# Stability and Tracking Performance of Dynamic Visual Feedback Control for Nonlinear Mechanical Systems

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**Abstract**

This paper investigates a robot motion control problem with visual information. Firstly the model of the relative rigid body motion and the nonlinear observer are shown in order to derive the visual feedback system. Secondly a design algorithm for the 3-D visual feedback control problem which contains the manipulator dynamics are considered. Finally we discuss stability and $L_2$-gain performance analysis for the 3-D visual feedback system contains the manipulator dynamics. $L_2$-gain performance analysis can be regarded as a tracking performance measure for the moving target object.

## 1 Introduction

By integrating control and vision, mechanical systems can move according to a dynamically changing working space. Tracking of a moving target by a robot manipulator is a typical example of this category in real situations. The combination of mechanical control with visual information, the so-called *visual feedback control* or *visual servo*, should become extremely important, when we consider a mechanical system working under *dynamic* environments. Recent research efforts toward this direction have been nicely collected in [1].

This paper deals with a robot motion control with visual information. This control problem is standard and important, and has gained much attention of researchers for many years [2]–[6]. The control objective is to track a moving target in a three-dimensional workspace by image information. The typical example is shown in Fig. 1. Hence the dynamics of the relative rigid motion is described by the nonlinear systems in a 3-D workspace. However few rigorous results have been obtained in terms of the nonlinear control aspects. For example, there exist no researches that explicitly show the *Lyapunov function* for the full 3-D visual feedback systems except the planar motion types [7]–[11].

In this paper, we discuss stability and $L_2$-gain performance analysis as a tracking performance measure for the full 3-D visual feedback system which consists the manipulator dynamics. In order to derive the full 3-D visual feedback system, we will consider a relative rigid body motion dynamics and a nonlinear observer. Stability and $L_2$-gain performance analysis of the 3-D visual feedback system *neglecting* the manipulator dynamics were carried out in [12]. This paper considers the further discussions of the authors' research concerning the visual feedback control, and deals with stability and $L_2$-gain performance analysis as a tracking performance measure *without neglecting* the manipulator dynamics.

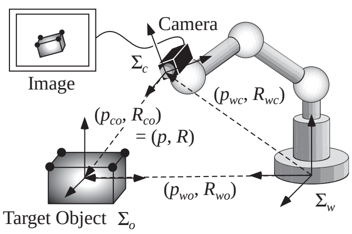


**Figure 1:** Eye-in-Hand Visual Feedback System

This paper is organized as follows. In Section 2, we consider a model of the relative rigid body motion. Section 3 shows a nonlinear observer which estimates the relative rigid body motion. The main theorems concerned with stability and $L_2$-gain performance analysis is derived in Section 4. Finally some comments are discussed in Section 6.

Let a rotation matrix $R_{ab} \in \mathcal{R}^{3\times 3}$ represent the change of the principle axes of a frame $b$ relative to a frame $a$. Then, $R_{ab}$ is known to become orthogonal with unit determinant. Such a matrix belongs to a Lie group of dimension three, called $SO(3) = \{R_{ab} \in \mathcal{R}^{3\times 3} | R_{ab}R_{ab}^T = R_{ab}^T R_{ab} = I, \det(R_{ab}) = +1\}$. The configuration space of the rigid body motion is the product space of $\mathcal{R}^3$ with $SO(3)$, which should be denoted as $SE(3)$ throughout this paper (see, e.g. [13]).

## 2 Relative Rigid Body Motion Model

Let us consider an eye-in-hand system depicted in Fig. 1. $\Sigma_w$ represents the world frame, $\Sigma_c$ represents the camera (end-effector) frame, $\Sigma_o$ represents the object frame, respectively. Let $p_{co} \in \mathcal{R}^3$ and $R_{co} \in \mathcal{R}^{3\times 3}$ denote the position vector and the rotation matrix from the camera frame $\Sigma_c$ to the object frame $\Sigma_o$. Then, the relative rigid body motion from $\Sigma_c$ to $\Sigma_o$ can be represented by $(p_{co}, R_{co}) \in SE(3)$. Similarly, we will define the rigid body motion $(p_{wc}, R_{wc})$ from $\Sigma_w$ to $\Sigma_c$, and $(p_{wo}, R_{wo})$ from $\Sigma_w$ to $\Sigma_o$, respectively, as in Fig. 1.

The objective of the visual feedback control is to bring the actual relative rigid body motion $(p_{co}, R_{co})$ to a given reference $(p_d, R_d)$. Our goal is to determine the camera's motion via the visual information for this purpose. The reference $(p_d, R_d)$ for the rigid motion $(p_{co}, R_{co})$ is assumed to be constant in the paper.

In this section, let us derive a model of the relative rigid body motion. The rigid body motion of the target object relative to the world frame $\Sigma_w$, $(p_{wo}, R_{wo})$, is given by

$$p_{wo} = p_{wc} + R_{wc}p_{co} \quad (1)$$
$$R_{wo} = R_{wc}R_{co} \quad (2)$$

which is a direct consequence of a transformation of the coordinates in Fig. 1. These rigid body transformations can be found in [13](Chapter 2, (2.3) and (2.22)). Using the property of a rotation matrix, i.e. $R^{-1} = R^T$, the relative rigid motion is now rewritten as

$$p_{co} = R_{wc}^T(p_{wo} - p_{wc}) \quad (3)$$
$$R_{co} = R_{wc}^T R_{wo}. \quad (4)$$

The dynamic model the relative rigid body motion involves velocities of each rigid body. To this aid, let us consider the velocity of rigid body as described in [13]. Let $\hat{\omega}_{wc}$ and $\hat{\omega}_{wo}$ denote the instantaneous body angular velocities from $\Sigma_w$ to $\Sigma_c$, and from $\Sigma_w$ to $\Sigma_o$, respectively [13] (Chap.2, Eq.(2.49)). Here the operator '$\wedge$' (wedge), from $\mathcal{R}^3$ to the set of $3\times 3$ skew-symmetric matrices $so(3)$, is defined as

$$\hat{a} = (a)^\wedge := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

While, the operator '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathcal{R}^3$. Recall that a skew-symmetric matrix corresponds to an axis of rotation (via the mapping $a \mapsto \hat{a}$). With these, it is possible to specify the velocities of each rigid body as follows [13] (Chap.2, Eq.(2.55)).

$$\dot{p}_{wc} = R_{wc}v_{wc}, \quad \dot{R}_{wc} = R_{wc}\hat{\omega}_{wc} \quad (5)$$
$$\dot{p}_{wo} = R_{wo}v_{wo}, \quad \dot{R}_{wo} = R_{wo}\hat{\omega}_{wo} \quad (6)$$

Using the property of $R$, i.e. $\dot{R}^T = -R^T\dot{R}R^T$, differentiating (3) and (4) with respect to time, we can obtain

$$\begin{aligned} \dot{p}_{co} &= -R_{wc}^T\dot{R}_{wc}R_{wc}^T(p_{wo} - p_{wc}) + R_{wc}^T(\dot{p}_{wo} - \dot{p}_{wc}) \\ &= -\hat{\omega}_{wc}p_{co} + R_{co}v_{wo} - v_{wc} \quad (7) \\ \dot{R}_{co} &= -R_{wc}^T\dot{R}_{wc}R_{wc}^T R_{wo} + R_{wc}^T R_{wo}\hat{\omega}_{wo} \\ &= -\hat{\omega}_{wc}R_{co} + R_{co}\hat{\omega}_{wo}. \quad (8) \end{aligned}$$

Now, let us denote the body velocity of the camera relative to the world frame $\Sigma_w$ as

$$u_c := [v_{wc}^T \ \omega_{wc}^T]^T. \quad (9)$$

Further, the body velocity of the target object relative to $\Sigma_w$ should be denoted as

$$V_{wo} := [v_{wo}^T \ \omega_{wo}^T]^T. \quad (10)$$

Then we can rearrange the above equations (7) and (8) in a matrix form as follows.

$$\begin{bmatrix} \dot{p} \\ (\dot{R}R^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} V_{wo}. \quad (11)$$

Here $(p, R)$ denotes $(p_{co}, R_{co})$ for short. We have presented the relative rigid motion $g = (p, R)$ and derived the relative rigid body motion model.

It should be noted that the body velocity of the camera, $u_c$, can be directly controlled if the joint velocity signals can be applied to the manipulator systems. There exists the manipulator Jacobian $J_b(q)$ [13] which relates the camera's body velocity $u_c$ to the joints angle velocity as follows.

$$u_c = J_b(q)\dot{q} \quad (12)$$

where $\dot{q}$ is the velocity of the joints.

## 3 Nonlinear Observer Design

The visual feedback control task requires information of the relative rigid motion $(p, R)$. However image information is only measured in the visual feedback systems. Hence, we propose a nonlinear observer which estimates the relative rigid motion.

Let us consider the following dynamic model which is similar to the dynamic model of the actual relative rigid body motion (11).

$$\begin{bmatrix} \dot{\bar{p}} \\ (\dot{\bar{R}}\bar{R}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} I & 0 \\ 0 & \bar{R} \end{bmatrix} u_e \quad (13)$$

where $(\bar{p}, \bar{R})$ is the estimated value of the relative rigid motion, and the new input $u_e$ is the estimated input which is constituted by image information in order to converge the estimated value to the actual relative rigid motion.

Next let us derive a pinhole camera model as shown in Fig. 2. Let $\lambda$ be a focal length, $p_{oi}$ and $p_{ci}$ be co-

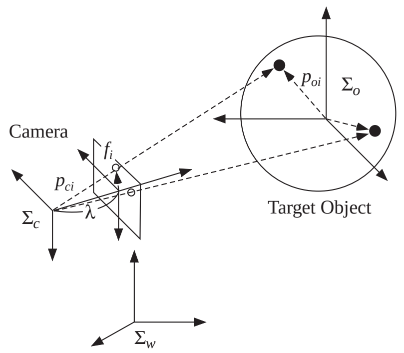


**Figure 2:** Pinhole camera model of the visual feedback system.

ordinates of the object's $i$-th feature point relative to $\Sigma_o$ and $\Sigma_c$, respectively. The perspective projection of the $i$-th feature point onto the image plane gives us the image plane coordinate $f_i$ as follows.

$$p_{ci} = p + Rp_{oi}, \tag{14}$$

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{15}$$

where $p_{ci} := [x_{ci}\ y_{ci}\ z_{ci}]^T$.

It is straightforward to extend this model to the $n$ image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T\ \cdots\ f_n^T]^T \in \mathcal{R}^{2n}$. Then the camera model is expressed by the mapping $\pi : SE(3) \to \mathcal{R}^{2n}$

$$f = \pi(p, R) \tag{16}$$

where $\pi$ is defined by the equations (14) and (15).

Now, we define the estimation error between the estimated value $(\bar{p}, \bar{R})$ and the actual relative rigid motion $(p, R)$ as

$$(p_{ee}, R_{ee}) := (p - \bar{p}, \bar{R}^T R). \tag{17}$$

As is already noted that, if $p = \bar{p}$ and $R = \bar{R}$, then it follows $p_{ee} = 0$ and $R_{ee} = I$.

Let the matrix $\mathrm{sk}(R)$ denote $\frac{1}{2}(R - R^T)$ and let

$$e_R(R) := \mathrm{sk}(R)^\vee \tag{18}$$

represent the error vector of the matrix $R$. Using the notation $e_R(R)$, the vector of the estimation error is given by

$$e_e := [p_{ee}^T\ e_R^T(R_{ee})]^T. \tag{19}$$

where $e_e = 0$ holds when $p_{ee} = 0$ and $R_{ee} = I$.

Next, we will derive the measurement equation from the camera model (16). Suppose the estimation error is *small* enough that we can let $R_{ee} \simeq I + \mathrm{sk}(R_{ee})$, the equation (14) becomes

$$p_{ci} = \bar{p}_{ci} - \bar{R}\hat{p}_{oi} e_R(R_{ee}) + p_{ee} \tag{20}$$

where $\bar{p}_{ci} := \bar{p} + \bar{R}p_{oi}$. Using Taylor expansion, the equation (15) can be written as

$$f_i = \bar{f}_i + \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}) \tag{21}$$

where $\bar{p}_{ci} = [\bar{x}_{ci}\ \bar{y}_{ci}\ \bar{z}_{ci}]^T$ and $\bar{f}_i := \frac{\lambda}{\bar{z}_{ci}}[\bar{x}_{ci}\ \bar{y}_{ci}]^T$.

An approximation of the nonlinear function $\pi$ around the estimated value $(\bar{p}, \bar{R})$ is given by

$$f - \bar{f} = J(\bar{p}, \bar{R}) e_e \tag{22}$$

where the matrix $J(\bar{p}, \bar{R}) : SE(3) \to \mathcal{R}^{2n \times 6}$ is defined as

$$J(\bar{p}, \bar{R}) := \begin{bmatrix} L(\bar{p}, \bar{R}; p_{o1}) \\ L(\bar{p}, \bar{R}; p_{o2}) \\ \vdots \\ L(\bar{p}, \bar{R}; p_{on}) \end{bmatrix} \tag{23}$$

$$L(\bar{p}, \bar{R}; p_{oi}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & -\bar{R}\hat{p}_{oi} \end{bmatrix}. \tag{24}$$

Note that the matrix $J(\bar{p}, \bar{R})$ is like as the image Jacobian which plays an important role in many researches of the visual feedback control [1].

The following assumption will be made.

**Assumption 1** For all $(\bar{p}, \bar{R}) \in SE(3)$, the matrix $J(\bar{p}, \bar{R})$ is full column rank.

Under the Assumption 1, the relative rigid body motion can be uniquely defined by the image feature vector. Moreover it is known that $n > 4$ is desirable for the visual feedback systems.

The above discussion shows that we can derive the vector of the estimation error $e_e$ from image information $f$ and the estimated value of the relative rigid motion $(\bar{p}, \bar{R})$,

$$e_e = J^\dagger(\bar{p}, \bar{R})(f - \bar{f}) \tag{25}$$

where $\dagger$ denotes the pseudo-inverse.

## 4 Dynamic Visual Feedback Control

In this section, we propose the visual feedback controller with the manipulator dynamics and construct Lyapunov and storage function. Stability and $L_2$-gain performance analysis as a tracking performance measure will be derived.

### 4.1 Visual Feedback System

Let us consider the visual feedback system without the manipulator dynamics. In order to derive the visual feedback system, we define the control error as follows.

$$(p_{ec}, R_{ec}) := (\bar{p} - p_d, \bar{R}R_d^T) \tag{26}$$

which represents the error between the estimated value $(\bar{p}, \bar{R})$ and the reference of the relative rigid motion $(p_d, R_d)$. Using the notation $e_R(R)$, we define the vector of the control error as

$$e_c := [p_{ec}^T \; e_R^T(R_{ec})]^T. \tag{27}$$

From the equations (13) and (26), the state equation of the control error can be given by

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} u_c + R_1 u_e \tag{28}$$

where $R_1 = \text{diag}\{I, \bar{R}\}$.

Next we consider the state equation of the estimation error. Using the equations (11), (13) and (17), the state equation of the estimation error can be obtained as follows.

$$\begin{bmatrix} \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} u_c - u_e + R_2 V_{wo} \tag{29}$$

where $R_2 = \text{diag}\{R, R_{ee}\}$.

Using the equations (28) and (29), the state equation of the visual feedback system without the manipulator dynamics can be derived as

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} u_c \\ u_e \end{bmatrix} + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo}. \tag{30}$$

Next, we consider the visual feedback system with manipulator dynamics based on the equation (30). Since the camera is mounted on the end effector of the manipulator, the control input $u_c$ is given by

$$J_b(q)\dot{q} = u_c \tag{31}$$

where $J_b(q)$ is the manipulator body Jacobian [13]. The velocity of the joints $\dot{q}$ is not directly controlled because there exists the manipulator dynamics based Euler Lagrange equation.

Before formulating the visual feedback control problem with the manipulator dynamics, we make the following assumption on the Jacobian $J_b(q)$.

**Assumption 2** The manipulator has 6 degrees of freedom, and the manipulator Jacobian $J_b(q)$ is the nonsingular matrix.

From this assumption, we can consider the visual feedback control problem without the kinematics problems.

Under the assumption, the equation (31) can be transformed into

$$\dot{q} = J_b^{-1}(q)u_c \tag{32}$$

where $\dot{q}$ is the velocity of the joints. Here let us consider the manipulator dynamics as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{33}$$

where $q$, $\dot{q}$, and $\ddot{q}$ are the joints angle, velocity, and acceleration, $\tau$ is the vector of the input torques. The joints velocity (32) can be regarded as the reference

$$\dot{q}_d := J_b^{-1}(q)u_c. \tag{34}$$

Now, we define the error vector with respect to the joints velocity of the manipulator dynamics as

$$\xi := \dot{q} - \dot{q}_d. \tag{35}$$

Let us consider the input torques as follows.

$$\tau = u_\xi + M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_b + g(q). \tag{36}$$

Substituting the equations (36) and (35) into (33) yields

$$M(q)\dot{\xi} + C(q,\dot{q})\xi = u_\xi \tag{37}$$

where the new input $u_\xi$ is to be determined in order to achieves the control objectives. Using the equations (30), (35) and (37), the visual feedback system with the manipulator dynamics can be derived as follows.

$$M(q)\dot{\xi} = -C(q,\dot{q})\xi + u_\xi \tag{38}$$

$$\begin{bmatrix} \dot{p}_{ec} \\ (R_{ec}^T\dot{R}_{ec})^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \\ 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} J_b(q)\xi + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo} + \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} u_c \\ u_e \end{bmatrix} \tag{39}$$

Let us define the error vector of the visual feedback system as

$$x := \left[\xi^T \; e_c^T \; e_e^T\right]^T.$$

### 4.2 Dynamic Visual Feedback Control Algorithm and Internal Stability Analysis

Let us consider the 3-D visual feedback control problem with the manipulator dynamics. Henceforth, we regard the error vector $x$ as the controlled output. Then the visual feedback control problem can be formulated as follows:

**Problem:** Given $\gamma > 0$, find a control input $u_\xi$, $u_c$ and $u_e$ such that the closed loop system satisfies the control objectives as follows :

1. (Internal stability) If the target object is static, i.e. $V_{wo} = 0$, then the equilibrium point $x = 0$ for the closed loop system is asymptotically stable.
2. (Tracking performance in the $L_2$-gain sense) The closed loop system has $L_2$-gain $\leq \gamma$.

We propose the following visual feedback controller

$$u_\xi = -J_b^T(q)B(p_d)e_c - K_\xi \xi \tag{40}$$

$$\left[\begin{array}{c} u_c \\ u_e \end{array}\right] = -\left[\begin{array}{cc} K_c & 0 \\ 0 & K_e \end{array}\right]\left[\begin{array}{cc} -B(p_d) & 0 \\ R_1^T & -I \end{array}\right] e \tag{41}$$

where $K_\xi$, $K_c$ and $K_e$ are $6 \times 6$ positive definite matrices. The matrix $B$ is defined by

$$B(a) = \left[\begin{array}{cc} I & 0 \\ \hat{a} & I \end{array}\right]$$

for any vector $a \in \mathcal{R}^3$. Further, the error vector $e$ is defined as follows.

$$e := \left[\begin{array}{cc} e_c^T & e_e^T \end{array}\right]^T$$

Note that the equation (41) has been proposed for the visual feedback system without the manipulator dynamics in [12].

The result with respect to asymptotic stability of the proposed controller (40) and (41) can be established as follows.

**Theorem 1** If $V_{wo} = 0$, then equilibrium point $x = 0$ for the closed loop system (38)-(41) has is asymptotically stable.

**Proof:** Let us consider the following positive definite function $V$ as the Lyapunov function candidate.

$$V = \frac{1}{2}\xi^T M(q)\xi + \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}) \tag{42}$$

where $\phi$ is the error function of the rotation matrix. We refer to Appendix A for the error function on $SO(3)$. It is well known that the inertia matrix $M(q)$ is the positive definite matrix for all joint $q$.

Differentiating (42) with respect to time along the trajectories of the system (38)-(41) yields

$$\begin{aligned}\dot{V} &= -\xi^T(J_b^T(q)B(p_d)e_c + K_\xi\xi) \\ &\quad + \frac{1}{2}\xi^T(\dot{M}(q) - 2C(q,\dot{q}))\xi + e_c^T B^T(p_d)J_b(q)\xi \\ &\quad + e^T\left[\begin{array}{cc} -B^T(p_d) & R_1 \\ 0 & -I \end{array}\right]\left[\begin{array}{c} u_c \\ u_e \end{array}\right] \\ &= -\xi^T K_\xi \xi - e^T K_{ce} e\end{aligned}$$

where

$$K_{ce} := \left[\begin{array}{cc} B^T(p_d) & -R_1 \\ 0 & I \end{array}\right]\left[\begin{array}{cc} K_c & 0 \\ 0 & K_e \end{array}\right]\left[\begin{array}{cc} B(p_d) & 0 \\ -R_1^T & I \end{array}\right].$$

Note that the matrix $\dot{M}(q) - 2C(q,\dot{q})$ is the skew-symmetric matrix for all $q$ and $\dot{q}$.

Differentiating the positive definite matrix $V$ with respect to time satisfies $V \leq 0$ for all $\xi$, $e_c$ and $e_e$. Hence the asymptotic stability can be confirmed. ∎

### 4.3 Tracking Performance Analysis of the Proposed Controller

In this subsection, we will discuss $L_2$-gain performance analysis as a tracking performance measure for the visual feedback system. Now, let us define

$$P = K_\xi - \frac{1}{2}I \tag{43}$$

$$Q = K_{ce} - \frac{1}{2}\left[\begin{array}{cc} I & 0 \\ 0 & \left(1 + \frac{1}{\gamma^2}I\right) \end{array}\right]. \tag{44}$$

where $\gamma \in \mathcal{R}$ is positive. Then we have the following theorem with respect to $L_2$-gain performance analysis as a tracking performance measure.

**Theorem 2** Given a positive scalar $\gamma$ and consider the visual feedback controller (40) and (41) with the gains $K_\xi$, $K_c$ and $K_e$ such that the matrices $P$ and $Q$ are positive semi-definite. Then the closed loop system (38)-(41) has $L_2$-gain $\leq \gamma$.

**Proof:** Differentiating the positive definite function $V$ along the trajectories of the system (38)-(41) yields

$$\dot{V} = -\xi^T K_\xi \xi - e^T K_{ce} e + e^T \left[\begin{array}{c} 0 \\ R_2 \end{array}\right] V_{wo}.$$

After the completing-the-square, the velocity of the target object (in the worst case) should be derived as

$$V_{wo} = \frac{1}{\gamma^2}\left[\begin{array}{cc} 0 & R_2^T \end{array}\right] e. \tag{45}$$

Hence for any disturbances $V_{wo}$ it can be verified that the inequality

$$\dot{V} + \frac{1}{2}\|x\|^2 - \frac{\gamma^2}{2}\|V_{wo}\|^2 \leq -\xi^T P \xi - e^T Q e \leq 0 \quad (46)$$

holds if the matrices $P$ and $Q$ are positive semi-definite. Integrating both sides of (46) from 0 to $T$ and noticing $V(T) \geq 0$, we have

$$\int_0^T \|x\|^2 dt \leq \gamma^2 \int_0^T \|V_{wo}\|^2 dt + 2V(0), \quad (47)$$

for all $T \geq 0$. This completes the proof. ∎

The positive definite function $V$ plays a role of the storage function for $L_2$-gain performance analysis. $L_2$-gain performance analysis as a tracking performance measure have been performed.

## 5 Conclusions

This paper has discussed the full 3-D visual feedback control which contains the manipulator dynamics from the theoretical standpoint. By using the representation of $SE(3)$, we have derived the relative motion dynamics between the moving target and the camera. The nonlinear observer has been proposed in order to derive the visual feedback system. Stability and $L_2$-gain performance analysis as a tracking performance measure have been performed. Especially, we made good use of the error function on $SO(3)$ as the Lyapunov or storage functions.

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## A Error Function on $SO(3)$

Let us introduce the notation of the error function.

$$\phi(R) := \frac{1}{2}\mathrm{tr}(I - R)$$

and, for any $3 \times 3$ matrix $A$, $\mathrm{sk}(A) := \frac{1}{2}(A - A^T)$. The error function $\phi$ has the following properties.

**Property 1** Let $R \in SO(3)$. The following properties hold.

1. $\phi(R) = \phi(R^T) \geq 0$ and $\phi(R) = 0$ if and only if $R = I$.
2. $\dot{\phi}(R) = e_R^T(R)(R^T\dot{R})^\vee = e_R^T(R)(\dot{R}R^T)^\vee$, where $e_R(R) := \mathrm{sk}(R)^\vee$.
3. there exist $b_1 \geq \frac{1}{2} \geq b_2 > 0$ such that

$$b_2\|e_R(R)\|^2 \leq \phi(R) \leq b_1\|e_R(R)\|^2$$

for any $R \in SO(3)$ which satisfy $\phi(R) < 1$.

These properties are proved in [14].