Visual Feedback Control of Nonlinear Robotics Systems via Stabilizing Receding Horizon Control Approach

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Abstract

This paper investigates a robot motion control with visual information via the nonlinear receding horizon control approach. Firstly the model of the relative rigid body motion and the nonlinear observer are considered in order to derive the visual feedback system. Secondly the stabilizing feedback control law for the closed-loop is discussed as a preparation for our main result. Finally we propose the stabilizing receding horizon control scheme for the 3-D visual feedback control problem by using an appropriate control Lyapunov function as the end point penalty. The proposed scheme employs the cost function as a Lyapunov function for establishing stability.

1 Introduction

Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision system to control the position of the robot end-effector in an efficient manner. The combination of mechanical control with visual information, so-called *visual feedback control* or *visual servo*, should become extremely important, when we consider a mechanical system working under *dynamical* environments. Recent research efforts toward this direction have been nicely collected in [1].

This paper deals with the relative rigid motion control of the target with respect to the camera frame. This control problem is standard and important, and has gained much attention of researchers for many years [1]–[4]. The control objective is to move the end effector of the manipulators in a three-dimensional workspace by visual information. The typical example is shown in Fig. 1. Hence the dynamics of the relative rigid motion is described by the nonlinear systems in a 3-D workspace. However few rigorous results have been obtained in terms of the nonlinear control aspects. For example, there exist no researches that explicitly show the *Lyapunov function* for the 3-D visual feedback systems except [4]–[7].

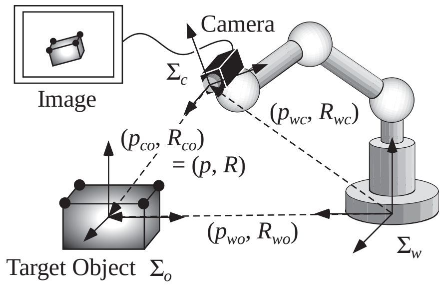

Figure 1: Eye-in-Hand Visual Feedback System

Recently there has been a rapidly growing interest in using receding horizon control, also known as model predictive control, for the nonlinear systems [8]. This interest is partly due to the availability of faster and cheaper computers as efficient numerical algorithms for solving optimization problems. In [9], Ohtsuka *et al.* have applied the nonlinear receding horizon control to obstacle avoidance of a space-vehicle model. While, stability is an overriding requirement and much of literature has suggested different methods to guarantee the closed-loop stability of the receding horizon scheme. In a recent paper by De Nicolao *et al.* [10], the receding horizon controller guarantees closed-loop stability by using a possibly non quadratic end point penalty. Jadbabaie *et al.* [11] have followed the method of De Nicolao *et al.* by using the control Lyapunov function as the end point penalty, and have shown that stability of the receding horizon scheme is guaranteed. Authors [12] have proposed the stabilizing receding horizon control scheme for the visual feedback control of the planar manipulators by using an appropriate control Lyapunov function as the end point penalty.

In this paper, we discuss stability for the 3-D visual feedback control problem based on a nonlinear receding horizon control scheme. The proposed scheme employs the cost function as a Lyapunov function for establishing stability. In order to derive the full 3-D visual feedback system, we will consider a relative rigid body motion dynamics and a nonlinear observer. A control Lyapunov function and a corresponding control law for the 3-D visual feedback system play a crucial role for the proposed scheme. This research continues the works presented in [6] which has investigated the full 3-D visual feedback control, and [12] which has proposed the stabilizing receding horizon control law for the planar motion types.

This paper is organized as follows. In Section 2, we consider the relative rigid body motion model. Section 3 derives the nonlinear observer which estimates the relative rigid body motion. Section 4 considers the 3-D visual feedback control as a preparation for our main result. In Section 5, we propose the stabilizing receding horizon control scheme for the visual feedback system by using an appropriate control Lyapunov function as the end point penalty. Finally the conclusion is shown in Section 6.

2 Relative Rigid Body Motion Model

Let us consider the eye-in-hand system [1] depicted in Fig. 1, where the coordinate frame Σ_w represents the world frame, Σ_c represents the camera (end-effector) frame, and Σ_o represents the object frame, respectively. Let $p_{co} \in \mathbb{R}^3$ and $R_{co} \in \mathbb{R}^{3\times 3}$ denote the position vector and the rotation matrix from the camera frame Σ_c to the object frame Σ_o. Then, the relative rigid body motion from Σ_c to Σ_o can be represented by $(p_{co}, R_{co}) \in SE(3)$. Similarly, we will define the rigid body motion (p_{wc}, R_{wc}) from Σ_w to Σ_c, and (p_{wo}, R_{wo}) from Σ_w to Σ_o, respectively, as in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion (p_{co}, R_{co}) to a given reference (p_d, R_d). Our goal is to determine the camera's motion via the visual information for this purpose. The reference (p_d, R_d) for the rigid motion (p_{co}, R_{co}) is assumed to be constant in the paper.

In this subsection, let us derive a model of the relative rigid body motion. The rigid body motion (p_{wo}, R_{wo}) of the target object, relative to the world frame Σ_w, is given by

$$p_{wo} = p_{wc} + R_{wc} p_{co} \tag{1}$$

$$R_{wo} = R_{wc} R_{co} \tag{2}$$

which is a direct consequence of a transformation of the coordinates in Fig. 1. These coordinate transformations can be found in [13] (Chap.2, Eq.(2.3) and (2.22)). Using the property of a rotation matrix, i.e. $R^{-1} = R^T$, the rigid body motion (1) and (2) is now rewritten as

$$p_{co} = R_{wc}^T (p_{wo} - p_{wc}) \tag{3}$$

$$R_{co} = R_{wc}^T R_{wo}. \tag{4}$$

The dynamic model of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [13]. Let $\hat{\omega}_{wc}$ and $\hat{\omega}_{wo}$ denote the instantaneous body angular velocities from Σ_w to Σ_c, and from Σ_w to Σ_o, respectively [13] (Chap.2, Eq.(2.49)). Here the operator '$\wedge$' (wedge), from $\mathbb{R}^3$ to the set of 3×3 skew-symmetric matrices $so(3)$, is defined as

$$\hat{a} = (a)^\wedge := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

While, the operator '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathbb{R}^3$. With these, it is possible to specify the velocities of each rigid body as follows [13] (Chap.2, Eq.(2.55)).

$$\dot{p}_{wc} = R_{wc} v_{wc}, \quad \dot{R}_{wc} = R_{wc} \hat{\omega}_{wc} \tag{5}$$

$$\dot{p}_{wo} = R_{wo} v_{wo}, \quad \dot{R}_{wo} = R_{wo} \hat{\omega}_{wo}. \tag{6}$$

Differentiating (3) and (4) with respect to time, we can obtain

$$\dot{p}_{co} = -v_{wc} + \hat{p}_{co} \omega_{wc} + R_{co} v_{wo} \tag{7}$$

$$\dot{R}_{co} = -\hat{\omega}_{wc} R_{co} + R_{co} \hat{\omega}_{wo}. \tag{8}$$

Now, let us denote the body velocity of the camera relative to the world frame Σ_w as

$$u_c := [v_{wc}^T \ \omega_{wc}^T]^T. \tag{9}$$

Further, the body velocity of the target object relative to Σ_w should be denoted as

$$V_{wo} := [v_{wo}^T \ \omega_{wo}^T]^T. \tag{10}$$

Then we can rearrange the above equations (7) and (8) in a matrix form as follows.

$$\begin{bmatrix} \dot{p} \\ (\dot{R} R^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} V_{wo}. \tag{11}$$

Here (p, R) denotes (p_{co}, R_{co}) for short. The equation (11) should be the model of the relative rigid body motion.

3 Nonlinear Observer Design

The visual feedback control task requires information of the relative rigid motion (p, R). However image information is only measured in the visual feedback systems. Thus, let us propose a nonlinear observer which estimates the relative rigid motion.

Now, we consider the following dynamic model which just comes from the relative rigid body motion (11).

$$\begin{bmatrix} \dot{\bar{p}} \\ (\dot{\bar{R}} \bar{R}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} I & 0 \\ 0 & \bar{R} \end{bmatrix} u_e \tag{12}$$

where $(\bar{p}, \bar{R})$ is the estimated value of the relative rigid motion, and u_e is the estimated input which is constituted by image information in order to converge the estimated value to the actual relative rigid motion.

Next let us derive a pinhole camera model as shown in Fig. 2. Let λ be a focal length, p_{oi} and p_{ci} be coordi-

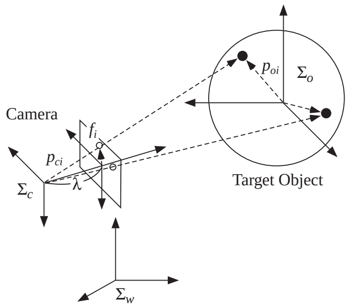

Figure 2: Pinhole camera model of the visual feedback system.

nates of the object's i-th feature point relative to Σ_o and Σ_c, respectively. Then, from a transformation of the coordinates, we have

$$p_{ci} = p + Rp_{oi} \tag{13}$$

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate f_i as follows.

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{14}$$

where $p_{ci} := [x_{ci} \ y_{ci} \ z_{ci}]^T$.

It is straightforward to extend this model to the n image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T \ \cdots \ f_n^T]^T \in \mathbb{R}^{2n}$.

Let us define the estimation error between the estimated value $(\bar{p}, \bar{R})$ and the actual relative rigid motion (p, R) as

$$(p_{ee}, R_{ee}) := (p - \bar{p}, \bar{R}^T R). \tag{15}$$

Let the matrix $\mathrm{sk}(R)$ denote $\frac{1}{2}(R - R^T)$ and let

$$e_R(R) := \mathrm{sk}(R)^{\vee} \tag{16}$$

represent the error vector of the matrix R. Using the notation $e_R(R)$, the vector of the estimation error is given by

$$e_e := [p_{ee}^T \ e_R^T(R_{ee})]^T. \tag{17}$$

Then we consider the measurement equation from the equations (13) and (14). Suppose the estimation error is *small* enough that we can let $R_{ee} \simeq I + \mathrm{sk}(R_{ee})$, the equation (13) becomes

$$p_{ci} = \bar{p}_{ci} - \bar{R}\hat{p}_{oi} e_R(R_{ee}) + p_{ee} \tag{18}$$

where $\bar{p}_{ci} := \bar{p} + \bar{R}p_{oi}$. Using Taylor expansion, the equation (14) can be written as

$$f_i = \bar{f}_i + \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}) \tag{19}$$

where $\bar{p}_{ci} = [\bar{x}_{ci} \ \bar{y}_{ci} \ \bar{z}_{ci}]^T$ and $\bar{f}_i := \frac{\lambda}{\bar{z}_{ci}}[\bar{x}_{ci} \ \bar{y}_{ci}]^T$.

An approximation of image information f around the estimated value $(\bar{p}, \bar{R})$ is given by

$$f - \bar{f} = J(\bar{p}, \bar{R})e_e \tag{20}$$

where the matrix $J(\bar{p}, \bar{R}) : SE(3) \to \mathbb{R}^{2n \times 6}$ is defined as

$$J(\bar{p}, \bar{R}) := \begin{bmatrix} L(\bar{p}, \bar{R}; p_{o1}) \\ L(\bar{p}, \bar{R}; p_{o2}) \\ \vdots \\ L(\bar{p}, \bar{R}; p_{on}) \end{bmatrix} \tag{21}$$

$$L(\bar{p}, \bar{R}; p_{oi}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & -\bar{R}\hat{p}_{oi} \end{bmatrix}. \tag{22}$$

Note that the matrix $J(\bar{p}, \bar{R})$ is like as the image Jacobian which plays an important role in many researches of the visual feedback control[1].

The following assumption will be made.

Assumption 1 For all $(\bar{p}, \bar{R}) \in SE(3)$, the matrix $J(\bar{p}, \bar{R})$ is full column rank.

Under the Assumption 1, the relative rigid motion can be uniquely defined by the image feature vector. Moreover it is known that $n > 4$ is desirable for the visual feedback systems.

The above discussion shows that we can derive the vector of the estimation error e_e from image information f and the estimated value of the relative rigid motion $(\bar{p}, \bar{R})$,

$$e_e = J^{\dagger}(\bar{p}, \bar{R})(f - \bar{f}) \tag{23}$$

where $\dagger$ denotes the pseudo-inverse.

4 Visual Feedback Control Algorithm and Internal Stability Analysis

In this section, we discuss stability of the visual feedback system as a preparation for our main result. In order to derive the visual feedback system, let us define the control error as follows.

$$(p_{ec}, R_{ec}) := (\bar{p} - p_d, \bar{R}R_d^T) \quad (24)$$

which represents the error between the estimated value $(\bar{p}, \bar{R})$ and the reference of the relative rigid motion (p_d, R_d). Using the notation $e_R(R)$, the vector of the control error is defined as

$$e_c := [p_{ec}^T \; e_R^T(R_{ec})]^T. \quad (25)$$

From the equations (12) and (24), the state equation of the control error can be given by

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} u_c + R_1 u_e \quad (26)$$

where $R_1 = \mathrm{diag}\{I, \bar{R}\}$.

Next, we consider the state equation of the estimation error. Using the equations (11), (12) and (15), the state equation of the estimation error can be obtained as follows.

$$\begin{bmatrix} \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} u_c - u_e + R_2 V_{wo} \quad (27)$$

where $R_2 = \mathrm{diag}\{R, R_{ee}\}$.

Using the equations (26) and (27), the state equation of the visual feedback system can be given by

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} u + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo} \quad (28)$$

where

$$u := \begin{bmatrix} u_c^T & u_e^T \end{bmatrix}^T \quad (29)$$

is defined as the input vector.

Let us define the error vector of the visual feedback system as

$$e := \begin{bmatrix} c_c^T & e_e^T \end{bmatrix}^T \quad (30)$$

which contains the control error vector e_c and the estimation error vector e_e. It should be remarked that the actual relative rigid motion (p, R) tends to the reference (p_d, R_d) if $e \to 0$.

Now, let us consider the following control input.

$$u = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I \end{bmatrix} e \quad (31)$$

where K_c and K_e are 6×6 positive definite matrices which are called the control gain and the estimation gain, respectively. B is defined as

$$B(a) = \begin{bmatrix} I & 0 \\ \hat{a} & I \end{bmatrix}$$

for any vector $a \in \mathbb{R}^3$.

The result with respect to stability of the control input (31) can be established as follows.

Lemma 1 If $V_{wo} = 0$, then the equilibrium point $e = 0$ for the closed loop system (28) and (31) is asymptotically stable.

Proof: Let us consider the following positive definite function

$$V := \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}) \quad (32)$$

where $\phi := \frac{1}{2}\mathrm{tr}(I - R)$, $R \in SO(3)$ is the error function of the rotation matrix and the following properties hold[14].

$$\phi(R) = \phi(R^T) \geq 0 \text{ and } \phi(R) = 0 \text{ iff } R = I$$
$$\dot{\phi}(R) = e_R^T(R)(R^T\dot{R})^\vee = e_R^T(R)(\dot{R}R^T)^\vee.$$

The positive definiteness of the function V is given by the property of the error function ϕ. Differentiating (32) with respect to time yields

$$\dot{V} = e^T \begin{bmatrix} \dot{p}_{ec} \\ (R_{ec}^T\dot{R}_{ec})^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix}. \quad (33)$$

Observing that the skew-symmetry of the matrices $\hat{p}_{ec}$ and $\hat{p}_{ee}$, i.e. $p_{ec}^T\hat{p}_{ec}\omega_{wc} = -p_{ec}^T\hat{\omega}_{wc}p_{ec} = 0$ and $p_{ee}^T\hat{p}_{ee}\omega_{wc} = -p_{ee}^T\hat{\omega}_{wc}p_{ee} = 0$, the above equation along the trajectories of the system (28) becomes

$$\begin{aligned} \dot{V} &= e^T \begin{bmatrix} -B^T(p_d) & R_1 \\ 0 & -I \end{bmatrix} u \\ &= -e^T K e \end{aligned} \quad (34)$$

where K is defined as

$$K := \begin{bmatrix} B^T(p_d) & -R_1 \\ 0 & I \end{bmatrix} \begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} B(p_d) & 0 \\ -R_1^T & I \end{bmatrix}. \quad (35)$$

Hence the asymptotic stability can be confirmed. ∎

Remark 1 The control input u contains the error vector e which consists of the vector of the control error e_c and the vector of the estimation error e_e. e_c is derived from the proposed nonlinear observer. While, e_e can also be obtained from the equation (23), Hence we can exploit the control input u.

5 Nonlinear Receding Horizon Control

In this section, we discuss stability of the visual feedback system via the nonlinear receding horizon control approach. Our approach is based on a control Lyapunov function and a corresponding feedback control law. The following lemma plays an important role in stability analysis performed below.

Lemma 2 The positive definite function (32) is a control Lyapunov function for the visual feedback system (28).

Proof: From equation (34), the time derivative of V along the trajectories to the system (28) can be derived as

$$\inf_u\{\dot{V}\} = \inf_u\left\{-e^T\begin{bmatrix} B^T(p_d) & -R_1 \\ 0 & I\end{bmatrix}u\right\} = -\infty \quad \text{if } e \neq 0. \tag{36}$$

Hence, the positive definite function (32) is a control Lyapunov function for the visual feedback system (28). This completes the proof. ■

Let us consider the Finite Horizon Optimal Control Problem (FHOCP) for the visual feedback system (28) which is based on the following cost function

$$J(t,e,T,u) = \int_t^{t+T} h(e(\tau),u(\tau))d\tau + \rho V(e(t+T)) \tag{37}$$

where $\rho \in \mathbb{R}$ is positive and ρV is a terminal penalty. Let denote the optimal cost as

$$J^*(t,e,T) = \inf_u J(t,e,T,u). \tag{38}$$

Now, we propose the following receding horizon optimal control scheme in order to ensure closed-loop stability.

$$h(e,u) = e^T\mathcal{Q}e + u^T\mathcal{R}u \tag{39}$$

$$V(e(t+T)) = \frac{1}{2}\|p_{ec}(t+T)\|^2 + \phi(R_{ec}(t+T)) + \frac{1}{2}\|p_{ee}(t+T)\|^2 + \phi(R_{ee}(t+T)) \tag{40}$$

$$e^*(t+T) = \phi_1\big(t,e,T,u^*(t,e,T)\big) \tag{41}$$

$$u^*(t,e,T) = \arg\inf_u J\big(t,e,T,u\big) \tag{42}$$

where $\mathcal{Q} > 0$ and $\mathcal{R} > 0$. V is a control Lyapunov function which has been proved in Lemma 2. At time t, the finite horizon optimal control problem is solved over $[t, t+T]$ and the corresponding optimal control law $u^*(\tau)$, $t \le \tau < t+T$ is computed (shown in Fig. 3).

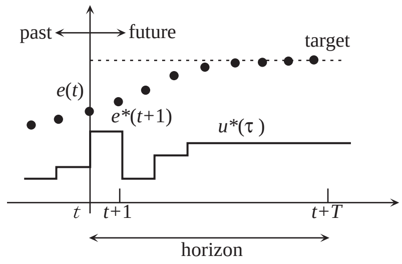

Figure 3: Receding Horizon Approach

Then, the optimal control trajectory is set equal to $u^*(t,e,T)$ and the current optimal control law is defined as $u^*(t)$. At the next time instant, the whole procedure will be repeated. ϕ_1 is the flow of the vector field along the open-loop receding horizon trajectory $u^*(t,e,T)$. If $V_{wo} = 0$, then we have the following theorem.

Theorem 1 Consider the FHOCP (37) for the visual feedback system (28) with the following control law

$$u_k = -K_k e \tag{43}$$

where K_k is defined as

$$K_k := \begin{bmatrix} K_c & 0 \\ 0 & K_e\end{bmatrix}\begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I\end{bmatrix}. \tag{44}$$

Then the receding horizon optimal control scheme (39)–(42) is asymptotically stabilizing.

Remark 2 u_k is a stabilizing control law for the visual feedback system which has been proved in Lemma 1. An important note is that the stabilizing control law u_k is never actually applied, but it is just used to compute the end point penalty.

Proof: Our goal is to prove that $J^*(t,e,T)$ will qualify as a Lyapunov function for the closed loop system. Let us consider the following sub-optimal strategy over the time interval $[t+\delta, t+T+\delta]$

$$\tilde{u} = \begin{cases} u^*(\tau) & \tau \in [t+\delta, t+T] \\ u_k(\tau) & \tau \in [t+T, t+T+\delta] \end{cases} \tag{45}$$

where u_k is a stabilizing feedback control law for the closed-loop system (28). Using the equation (45), $J^*(t,e,T)$ can be transformed into

$$\begin{aligned} &J^*(t,e,T) \\ &= J(t+\delta, e^*(t+\delta), T, \tilde{u}) + \int_t^{t+\delta} h(e^*,u^*)d\tau \\ &+\rho[V(e^*(t+T)) - V(\phi_2(t+T+\delta; e^*(t+T), u_k)] \\ &-\int_{t+T}^{t+T+\delta} h(\phi_2(t+T+\delta; e^*(t+T), u_k), u_k)d\tau. \end{aligned} \tag{46}$$

Since $\tilde{u}$ is sub-optimal strategy over the time interval $[t+\delta, t+T+\delta]$,

$$J^*(t+\delta, x^*(t+\delta), T) \leq J(t+\delta, x^*(t+\delta), T, \tilde{u}) \quad (47)$$

holds. Substituting the equation (46) into (47) yields

$$\begin{aligned} &J^*(t+\delta, x^*(t+\delta), T) - J^*(t, e, T) \\ &\leq -\int_t^{t+\delta} h(e^*, u^*) d\tau \\ &+\rho[V(\phi_2(t+T+\delta; e^*(t+T), u_k) - V(e^*(t+T))] \\ &+\int_{t+T}^{t+T+\delta} h(\phi_2(t+T+\delta; e^*(t+T), u_k), u_k) d\tau. \quad (48) \end{aligned}$$

Dividing both sides of the above equation by δ and taking the limit as $\delta \to 0$, we have

$$\begin{aligned} \dot{J}^*(t, e, T) = &-e^T \mathcal{Q} e - {u^*}^T \mathcal{R} u^* + \rho \dot{V}(e_T^*) \\ &+{e_T^*}^T \mathcal{Q} e_T^* + u_k^T \mathcal{R} u_k. \quad (49) \end{aligned}$$

Here, for the sake of simplicity, e_T^* is defined as $e^*(t+T)$. Evaluating the time derivative of the control Lyapunov function (40) along the trajectories to the system (28) gives us

$$\dot{V}(e_T^*) = -{e_T^*}^T K e_T^*. \quad (50)$$

Using the equations (43) and (50), the equation (49) can be transformed into

$$\dot{J}^*(t, e, T) = -{e_T^*}^T \mathcal{P} e_T^* - e^T \mathcal{Q} e - {u^*}^T \mathcal{R} u^* \quad (51)$$

where

$$\mathcal{P} := \rho K - \mathcal{Q} - K_k^T \mathcal{R} K_k. \quad (52)$$

There exists a $\rho > 0$ such that $\mathcal{P}$ is positive definite. Hence the total derivative of $J^*(t, e, T)$ is negative definite. This completes the proof. ∎

In this section, we have derived the stabilizing receding horizon control scheme for the visual feedback system. The proposed scheme has employed the cost function as a Lyapunov function for establishing stability. Our proposed scheme is based on the control Lyapunov function and the corresponding feedback control law.

6 Conclusions

This paper has discussed stability for the full 3-D visual feedback control via receding horizon control approach. By using the representation of $SE(3)$, we have derived the relative rigid body motion dynamics between the target object and the camera. The nonlinear observer has been proposed in order to derive the visual feedback system. Based on the control Lyapunov function and the corresponding feedback control law, we have proposed the stabilizing receding horizon control scheme for the visual feedback system. The proposed scheme has employed the cost function as a Lyapunov function for establishing stability.

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