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Observer Based Dynamic Visual Servoing of Nonlinear Robotic Systems : Stability and L_2-gain Performance Analysis

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Abstract. This paper investigates a vision-based robot motion control using position measurements and visual information. Firstly the model of relative rigid body motion (positions and rotations) and the method for estimation of the relative rigid body motion are presented in order to derive the visual feedback system. Next, we consider the velocity observer and derive the dynamic visual feedback system which contains the manipulator dynamics. Finally the main results with respect to stability and L_2-gain performance analysis for the proposed dynamic visual feedback control are discussed.

1 Introduction

Vision-based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision system to control the position of the robot end-effector in an efficient manner [1]. The combination of mechanical control with visual information, so-called *visual feedback control* or *visual servoing*, should become extremely important, when a mechanical system is operating in an unstructured environment. In this approach, the control objective is to track the target object in a three-dimensional workspace by using image information. An interesting historical review of the visual servoing can be found in [1].

This paper deals with a robot motion control with visual information in the eye-in-hand configuration as depicted in Figure 1. Classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop [2][3]. This assumption is invalid for high speed tasks, while it holds for kinematic control problems. Kelly *et al.* [4] have considered a simple image-based controller which guaranteed local asymptotic stability under an assumption that objects' depths are known. In recent papers [5][6], the authors have proposed the 3-D visual feedback control which has guaranteed local asymptotic stability without the known objects' depths from the theoretical standpoint. While several researches [8][9] have pointed out a problem of global stability, local asymptotic stability is very important in case of the image feature points exist in the neighborhood of the equilibrium point. However, performance measures for the visual feedback systems have not been obtained in the previous works. Since one of the control objective is to track the moving target object, the tracking performance measures are important for the visual feedback systems. In the area of vision and control, it has been expected to investigate not only stability but performance analyses based on theoretical approaches.

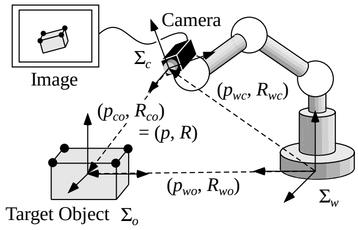

Figure 1: Eye-in-hand visual feedback system.

In this paper, we propose the vision-based control of robots using position measurements only and discuss stability and L_2-gain performance analysis for the visual feedback system from the theoretical standpoint. The key idea of our proposed method is based on a structural passivity-like property of the visual feedback system. Our previous research [6] has proposed state feedback control which guaranteed local stability and L_2-gain performance analysis. This work is a continuation of our previous researches [5]–[7].

This paper is organized as follows. In Section 2, we consider a model of the relative rigid body motion. Section 3 presents a method for the estimation of the relative rigid body motion and leads the visual feedback system. Stability and L_2-gain performance analysis for the visual feedback system considering the proposed velocity observer are discussed in Section 4. Finally, we offer some conclusions in Section 5.

Throughout thie paper, A_m and A_M represent the minimum and maximum eigenvalue of $A(x)$, respectively. The norm of a vector x and a matrix A are defined as $\|x\| = \sqrt{x^T x}$ and $\|A\| = \sqrt{\lambda_{max}(A^T A)}$, respectively. λ_{max} denotes the maximum eigenvalue.

2 Relative Rigid Body Motion

Let us consider the eye-in-hand system [1] depicted in Figure 1, where the coordinate frame Σ_w represents the world frame, Σ_c represents the camera (end-effector) frame, and Σ_o represents the object frame, respectively. Let $p_{co} \in \mathcal{R}^3$ and $R_{co} \in \mathcal{R}^{3\times 3}$ denote the position vector and the rotation matrix from the camera frame Σ_c to the object frame Σ_o. Then, the relative rigid body motion from Σ_c to Σ_o can be represented by $(p_{co}, R_{co}) \in SE(3)$. Similarly, we will define the rigid body motion (p_{wc}, R_{wc}) from Σ_w to Σ_c, and (p_{wo}, R_{wo}) from Σ_w to Σ_o, respectively, as in Figure 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion (p_{co}, R_{co}) to a given reference (p_d, R_d) (see, e.g. [1]). The reference (p_d, R_d) for the rigid motion (p_{co}, R_{co}) is assumed to be constant in this paper.

In this section, let us derive a model of the relative rigid body motion. The rigid body motion (p_{wo}, R_{wo}) of the target object, relative to the world frame Σ_w, is given by

$$p_{wo} = p_{wc} + R_{wc}p_{co} \tag{1}$$
$$R_{wo} = R_{wc}R_{co} \tag{2}$$

which is a direct consequence of a transformation of the coordinates [11] in Figure 1. Using the property of a rotation matrix, i.e. $R^{-1} = R^T$, the rigid body motion (1) and (2) is now rewritten as

$$p_{co} = R_{wc}^T(p_{wo} - p_{wc}) \tag{3}$$
$$R_{co} = R_{wc}^T R_{wo}. \tag{4}$$

The dynamic model of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body [11]. Let $\hat{\omega}_{wc}$ and $\hat{\omega}_{wo}$ denote the instantaneous body angular velocities from Σ_w to Σ_c, and from Σ_w to Σ_o, respectively. Here the operator '$\wedge$' (wedge), from $\mathcal{R}^3$ to the set of 3×3 skew-symmetric matrices $so(3)$, is defined as

$$\hat{a} = (a)^\wedge := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

The operator '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e. $so(3) \to \mathcal{R}^3$. With these, it is possible to specify the velocities of each rigid body as follows [11](Chap.2, (2.55)).

$$\dot{p}_{wc} = R_{wc}v_{wc}, \quad \dot{R}_{wc} = R_{wc}\hat{\omega}_{wc} \tag{5}$$
$$\dot{p}_{wo} = R_{wo}v_{wo}, \quad \dot{R}_{wo} = R_{wo}\hat{\omega}_{wo}. \tag{6}$$

Differentiating (3) and (4) with respect to time, we can obtain

$$\dot{p}_{co} = -v_{wc} + \hat{p}_{co}\omega_{wc} + R_{co}v_{wo} \tag{7}$$
$$\dot{R}_{co} = -\hat{\omega}_{wc}R_{co} + R_{co}\hat{\omega}_{wo}. \tag{8}$$

Now, let us denote the body velocity of the camera relative to the world frame Σ_w as $V_{wc} := [v_{wc}^T \ \omega_{wc}^T]^T$. Further, the body velocity of the target object relative to Σ_w should be denoted as $V_{wo} := [v_{wo}^T \ \omega_{wo}^T]^T$.

Then we can rearrange (7) and (8) in a matrix form as follows
(Relative Rigid Body Motion : RRBM)

$$\begin{bmatrix} \dot{p} \\ (\dot{R}R^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \end{bmatrix} V_{wc} + \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} V_{wo}. \tag{9}$$

Here (p, R) denotes (p_{co}, R_{co}) for short. Equation (9) should be the model of the relative rigid body motion [5].

3 Visual Feedback System

3.1 Estimation of Relative Rigid Body Motion

The visual feedback control task should require the information of the relative rigid body motion (p, R). However, the available information that can be measured in the visual feedback systems is only image information. Hence, let us consider a nonlinear observer which will estimate the relative rigid body motion via image information.

We shall consider the following dynamic model which just comes from the actual relative rigid body motion (9).

$$\begin{bmatrix} \dot{\bar{p}} \\ (\dot{\bar{R}}\bar{R}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} V_{wc} + \begin{bmatrix} I & 0 \\ 0 & \bar{R} \end{bmatrix} u_e \tag{10}$$

where $(\bar{p}, \bar{R})$ is the estimated value of the relative rigid body motion, and a new input u_e for the estimation is to be determined in order to converge the estimated value to the actual relative rigid body motion.

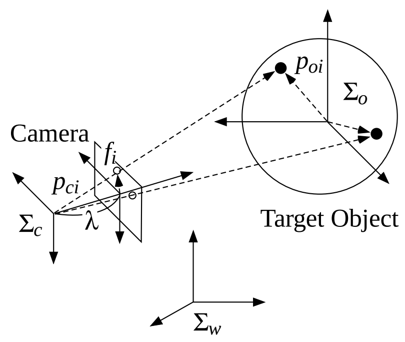

Figure 2: Pinhole camera model.

Next let us derive a pinhole camera model as shown in Figure 2. Let λ be a focal length. Let p_{oi} and p_{ci} be coordinates of the target object's i-th feature point relative to Σ_o and Σ_c, respectively. From a transformation of the coordinates, we have

$$p_{ci} = p + Rp_{oi}. \tag{11}$$

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate as follows

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{12}$$

where $p_{ci} := [x_{ci}\ y_{ci}\ z_{ci}]^T$. It is straightforward to extend this model to the n feature points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T \ \cdots \ f_n^T]^T = \pi(p, R) \in \mathcal{R}^{2n}$.

Now, we define the estimation error between the estimated value $(\bar{p}, \bar{R})$ and the actual relative rigid motion (p, R) as

$$(p_{ee}, R_{ee}) := (p - \bar{p}, \bar{R}^T R). \tag{13}$$

Note that, if $p = \bar{p}$ and $R = \bar{R}$, then it follows $p_{ee} = 0$ and $R_{ee} = I$. Let the matrix $\mathrm{sk}(R)$ denote $\frac{1}{2}(R - R^T)$ and let $e_R(R) := \mathrm{sk}(R)^{\vee}$ represent the error vector of the rotation matrix R. Then the vector of the estimation error is given by $e_e := [p_{ee}^T \ e_R^T(R_{ee})]^T$. Note that $e_e = 0$ holds when $p_{ee} = 0$ and $R_{ee} = I$.

Next, we will derive the measurement equation from (11) and (12). Suppose the estimation error is *small* enough that we can let $R_{ee} \simeq I + \mathrm{sk}(R_{ee})$, then (11) becomes

$$p_{ci} = \bar{p}_{ci} - \bar{R}\hat{p}_{oi} e_R(R_{ee}) + p_{ee} \tag{14}$$

where $\bar{p}_{ci} := \bar{p} + \bar{R}p_{oi}$. Using Taylor expansion, (12) can be written as

$$f_i = \bar{f}_i + \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}) \tag{15}$$

where $\bar{p}_{ci} = [\bar{x}_{ci} \ \bar{y}_{ci} \ \bar{z}_{ci}]^T$ and $\bar{f}_i := \frac{\lambda}{\bar{z}_{ci}}[\bar{x}_{ci} \ \bar{y}_{ci}]^T$.

An approximation of image information around the estimated value $(\bar{p}, \bar{R})$ is given by

$$f - \bar{f} = J(\bar{p}, \bar{R}) e_e \tag{16}$$

where the matrix $J(\bar{p}, \bar{R})$ is defined as

$$J(\bar{p}, \bar{R}) := \begin{bmatrix} L(\bar{p}, \bar{R}; p_{o1}) \\ L(\bar{p}, \bar{R}; p_{o2}) \\ \vdots \\ L(\bar{p}, \bar{R}; p_{on}) \end{bmatrix}, \quad L(\bar{p}, \bar{R}; p_{oi}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & -\bar{R}\hat{p}_{oi} \end{bmatrix}. \tag{17}$$

Note that the matrix $J(\bar{p}, \bar{R})$ can be interpreted as the image Jacobian. It is known that the appropriate pseudo-inverse of the image Jacobian exists in case of $n \geq 4$ [1].

We consider the state equation of the estimated relative rigid body motion error. Using (9), (10) and (13), the state equation of the estimated RRBM error can be obtained as follows.

$$\begin{bmatrix} \dot{p}_{ee} \\ (\dot{R}_{ee} R_{ee}^T)^{\vee} \end{bmatrix} = \begin{bmatrix} 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} V_{wc} - u_e + R_2 V_{wo} \tag{18}$$

where $R_2 = \mathrm{diag}\{R, R_{ee}\}$.

3.2 Visual Feedback System

Let us derive a model of the visual feedback system. First, we define the relative rigid body motion error which represents the error between the estimated value $(\bar{p}, \bar{R})$ and the reference of the relative rigid body motion (p_d, R_d) as follows.

$$(p_{ec}, R_{ec}) := (\bar{p} - p_d, \bar{R}R_d^T) \tag{19}$$

It should be remarked that $p_d = \bar{p}$ and $R_d = \bar{R}$ iff $p_{ec} = 0$ and $R_{ec} = I$. Using the notation $e_R(R)$, the vector of the RRBM error is defined as $e_c := [p_{ec}^T \ e_R^T(R_{ec})]^T$. Note that $e_c = 0$ iff $p_{ec} = 0$ and $R_{ec} = I$.

From (10) and (19), the state equation of the RRBM error can be given by

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \end{bmatrix} = -B^T(\bar{p})u_c + R_1 u_e, \quad B(a) := \begin{bmatrix} I & 0 \\ \hat{a} & I \end{bmatrix} \quad \forall a \in \mathcal{R}^3 \tag{20}$$

where $R_1 = \mathrm{diag}\{I, \bar{R}\}$. Since the camera velocity V_{wc} is considered as an input, the notation u_c is used instead of V_{wc}.

Using (18) and (20), the state equation of the visual feedback system can be derived as

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} u_r + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo} \tag{21}$$

where $u_r := [u_c^T \ u_e^T]^T$ denotes the input of the visual feedback system.

Let us define the error vector of the visual feedback system as $e := [e_c^T \ e_e^T]^T$ which consists of the RRBM error vector e_c and the estimated RRBM error vector e_e. It should be noted that the actual relative rigid body motion (p, R) tends to the reference (p_d, R_d) when $e \to 0$.

The following control input has been proposed in [5].

$$u_r = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I \end{bmatrix} e \tag{22}$$

where K_c and K_e are 6×6 positive definite matrices. Stability and L_2-gain performance analysis for the visual feedback system have been discussed in [5]. Figure 3 shows a block diagram of the visual feedback system.

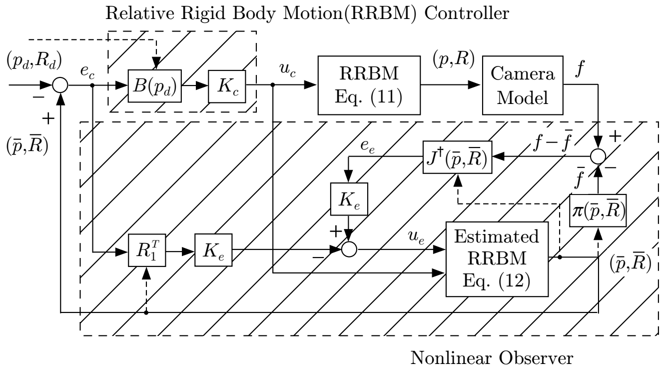

Figure 3: Block diagram of the visual feedback system.

4 Vision-based Robot Control

4.1 Visual Feedback System with Manipulator Dynamics

The manipulator dynamics can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{23}$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angles, velocities and accelerations, respectively. τ is the vector of the input torques.

Since the camera is mounted on the end-effector of the manipulator in the eye-in-hand configuration, the body velocity of the camera V_{wc} is given by $V_{wc} = J_b(q)\dot{q}$. $J_b(q)$ is the manipulator body Jacobian [11]. We assume that the manipulator Jacobian $J_b(q)$ is the nonsingular matrix. Under this assumption, we define the reference of the joint velocities as $\dot{q}_d := J_b^{-1}(q)u_c$. u_c represents an ideal body velocity of the camera.

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as $s_1 := \dot{q} - \dot{q}_d$. Now, we consider the velocity observer approach [10] in order to estimate the joint velocities. Here we propose the control law as the input torques and the velocity observer for the dynamic visual feedback system as follows.

$$\text{Controller} \left\{ \begin{array}{rcl} \tau &=& M(q)\ddot{q}_d + C(q,\dot{q}_0)\dot{q}_d + g(q) + u_s \\ \dot{q}_0 &=& \dot{\bar{q}} - \Lambda\tilde{q} \end{array} \right. \tag{24}$$

$$\begin{array}{c}\text{Velocity}\\ \text{Observer}\end{array} \left\{ \begin{array}{rcl} \dot{\bar{q}} &=& z + L_d\tilde{q} \\ \dot{z} &=& \ddot{q}_d + L_p\tilde{q} \end{array} \right. \tag{25}$$

where $[\bar{q}^T \ z^T]^T$ is the observer state, $\dot{\bar{q}}$ denotes the estimated velocities, $\Lambda = \Lambda^T > 0$, $L_d = l_dI + \Lambda > 0$, $L_p = l_d\Lambda > 0$, $l_d > 0$ is scalar. The new input u_s is to be determined in order to achieve the control objectives. Moreover, $\tilde{q}$ is defined as $\tilde{q} := q - \bar{q}$ which represents the error between the actual joint angles and the estimated joint angles. Now we assume that the desired velocities $\dot{q}_d$ are bounded by V_M, i.e. $V_M = \sup_t \|\dot{q}_d(t)\|$.

Here we consider the error dynamics of the manipulator. Substituting (24) into (23) yields

$$M(q)\dot{s}_1 = -C(q,\dot{q})s_1 - C(q,s_2)\dot{q}_d + u_s \tag{26}$$

where s_2 is defined as $s_2 := \dot{q} - \dot{q}_0 = \dot{\tilde{q}} + \Lambda\tilde{q}$.

The error dynamics of the velocity observer can be written as

$$M(q)\dot{s}_2 = -M(q)l_ds_2 + C(q,s_2-\dot{q})s_1 - C(q,\dot{q})s_2 + u_s. \tag{27}$$

where $\ddot{q}_0 - \ddot{q}_d = l_ds_2$ is used. Using (18), (20), (26) and (27), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows

$$M(q)\dot{s}_1 = -C(q,\dot{q})s_1 - C(q,s_2)\dot{q}_d + u_s \tag{28}$$

$$M(q)\dot{s}_2 = -M(q)l_ds_2 + C(q,s_2-\dot{q})s_1 - C(q,\dot{q})s_2 + u_s \tag{29}$$

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \\ 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} J_b(q)s_1 + \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} u_c \\ u_e \end{bmatrix} + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo}. \tag{30}$$

Equations (28) and (29) represent the manipulator dynamics with the velocity observer. Equation (30) denotes the relative rigid body motion with the nonlinear observer. Figure 4 shows a block diagram of the visual feedback system with the manipulator dynamics.

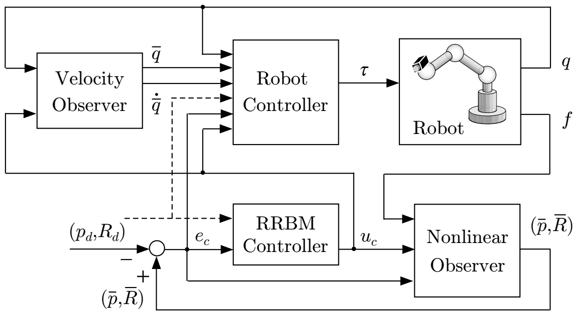

Figure 4: Block diagram of the dynamic visual feedback system.

4.2 Stability of Vision-based Robot Control

Let us define the error vector of the dynamic visual feedback system as $x := \left[s_1^T\ s_2^T\ e_c^T\ e_e^T\right]^T$. Then the dynamic visual feedback control problem can be formulated as follows.

Problem : Find a input vector $u = [u_s^T\ u_c^T\ u_e^T]^T$ such that the closed-loop system satisfies the control objectives as follows:

(Internal stability) If the target object is static, i.e. $V_{wo} = 0$, then the equilibrium point $x = 0$ for the closed-loop system is asymptotically stable.

(L_2-gain performance analysis) The closed-loop system has L_2-gain $\leq \gamma$.

We propose the following dynamic visual feedback control law

$$u_s = -K_s(s_1 - s_2) - J_b^T(q)B(p_d)e_c \tag{31}$$

$$\begin{bmatrix} u_c \\ u_e \end{bmatrix} = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix}\begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I \end{bmatrix} e \tag{32}$$

where K_s, K_c and K_e are 6×6 positive definite matrices. Note that s_1 and s_2 cannot be realized, whereas the difference $s_1 - s_2$ can be obtained from known signals, i.e., $s_1 - s_2 = \dot{\bar{q}} - \Lambda\tilde{q} - \dot{q}_d$.

Now, let us define the following matrices.

$$K_{ce} := \begin{bmatrix} B^T(p_d) & -R_1 \\ 0 & I \end{bmatrix}\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix}\begin{bmatrix} B(p_d) & 0 \\ -R_1^T & I \end{bmatrix} \tag{33}$$

$$K_J(q) := (J_b^T(q)B(p_d))^T(J_b^T(q)B(p_d)). \tag{34}$$

The result with respect to asymptotic stability for the closed-loop system can be established as follows.

Theorem 1 *[7] Suppose that the following conditions hold.*

$$K_{s,m} > C_M V_M, \quad l_d > M_m^{-1}(K_{s,M} + \frac{1}{2} + C_M V_M), \quad K_{ce,m} > \frac{1}{2}K_{J,M}$$

If $V_{wo} = 0$, then the equilibrium point $x = 0$ for the closed-loop system (28)-(32) is asymptotically stable. Moreover, a region of attraction is given by

$$D_1 = \left\{ x \in \mathcal{R}^{24} \;\middle|\; \frac{2l_d M_m - 2K_{s,M} - 1}{2C_M} - V_M > \|s_1\|,\ \frac{K_{s,m}}{C_M} - V_M > \|s_2\| \right\}. \tag{35}$$

4.3 L_2-Gain Performance Analysis

In case the target object is moving, we consider L_2-gain performance analysis as a tracking performance measure for the dynamic visual feedback system.

Theorem 2 *Given a positive scalar γ and suppose that the following conditions hold.*

$$K_{s,m} > C_M V_M + \frac{1}{2},\ l_d > M_m^{-1}(K_{s,M} + 1 + C_M V_M),\ K_{ce,m} > \frac{1}{2}\Big(K_{J,M} + 1 + \frac{1}{\gamma^2}\Big) \tag{36}$$

Then the closed-loop system (28)-(32) has L_2-gain $\leq \gamma$. Moreover, a region of attraction is given by

$$D_2 = \left\{ x \in \mathcal{R}^{24} \;\middle|\; \frac{l_d M_m - K_{s,M} - 1}{C_M} - V_M > \|s_1\|,\ \frac{2K_{s,m} - 1}{2C_M} - V_M > \|s_2\| \right\}. \tag{37}$$

(proof) Consider the following positive definite function

$$V = \frac{1}{2} s_1^T M(q) s_1 + \frac{1}{2} s_2^T M(q) s_2 + \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}). \tag{38}$$

Differentiating the positive definite function V along the trajectory of the closed-loop system and completing the squares yields

$$\begin{aligned} \dot{V} \leq\ & \frac{\gamma^2}{2}\|V_{wo}\|^2 - \frac{\gamma^2}{2}\Big\|V_{wo} - \frac{1}{\gamma^2}\left[0\ R_2^T\right] e\Big\|^2 + \frac{1}{2\gamma^2} e^T \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} e - \big(K_{ce,m} - \frac{1}{2}K_{J,M}\big)\|e\|^2 \\ & - \big(K_{s,m} - C_M(V_M + \|s_2\|)\big)\|s_1\|^2 - \big(l_d M_m - K_{s,M} - \frac{1}{2} - C_M(V_M + \|s_1\|)\big)\|s_2\|^2 \end{aligned} \tag{39}$$

Hence for any V_{wo} it can be verified that the inequality

$$\begin{aligned} \dot{V} + \frac{1}{2}\|x\|^2 - \frac{\gamma^2}{2}\|V_{wo}\|^2 \leq & -\big(K_{s,m} - C_M(V_M + \|s_2\|) - \frac{1}{2}\big)\|s_1\|^2 \\ & -\big(l_d M_m - K_{s,M} - 1 - C_M(V_M + \|s_1\|)\big)\|s_2\|^2 \\ & -\big(K_{ce,m} - \frac{1}{2}K_{J,M} - \frac{1}{2} - \frac{1}{2\gamma^2}\big)\|e\|^2 \leq 0 \end{aligned} \tag{40}$$

holds under the conditions (36). Integrating (40) from 0 to T and noticing $V(x(T)) \geq 0$, we have

$$\int_0^T \|x\|^2 dt \leq \gamma^2 \int_0^T \|V_{wo}\|^2 dt + 2V(x(0)),\ \forall T > 0. \tag{41}$$

This completes the proof.

L_2-gain performance analysis can be regarded as a tracking performance measure for the moving target object. The positive definite function V plays the role of the storage function for L_2-gain performance analysis.

5 Conclusions

This paper has presented the vision-based control of robots using position measurements only. Stability and L_2-gain performance analysis for the dynamic visual feedback system have been discussed. Our proposed method has been based on the structural passivity-like property of the visual feedback system. The nonlinear observer has been employed in order to estimate the relative rigid body motion. Furthermore, we have proposed the velocity observer with the aim of obtaining the joint velocities.

Our proposed passivity approach will enrich the field of visual servoing, although there are some problems. In particular, motion planning based on optimal control for the proposed framework is an important direction for our future work.

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