

Abstract

This paper investigates vision–based control for the relative rigid body motion (positions and rotations) using the adjoint transformation. Firstly the relation between the rigid body velocity and the adjoint transformation is stated. Secondly the model of the relative rigid body motion and the nonlinear observer with the adjoint transformation are considered in order to derive the visual feedback system. The property of estimated error dynamics can be derived using the energy function. Finally stability and L_2-gain performance analysis are discussed based on the property of the visual feedback system which is similar to passivity.

1 Introduction

Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision system to control the position of the robot end-effector in an efficient manner. The combination of mechanical control with visual information, so-called *visual feedback control* or *visual servo*, should become extremely important, when we consider a mechanical system working under *dynamical* environments [1, 2]. Recently, autonomous injection of biological cells has been discussed using visual feedback control [3] and fields which need visual feedback control are increasing.

This paper deals with the relative rigid body motion control of a moving target object with respect to the camera frame. This control problem is standard and important, and has gained much attention of researchers for many years. Kelly et al. [4] have considered a simple image-based controller for the 3-D visual feedback system under the assumption that the objects' depths are known. Their controller has guaranteed that the overall closed-loop system is stable by invoking the Lyapunov direct method. Several approaches have been proposed to guarantee global stability [5, 6, 7]. Visual feedback systems of the eye-in-hand configuration typically use three coordinate frames which consist of a world frame, a target object frame and a camera (end-effector) frame. In this control strategy, one of the control objectives is to track the moving target object in a three-dimensional workspace by image information. Hence the model of the relative rigid body transformation, which represents the position and orientation of the target object frame relative to the camera frame, can be described by the nonlinear systems on the group of rigid motions, which shall be denoted as $SE(3)$. The typical example is shown in Fig. 1. Hence the dynamics of the relative rigid body motion is described by the nonlinear systems in a 3-D workspace. Nevertheless, previous works hardly regard the rotation between the camera frame and the object frame as the state of the 3-D visual feedback system.

In this paper, we investigate the rigid body motion (involving both translation and rotation) control problem of vision-based robotic systems. In order to regard the rotation as the state for visual feedback system, we derive the relative rigid body motion dynamics between the moving target object and the camera using the homogeneous representation and the

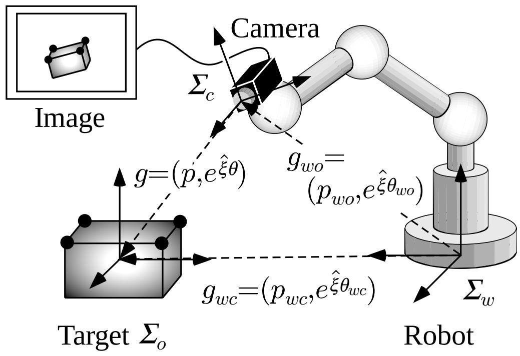

Figure 1: Eye-in-Hand visual feedback system

adjoint transformation. Further, we consider a nonlinear observer for constructing a 3-D visual feedback system. Stability and L_2-gain performance analysis for the full 3-D visual feedback system will be discussed based on passivity with an energy function. The main contribution of this paper is that the interpretation of our proposed strategy has been given based on a geometric framework. This approach continues the research originally presented in [8, 9].

This paper is organized as follows. In Section 2, we consider a model of the relative rigid body motion using the homogeneous representation and the adjoint transformation. Section 3 shows a nonlinear observer which estimates the relative rigid body motion. Stability and L_2-gain performance analysis for the visual feedback system are derived in Section 4. Finally, we offer some conclusions in Section 5.

Let a rotation matrix $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times3}$ represent the change of the principle axes of a frame b relative to a frame a. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. Here $\hat{\xi}\theta_{ab}$ denotes $\hat{\xi}_{ab}\theta_{ab}$ for simplicity of notation. The operator '$\wedge$' (wedge) will be defined later. Then, $e^{\hat{\xi}\theta_{ab}}$ is known to become orthogonal with unit determinant. Such a matrix belongs to a Lie group of dimension three, called $SO(3) = \{e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times3} | e^{\hat{\xi}\theta_{ab}} e^{-\hat{\xi}\theta_{ab}} = e^{-\hat{\xi}\theta_{ab}} e^{\hat{\xi}\theta_{ab}} = I, \det(e^{\hat{\xi}\theta_{ab}}) = +1\}$. The configuration space of the rigid body motion is the product space of $\mathcal{R}^3$ with $SO(3)$, which should be denoted as $SE(3)$ throughout this paper (see, e.g. [10]). We use the 4×4 matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$.

2 Relative Rigid Body Motion Model

We consider the eye-in-hand system [1] depicted in Fig. 1, where the coordinate frame Σ_w represents the world frame, Σ_c represents the camera (end-effector) frame, and Σ_o represents the object frame, respectively. Let $p_{co} \in \mathcal{R}^3$ and $e^{\hat{\xi}\theta_{co}} \in \mathcal{R}^{3\times3}$ denote the position vector and the rotation matrix from the camera frame Σ_c to the object frame Σ_o. Then, the relative rigid body motion from Σ_c to Σ_o can be represented by $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}}) \in SE(3)$. Similarly, we will define the rigid body motion $g_{wc} = (p_{wc}, e^{\hat{\xi}\theta_{wc}})$ from Σ_w to Σ_c, and $g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}})$ from Σ_w to Σ_o, respectively, as in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion

$g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ to a given reference $g_d = (p_d, e^{\hat{\xi}\theta_d})$ (see, e.g. [1]). Our goal is to determine the camera's motion via the visual information for this purpose. The reference $g_d = (p_d, e^{\hat{\xi}\theta_d})$ for the rigid motion $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ is assumed to be constant in the paper.

In this subsection, let us derive a model of the relative rigid body motion. The rigid body motion $g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}})$ of the target object, relative to the world frame Σ_w, is given by

$$g_{wo} = g_{wc} g_{co} \tag{1}$$

which is a direct consequence of a transformation of the coordinates in Fig. 1. These coordinate transformations can be found in [10] (Chap. 2, eq. (2.24)). Using the notation g_{ab}^{-1} as inverse of g_{ab}, the rigid motion (1) can be rewritten as

$$g_{co} = g_{wc}^{-1} g_{wo} \tag{2}$$

where g_{ab}^{-1} is determined by straightforward matrix inversion to be

$$g_{ab}^{-1} = \begin{bmatrix} e^{-\hat{\xi}\theta_{ab}} & -e^{-\hat{\xi}\theta_{ab}} p_{ab} \\ 0 & 1 \end{bmatrix} \in SE(3).$$

The dynamic model of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [10]. Now, let us denote the body velocity of the camera relative to the world frame Σ_w as

$$\hat{V}_{wc}^b := g_{wc}^{-1} \dot{g}_{wc} = \begin{bmatrix} \hat{\omega}_{wc} & v_{wc} \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4\times 4}, \quad V_{wc}^b = \begin{bmatrix} v_{wc} \\ \omega_{wc} \end{bmatrix} \in \mathcal{R}^6. \tag{3}$$

where v_{wc} and ω_{wc} denotes the velocity of the origin and the instantaneous body angular velocities from Σ_w to Σ_c, respectively([10] Chap. 2, eq. (2.49)). Here the operator '$\wedge$' (wedge), from $\mathcal{R}^3$ to the set of 3×3 skew-symmetric matrices $so(3)$, is defined as

$$\hat{a} = (a)^\wedge := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

The operator '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathcal{R}^3$. Recall that a skew-symmetric matrix corresponds to an axis of rotation (via the mapping $a \mapsto \hat{a}$).

Further, the body velocity of the target object relative to Σ_w should be represented as

$$\hat{V}_{wo}^b := g_{wo}^{-1} \dot{g}_{wo} = \begin{bmatrix} \hat{\omega}_{wo} & v_{wo} \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4\times 4}, \quad V_{wo}^b = \begin{bmatrix} v_{wo} \\ \omega_{wo} \end{bmatrix} \in \mathcal{R}^6 \tag{4}$$

where v_{wo} and ω_{wo} denotes the velocity of the origin and the instantaneous body angular velocities from Σ_w to Σ_o, respectively.

Differentiating (2) with respect to time, we have

$$\dot{g} = \dot{g}_{wc}^{-1} g_{wo} + g_{wc}^{-1} \dot{g}_{wo} = -g_{wc}^{-1} \dot{g}_{wc} g_{wc}^{-1} g_{wo} + g_{wc}^{-1} g_{wo} g_{wo}^{-1} \dot{g}_{wo}. \tag{5}$$

By substituting (3) and (4) into the above equation, we can obtain

$$\dot{g} = -\hat{V}_{wc}^b g + g \hat{V}_{wo}^b \tag{6}$$

Here $g = (p, e^{\hat{\xi}\theta})$ denotes $g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$ for short. We multiply (6) by g^{-1} from left side to obtain

$$g^{-1}\dot{g} = -g^{-1}\hat{V}^b_{wc}g + \hat{V}^b_{wo}. \tag{7}$$

In order to derive the model of relative rigid body motion, the adjoint transformation[10] can be introduced. The adjoint transformation associated with g_{ab}, written $\mathrm{Ad}_{g_{ab}}$, is given as

$$\mathrm{Ad}_{g_{ab}} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & \hat{p}_{ab}e^{\hat{\xi}\theta_{ab}} \\ 0 & e^{\hat{\xi}\theta_{ab}} \end{bmatrix}. \tag{8}$$

The following property concerning the adjoint transformation is important for the rigid body motion. If $\hat{V}' = g_{ab}\hat{V}g_{ab}^{-1}$, then

$$V' = \mathrm{Ad}_{g_{ab}}V \tag{9}$$

holds([10], Chap. 2, pp. 60, eq. (2.64)). Using the above property, eq. (7) can be rewritten as

$$V^b = -\mathrm{Ad}_{(g^{-1})}V^b_{wc} + V^b_{wo}. \tag{10}$$

Eq. (10) should be the model of the relative rigid body motion(RRBM). Fig. 2 shows the block diagram of relative rigid body motion.

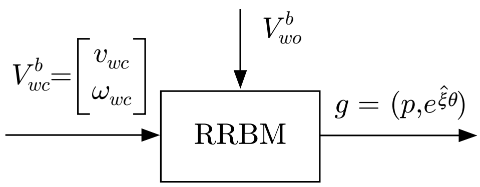

Figure 2: Block diagram of relative rigid body motion.

Next, we derive a model of a pinhole camera as shown in Fig. 3. Let λ be a focal length, $p_{oi} \in \mathcal{R}^3$ and $p_{ci} \in \mathcal{R}^3$ be coordinates of the target object's i-th feature point relative to Σ_o and Σ_c, respectively. Using a transformation of the coordinates, we have

$$p_{ci} = gp_{oi}, \tag{11}$$

where p_{ci} and p_{oi} should be regarded as $[p_{ci}^T\ 1]^T$ and $[p_{oi}^T\ 1]^T$, respectively. These representations, i.e. a point vector is appended 1, are called the homogeneous coordinates of the point p_{ab}. In case of using the homogeneous transformation, we will treat p_{ab} as $[p_{ab}^T\ 1]^T$ without confusion.

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate f_i as follows.

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{12}$$

where $p_{ci} := [x_{ci}\ y_{ci}\ z_{ci}]^T$. It is straightforward to extend this model to the m image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T\ \cdots\ f_m^T]^T \in$

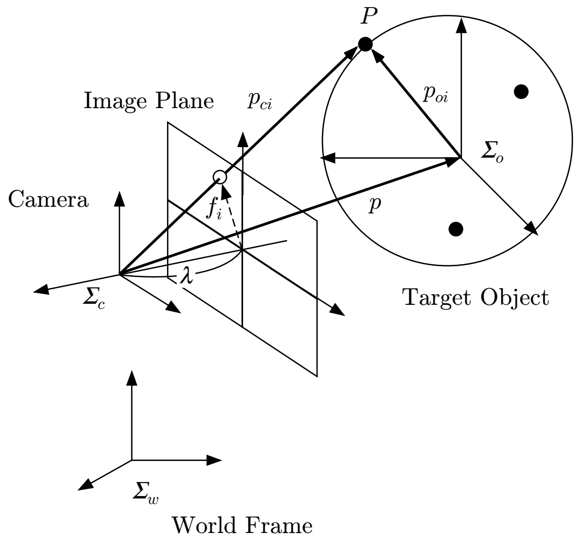

Figure 3: Pinhole camera

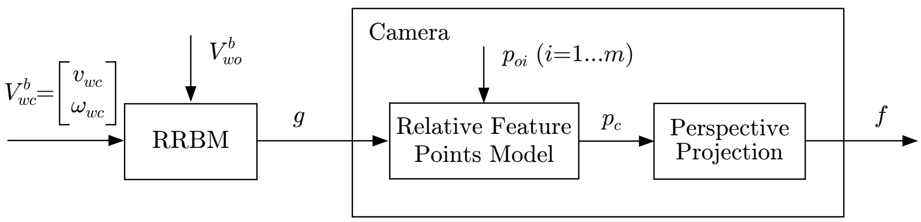

Figure 4: Block diagram of camera model with RRBM.

$\mathcal{R}^{2m}$. Fig. 4 depicts a block diagram of the camera model with the relative rigid body motion. Unfortunately the relative rigid body motion g can not be obtained directly in the visual feedback system, the visual information f which includes the relative rigid body motion can be exploited.

3 Nonlinear Observer

The visual feedback control task should require information of the relative rigid body motion g. Since the measurable information is only the image information in the visual feedback systems, we consider a nonlinear observer in order to estimate the relative rigid body motion from the image information.

First, we shall consider the following dynamic model which just comes from the actual relative rigid body motion model (10).

$$\bar{V}^b = -\mathrm{Ad}_{(\bar{g}^{-1})} V^b_{wc} + u_e \tag{13}$$

where $\bar{g} = (\bar{p}, e^{\hat{\bar{\xi}}\bar{\theta}})$ and $\bar{V}^b$ are the estimated value of the relative rigid body motion and the estimated body velocity, respectively. The new input u_e is to be determined in order to converge the estimated value to the actual relative rigid body motion. Because the design of u_e needs a property of the whole visual feedback system, we will propose u_e in Section 4

Similarly to (11) and (12), the estimated image feature point $\bar{f}_i$ $(i = 1, \cdots, m)$ should be described as

$$\bar{p}_{ci} = \bar{g} p_{oi} \tag{14}$$

$$\bar{f}_i = \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix} \tag{15}$$

where $\bar{p}_{ci} := [\bar{x}_{ci}\ \bar{y}_{ci}\ \bar{z}_{ci}]^T$. $\bar{f} := [f_1^T\ \cdots\ f_m^T]^T \in \mathcal{R}^{2m}$ means the m image points case. Fig. 5 shows a block diagram of the model of the estimated relative rigid body motion.

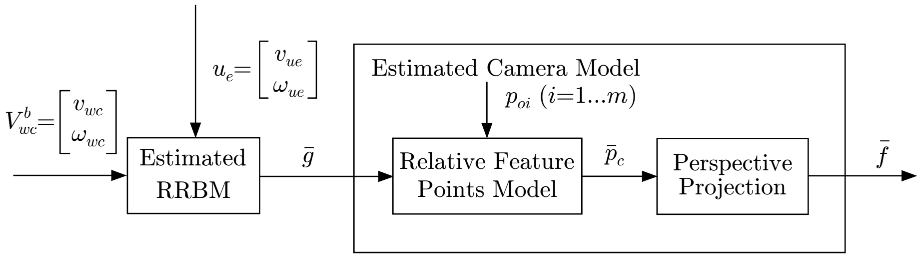

Figure 5: Block diagram of estimated model.

Now, we define the estimation error between the estimated value $\bar{g}$ and the actual relative rigid motion g as

$$g_{ee} = \bar{g}^{-1} g, \tag{16}$$

in other words, $p_{ee} = e^{-\hat{\bar{\xi}}\bar{\theta}}(p - \bar{p})$ and $e^{\hat{\xi}\theta_{ee}} = e^{-\hat{\bar{\xi}}\bar{\theta}} e^{\hat{\xi}\theta}$. Note that $p = \bar{p}$ and $e^{\hat{\xi}\theta} = e^{-\hat{\bar{\xi}}\bar{\theta}}$ iff $g = I$, i.e. $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I$. Let the matrix $\mathrm{sk}(e^{\hat{\xi}\theta})$ denote $\frac{1}{2}(e^{\hat{\xi}\theta} - e^{-\hat{\xi}\theta})$ and let

$$e_R(e^{\hat{\xi}\theta}) := \mathrm{sk}(e^{\hat{\xi}\theta})^\vee \tag{17}$$

represent an error vector of the rotation matrix $e^{\hat{\xi}\theta}$. Using the notation $e_R(e^{\hat{\xi}\theta})$ defined in the above eq. (17), the vector of the estimation error is given by

$$e_e := \begin{bmatrix} p_{ee}^T & e_R^T(e^{\hat{\xi}\theta_{ee}}) \end{bmatrix}^T. \tag{18}$$

Note that $e_e = 0$ iff $p_{ee} = 0$ and $e^{\hat{\xi}\theta_{ee}} = I$.

Next, we will derive an estimation error system. Differentiating (16) with respect to time, we can obtain

$$\dot{g}_{ee} = -(-\bar{g}^{-1}\hat{V}_{wc}^b\bar{g} + \hat{u}_e)g_{ee} + g_{ee}(-g^{-1}\hat{V}_{wc}^b g + \hat{V}_{wo}^b) \tag{19}$$

where detailed derivation can be found in Appendix A. We multiply both sides of (19) by g_{ee}^{-1} to obtain

$$\begin{aligned} g_{ee}^{-1}\dot{g}_{ee} &= -g_{ee}^{-1}(-\bar{g}^{-1}\hat{V}_{wc}^b\bar{g} + \hat{u}_e)g_{ee} + (-g^{-1}\hat{V}_{wc}^b g + \hat{V}_{wo}^b) \\ &= -g_{ee}^{-1}\hat{u}_e g_{ee} + \hat{V}_{wo}^b. \end{aligned} \tag{20}$$

Furthermore, using the property concerning the adjoint transformation described in (9), the above equation can be transformed into the following

$$V_{ee}^b = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \tag{21}$$

Eq. (21) represents the model of the estimation error system. Then the following lemma can be obtained.

Lemma 1 *If the target object is static, i.e.* $V_{wo}^b = 0$*, then the following inequality holds for the system (21).*

$$\int_0^T u_e^T(-e_e)d\tau \geq -\gamma_e^2 \tag{22}$$

where γ_e *is a positive scalar.*

(Proof:) Consider the positive definite function defined as

$$V_e = \frac{1}{2}\|p_{ee}\|^2 + \phi(e^{\hat{\xi}\theta_{ee}}) \tag{23}$$

where ϕ is the error function of the rotation matrix and introduced in [12]. We refer to Appendix B for this error function on $SO(3)$.

From the property of '$\wedge$' (wedge), i.e. '$\wedge$' is the cross product operator and $\hat{a}$ is a 3×3 skew-symmetric matrix, we have $p_{ee}^T \hat{p}_{ee}\omega_{ue} = -p_{ee}^T \hat{\omega}_{ue} p_{ee} = 0$. Using this fact and evaluating the time derivative of V_e gives us

$$\begin{aligned}\dot{V}_e &= p_{ee}^T e^{\hat{\xi}\theta_{ee}} e^{-\hat{\xi}\theta_{ee}} \dot{p}_{ee} + e_R^T(e^{\hat{\xi}\theta_{ee}}) e^{\hat{\xi}\theta_{ee}} \omega_{ee} \\ &= e_e^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} V_{ee}^b = -e_e^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} \mathrm{Ad}_{(g_{ee}^{-1})} u_e \\ &= -e_e^T \mathrm{Ad}_{(-p_{ee})} u_e = u_e^T(-e_e)\end{aligned} \tag{24}$$

Integrating (24) from 0 to T yields

$$\int_0^T u_e^T(-e_e)d\tau = V_e(T) - V_e(0) \geq -\gamma_e^2 \tag{25}$$

This completes the proof. □

Remark 1 In the estimation error system (9), $p_{ee}^T \hat{\omega}_{ue} p_{ee} = 0$ holds. This property is analogous to the one of the robot dynamics, i.e. $x^T(\dot{M} - 2C)x = 0,\ \forall x \in \mathcal{R}^n$ (where $M \in \mathcal{R}^{n\times n}$ is the manipulator inertia matrix and $C \in \mathcal{R}^{n\times n}$ is the Coriolis matrix [10]). Moreover, let us take u_e as the input and e_e as its output. Then, Lemma 1 would suggest that the system (9) is *passive* from the input u_e to the output $-e_e$ just formally as in the definition in [11].

Next, we will derive a relation between the actual image information and the estimated one. Suppose the estimation error is *small* enough that we can let $e^{\hat{\xi}\theta_{ee}} \simeq I + \mathrm{sk}(e^{\hat{\xi}\theta_{ee}})$, then the following relation between the actual feature point p_{ci} and the estimated one $\bar{p}_{ci}$ holds.

$$p_{ci} - \bar{p}_{ci} = \begin{bmatrix} I & -(e^{\hat{\bar{\xi}}\bar{\theta}} p_{oi})^\wedge \end{bmatrix} \begin{bmatrix} e^{\hat{\bar{\xi}}\bar{\theta}} & 0 \\ 0 & e^{\hat{\bar{\xi}}\bar{\theta}} \end{bmatrix} \begin{bmatrix} p_{ee} \\ e_R(e^{\hat{\xi}\theta_{ee}}) \end{bmatrix} \tag{26}$$

where the above equation has been described in more detail in Appendix C.

Using Taylor expansion with the first order approximation, the relation between the actual image information and the estimated one can be derived as

$$f_i - \bar{f}_i = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}). \tag{27}$$

From the above equation, the relation between the actual image information and the estimated one can be given by

$$f - \bar{f} = J(\bar{g}) e_e, \tag{28}$$

where $J(\bar{g}) : SE(3) \to \mathcal{R}^{2m\times 6}$ is defined as

$$J(\bar{g}) := \begin{bmatrix} J_1(\bar{g}) \\ J_2(\bar{g}) \\ \vdots \\ J_m(\bar{g}) \end{bmatrix} \begin{bmatrix} e^{\hat{\bar{\xi}}\bar{\theta}} & 0 \\ 0 & e^{\hat{\bar{\xi}}\bar{\theta}} \end{bmatrix} \tag{29}$$

$$J_i(\bar{g}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & -(e^{\hat{\bar{\xi}}\bar{\theta}} \hat{p}_{oi})^{\wedge} \end{bmatrix}, \quad i = 1, \cdots, m. \tag{30}$$

Note that the matrix $J(\bar{p}, e^{\hat{\bar{\xi}}\bar{\theta}})$ is like as the image Jacobian which plays an important role in many researches of the visual feedback control [1]. We assume that the matrix $J(\bar{g})$ is full column rank for all $\bar{g} \in SE(3)$. Then, the relative rigid body motion can be uniquely defined by the image feature vector. Because this may not hold in some cases when $n = 3$, it is known that $n \geq 4$ is desirable for the full column rank of the image Jacobian [13].

The above discussion shows that we can derive the vector of the estimation error e_e from image information f and the estimated value of the relative rigid body motion $(\bar{p}, e^{\hat{\bar{\xi}}\bar{\theta}})$,

$$e_e = J^{\dagger}(\bar{g})(f - \bar{f}) \tag{31}$$

where † denotes the pseudo-inverse. Therefore the estimation error e_e can be exploited in the 3D visual feedback control law using image information f obtained from the camera. Fig. 6 shows the block diagram of estimation error system.

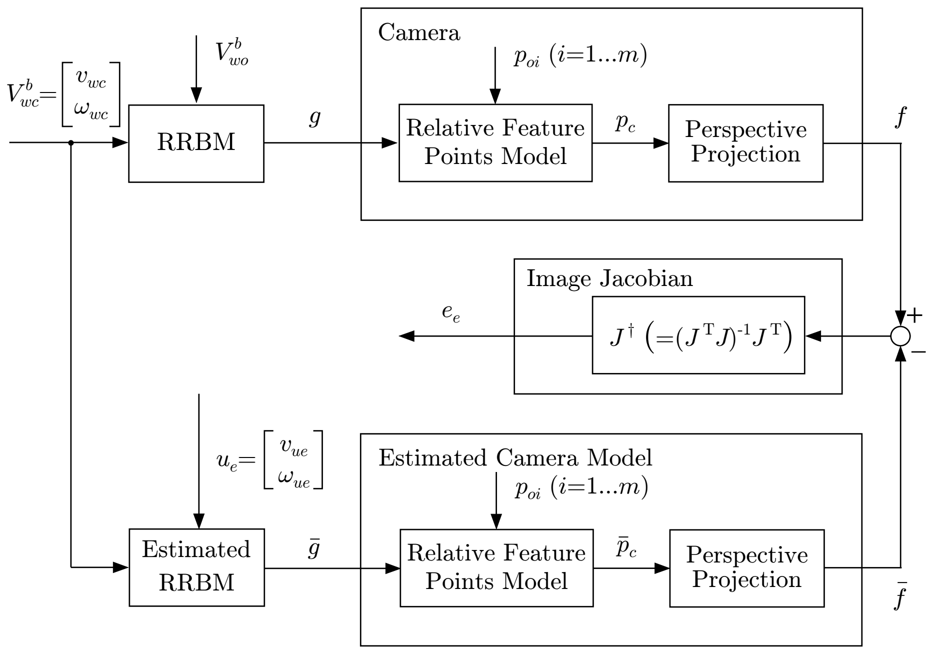

Figure 6: Block diagram of estimation error system.

4 Visual Feedback Control

4.1 Model of Visual Feedback System

In this paper, we rigorously discuss stability and control performance analysis of the visual feedback system with the nonlinear observer. Let us derive a model of the visual feedback system. First, we define the control error as follows.

$$g_{ec} = g_d^{-1}\bar{g}, \tag{32}$$

which represents the error between the estimated value $\bar{g}$ and the reference of the relative rigid body motion g_d. It should be remarked that $p_d = \bar{p}$ and $e^{\hat{\xi}\theta_d} = e^{\hat{\bar{\xi}}\bar{\theta}}$ iff $g_{ec} = I$. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the control error is defined as

$$e_c := \left[\begin{array}{cc} p_{ec}^T & e_R^T(e^{\hat{\xi}\theta_{ec}}) \end{array} \right]^T. \tag{33}$$

Note that $e_c = 0$ iff $p_{ec} = 0$ and $e^{\hat{\xi}\theta_{ec}} = I$.
Similarly to (21), the model of the control error can be obtained as

$$V_{ec}^b = -\mathrm{Ad}_{(\bar{g}^{-1})} V_{wc}^b + u_e. \tag{34}$$

Using (21) and (34), the visual feedback system can be derived as

$$\left[\begin{array}{c} V_{ec}^b \\ V_{ee}^b \end{array} \right] = \left[\begin{array}{cc} -\mathrm{Ad}_{(\bar{g}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{array} \right] u_{ce} + \left[\begin{array}{c} 0 \\ I \end{array} \right] V_{wo}^b \tag{35}$$

where

$$u_{ce} := \left[\begin{array}{cc} u_c^T & u_e^T \end{array} \right]^T \tag{36}$$

denotes the control input. Since the camera velocity V_{wc}^b is considered as an input, the notation u_c will be used instead of V_{wc}^b. Let us define the error vector of the visual feedback system as

$$e := \left[\begin{array}{cc} e_c^T & e_e^T \end{array} \right]^T \tag{37}$$

which contains of the control error vector e_c and the estimation error vector e_e. It should be noted that the actual relative rigid body motion g tends to the reference g_d when $e \to 0$.

4.2 Visual Feedback Control and Stability Analysis

Before deriving the visual feedback control law, we show an important lemma.

Lemma 2 *If $V_{wo}^b = 0$, then the visual feedback system (35) satisfies*

$$\int_0^T u_{ce}^T \nu d\tau \geq -\gamma_{ce}, \quad \forall T > 0 \tag{38}$$

where ν is

$$\nu := \left[\begin{array}{cc} -\mathrm{Ad}_{(g_d^{-1})}^T & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{array} \right] e. \tag{39}$$

(Proof:) Consider the following positive definite function

$$V = \frac{1}{2}\|p_{ec}\|^2 + \phi(e^{\hat{\xi}\theta_{ec}}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(e^{\hat{\xi}\theta_{ee}}) \tag{40}$$

which utilizes the error function ϕ. The positive definiteness of the function V can be given by the property of the error function ϕ. Differentiating (40) with respect to time yields

$$\begin{aligned} \dot{V} &= p_{ec}^T e^{\hat{\xi}\theta_{ec}} e^{-\hat{\xi}\theta_{ec}} \dot{p}_{ec} + e_R^T(e^{\hat{\xi}\theta_{ec}}) e^{\hat{\xi}\theta_{ec}} \omega_{ec} + p_{ee}^T e^{\hat{\xi}\theta_{ee}} e^{-\hat{\xi}\theta_{ee}} \dot{p}_{ee} + e_R^T(e^{\hat{\xi}\theta_{ee}}) e^{\hat{\xi}\theta_{ee}} \omega_{ee} \\ &= \left[\begin{array}{cccc} p_{ec}^T & e_R^T(e^{\hat{\xi}\theta_{ec}}) & p_{ee}^T & e_R^T(e^{\hat{\xi}\theta_{ee}}) \end{array}\right] \left[\begin{array}{cc} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} \end{array}\right] \left[\begin{array}{c} V_{ec}^b \\ V_{ee}^b \end{array}\right]. \end{aligned} \tag{41}$$

Observing that the skew-symmetry of the matrices $\hat{p}_{ec}$ and $\hat{p}_{ee}$, i.e. $p_{ec}^T \hat{p}_{ec} \omega_{wc} = -p_{ec}^T \hat{\omega}_{wc} p_{ec} = 0$ and $p_{ee}^T \hat{p}_{ee} \omega_{wc} = -p_{ee}^T \hat{\omega}_{wc} p_{ee} = 0$, the above equation along the trajectories of the system (35) can be transformed into

$$\begin{aligned} \dot{V} &= e^T \left[\begin{array}{cc} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} \end{array}\right] \left[\begin{array}{cc} -\mathrm{Ad}_{(\bar{g}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{array}\right] u_{ce} \\ &= e^T \left[\begin{array}{cc} -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_d})} \mathrm{Ad}_{(-\bar{p})} & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} \\ 0 & -\mathrm{Ad}_{(-p_{ee})} \end{array}\right] u_{ce} \\ &= e^T \left[\begin{array}{cc} -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_d})} \mathrm{Ad}_{(-e^{\hat{\xi}\theta_d} p_{ec} - p_d)} & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} \\ 0 & -I \end{array}\right] u_{ce} \\ &= e^T \left[\begin{array}{cc} -\mathrm{Ad}_{(g_d^{-1})} & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} \\ 0 & -I \end{array}\right] u_{ce} = u_{ce}^T \nu \end{aligned} \tag{42}$$

Integrating (42) from 0 to T, we can obtain

$$\int_0^T u_{ce}^T \nu d\tau = V(T) - V(0) \geq -\gamma_{ce}. \tag{43}$$

This completes the proof. □

The block diagram of the visual feedback system is shown in Fig. 7. It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

$$u_{ce} = -\left[\begin{array}{cc} K_c & 0 \\ 0 & K_e \end{array}\right] \nu \tag{44}$$

where K_c and K_e are 6×6 positive definite matrices called the control gain and the estimation gain, respectively. The result with respect to exponential stability of the proposed control input (44) can be established as follows.

Theorem 1 *If $V_{wo} = 0$ and the initial state $e(0)$ belongs to $\{e | V(e(0)) < 1\}$, then the equilibrium point $e = 0$ for the closed-loop system (35) and (44) is exponentially stable.*

(Proof:) In the proof of Lemma 2, we have already derived that the time derivative of V along the trajectory of the system (35) is formulated as (42). Using the control input (44), eq. (42) can be transformed into

$$\dot{V} = -e^T K e \leq -\beta_k \|e\|^2 \tag{45}$$

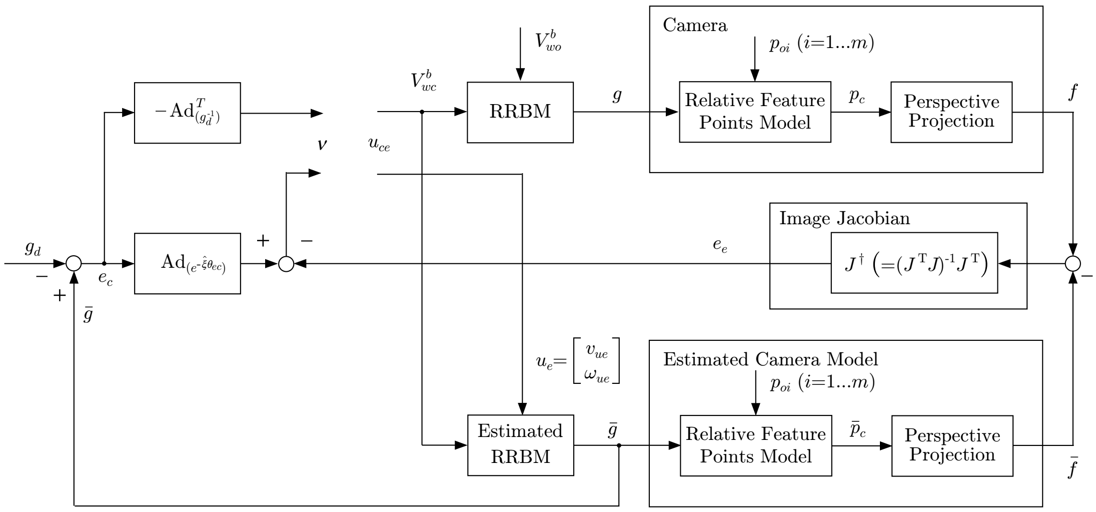

Figure 7: Block diagram of the visual feedback system

where K is defined as

$$K := \begin{bmatrix} \mathrm{Ad}_{(g_d^{-1})} & -\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} \\ 0 & I \end{bmatrix} \begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} \mathrm{Ad}^T_{(g_d^{-1})} & 0 \\ -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & I \end{bmatrix}.$$

Because K is a positive definite matrix and $e^{\hat{\xi}\theta_{ec}} \in SO(3)$, there exists a bounded positive scalar β_k which does not depend on the state e. β_k is given by

$$\beta_k = \sigma_{max}(K) \tag{46}$$

where σ_{max} denotes the maximum singular value. Since V is a non-increasing function and $V(0) < 1$, the function $V(t)$ verifies

$$\beta_2 \|e\|^2 \leq V(t) \leq \beta_1 \|e\|^2 \tag{47}$$

for some bounded constants $\beta_1 \geq \beta_2 > 0$. From (45) and (47), we have

$$\frac{\dot{V}}{V} \leq -\frac{\beta_k \|e\|^2}{\beta_1 \|e\|^2} := -\zeta \tag{48}$$

where $\zeta > 0$ is a bounded constant. Integrating both sides of (48) yields

$$\ln \frac{V(t)}{V(0)} \leq -\zeta t \tag{49}$$

which leads to the inequality

$$V(t) \leq V(0) e^{-\zeta t}. \tag{50}$$

This completes the proof. □

Remark 2 Lemma 2 can be interpreted as follows: The visual feedback system (35) is *passive* from the input u to the output ν just formally as in the definition in [11].

4.3 L_2-Gain Performance Analysis

In this section, we consider L_2-gain performance analysis of the visual feedback system. Now, let us define

$$P := K - \frac{1}{2}\begin{bmatrix} I & 0 \\ 0 & \left(1+\frac{1}{\gamma^2}\right) I \end{bmatrix} \tag{51}$$

where $\gamma \in \mathcal{R}$ is positive. Then we have the following theorem.

Theorem 2 *Given a positive scalar γ and consider the control input (44) with the gains K_c and K_e such that the matrix P is positive semi-definite, then the closed-loop system (35) and (44) has L_2-gain $\leq \gamma$.*

(Proof:) Differentiating the positive definite function V defined in (40) along the trajectory of the closed-loop system and completing the squares yields

$$\begin{aligned} \dot{V} &= -e^T K e + e_e^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} V_{wo}^b \\ &= \frac{\gamma^2}{2}\|V_{wo}^b\|^2 - \frac{1}{2}\|e\|^2 - \frac{\gamma^2}{2}\left\|V_{wo}^b - \frac{1}{\gamma^2}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} e_e\right\|^2 \\ &\quad -e^T K e + \frac{1}{2\gamma^2} e^T \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} e + \frac{1}{2}\|e\|^2. \end{aligned} \tag{52}$$

Then the velocity of the target object (in the worst case) should be derived as

$$V_{wo}^b = \frac{1}{\gamma^2}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} e_e. \tag{53}$$

Hence for any V_{wo}^b it can be verified that the inequality

$$\dot{V} + \frac{1}{2}\|e\|^2 - \frac{\gamma^2}{2}\|V_{wo}^b\|^2 \leq -e^T P e \leq 0 \tag{54}$$

holds if P is positive semi-definite. Integrating (54) from 0 to T and noticing $V(T) \geq 0$, we have

$$\int_0^T \|e\|^2 dt \leq \gamma^2 \int_0^T \|V_{wo}^b\|^2 dt + 2V(0), \ \forall T > 0. \tag{55}$$

This completes the proof. □

The positive definite function V plays the role of the storage function for L_2-gain performance analysis.

5 Conclusions

This paper investigates the rigid body motion (involving both translation and rotation) control problem of vision-based robotic systems. The main contribution of this paper is that the interpretation of our proposed strategy has been given based on a geometric framework. By using the homogeneous representation and the adjoint transformation, we have derived the relative rigid body motion dynamics between the moving target object and the camera. The nonlinear observer has been proposed in order to derive the visual feedback system.

Stability and L_2-gain performance analysis for the full 3-D visual feedback system have been discussed based on passivity with the energy function.

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Appendix

A Model of Estimation Error System

We shall derive the model of the estimation error system presented in Section 3. The property of the adjoin transformation, i.e. $gg^{-1} = I$, gives us

$$\begin{aligned} \dot{g}_{ee} &= \dot{\bar{g}}^{-1} g + \bar{g}^{-1}\dot{g} = -\bar{g}^{-1}\dot{\bar{g}}\bar{g}^{-1} g + \bar{g}^{-1} g g^{-1}\dot{g} \\ &= -\bar{g}^{-1}\dot{\bar{g}} g_{ee} + g_{ee} g^{-1}\dot{g}. \end{aligned} \tag{56}$$

Using the property concerning the adjoint transformation (8), the estimated relative rigid body motion (13) can be transformed into

$$\hat{\hat{V}}^b = \bar{g}^{-1}\dot{g} = -(\bar{g}^{-1})\hat{V}^b_{wc}\bar{g} + \hat{u}_e. \tag{57}$$

By substituting (7) and (57) into (56), we can obtain

$$\dot{g}_{ee} = -(-\bar{g}^{-1}\hat{V}^b_{wc}\bar{g} + \hat{u}_e)g_{ee} + g_{ee}(-g^{-1}\hat{V}^b_{wc} g + \hat{V}^b_{wo}). \tag{58}$$

B Error Function on $SO(3)$

Let us introduce the notation of the error function.

$$\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta}), \tag{59}$$

and, for any 3×3 matrix A, $\mathrm{sk}(A) := \frac{1}{2}(A - A^T)$. The error function ϕ has the following properties.

Property 1 Let $e^{\hat{\xi}\theta} \in SO(3)$. The following properties hold.

1. $\phi(e^{\hat{\xi}\theta}) = \phi(e^{-\hat{\xi}\theta}) \geq 0$ and $\phi(e^{\hat{\xi}\theta}) = 0$ if and only if $e^{\hat{\xi}\theta} = I$.
2. $\dot{\phi}(e^{\hat{\xi}\theta}) = e_R^T(e^{\hat{\xi}\theta})\omega = e_R^T(e^{\hat{\xi}\theta})e^{\hat{\xi}\theta}\omega$, where $e_R(e^{\hat{\xi}\theta}) := \mathrm{sk}(e^{\hat{\xi}\theta})^\vee$.

These properties are proved in [12].

C Relation between Feature Point and Estimated one

The relation between the feature point and the estimated one will be derived. The feature point can be rewritten as

$$\begin{aligned} p_{ci} - \bar{p}_{ci} &= (p - \bar{p}) + (e^{\hat{\xi}\theta} - e^{\hat{\bar{\xi}}\bar{\theta}})p_{oi} \\ &= e^{\hat{\bar{\xi}}\bar{\theta}}p_{ee} + e^{\hat{\bar{\xi}}\bar{\theta}}(e^{-\hat{\bar{\xi}}\bar{\theta}}e^{\hat{\xi}\theta} - I)p_{oi}. \end{aligned} \tag{60}$$

Suppose the estimation error is *small* enough that we can let $e^{\hat{\xi}\theta_{ee}} \simeq I + \mathrm{sk}(e^{\hat{\xi}\theta_{ee}})$ (which is derived from *Rodrigues' formula* in [10]), then the equation (60) becomes

$$\begin{aligned} p_{ci} - \bar{p}_{ci} &= e^{\hat{\bar{\xi}}\bar{\theta}}p_{ee} + e^{\hat{\bar{\xi}}\bar{\theta}}\mathrm{sk}(e^{\hat{\xi}\theta_{ee}})p_{oi} = e^{\hat{\bar{\xi}}\bar{\theta}}p_{ee} + (e^{\hat{\bar{\xi}}\bar{\theta}}\mathrm{sk}(e^{\hat{\xi}\theta_{ee}})e^{-\hat{\bar{\xi}}\bar{\theta}})e^{\hat{\bar{\xi}}\bar{\theta}}p_{oi} \\ &= e^{\hat{\bar{\xi}}\bar{\theta}}p_{ee} + (e^{\hat{\bar{\xi}}\bar{\theta}}\mathrm{sk}(e^{\hat{\xi}\theta_{ee}})^\vee)^\wedge e^{\hat{\bar{\xi}}\bar{\theta}}p_{oi}. \end{aligned} \tag{61}$$

It should be noted that $\hat{a}b = -\hat{b}a$, $\forall a, b \in \mathcal{R}^3$ and $e_R(e^{\hat{\xi}\theta}) = \mathrm{sk}(e^{\hat{\xi}\theta})^\vee, \forall e^{\hat{\xi}\theta} \in SO(3)$ hold, then we have

$$\begin{aligned} p_{ci} - \bar{p}_{ci} &= e^{\hat{\bar{\xi}}\bar{\theta}}p_{ee} - (e^{\hat{\bar{\xi}}\bar{\theta}}p_{oi})^\wedge(e^{\hat{\bar{\xi}}\bar{\theta}}e_R(e^{\hat{\xi}\theta_{ee}})) \\ &= \left[\begin{array}{cc} I & -(e^{\hat{\bar{\xi}}\bar{\theta}}p_{oi})^\wedge \end{array}\right]\left[\begin{array}{cc} e^{\hat{\bar{\xi}}\bar{\theta}} & 0 \\ 0 & e^{\hat{\bar{\xi}}\bar{\theta}} \end{array}\right]\left[\begin{array}{c} p_{ee} \\ e_R(e^{\hat{\xi}\theta_{ee}}) \end{array}\right] \end{aligned} \tag{62}$$

Hence, (26) holds.

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