Visual Feedback Control of Planar Manipulators Based on Nonlinear Receding Horizon Control Approach

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Abstract

This paper deals with the vision-based robot motion control via the nonlinear receding horizon control approach. The visual feedback system consists of the manipulator dynamics and the image dynamics which is derived from the camera model. We propose the stabilizing receding horizon control scheme which is based on a control Lyapunov function and a corresponding feedback control law. The control Lyapunov function is constructed by the full Lagrangian dynamics based on the image feature parameter potential. The proposed scheme employs the cost function as a Lyapunov function for establishing stability of nonlinear receding horizon control. The effectiveness of the proposed scheme is illustrated by applying this approach to the planar model of visual feedback system.

1 Introduction

Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision systems to control the position of the robot end-effector. The combination of mechanical control with visual information should become extremely important, when we consider the mechanical systems working with targets whose position is unknown. Non-contact sensing is useful in the achievement of many kinds of the robotics tasks. Recent research efforts toward this direction have been nicely collected in [1] and [2].

This paper deals with the eye-in-hand approach to the visual feedback control. The typical example is shown in Fig. 1. This control problem is important, and has gained much attention of researchers for recent years [1]. However, much of previous works assume that the manipulator dynamics do not interact with the visual feedback loop. Although this assumption is valid for slow robot motion, it does not hold for high-speed tasks where the manipulator dynamics is not neglectable. Hence, it is important to deal with the visual feedback control problems in terms of the nonlinear dynamical control aspects [1], [3]. In particular, the Lyapunov function for the visual feedback systems has been shown explicitly in [4]–[11].

While, recently there has been a rapidly growing interest in using receding horizon control schemes for control of the nonlinear systems [12]. In [13], Ohtsuka *et al.* have applied the nonlinear receding horizon control to obstacle avoidance of a space-vehicle model. This interest is partly due to the availability of faster and cheaper computers as efficient numerical algorithms for solving optimization problems. In the area of the chemical process control, receding horizon control methods have been widely successful. This is due to the fact that many important industrial chemical processes are open-loop stable so that stability is not a primary concern for these methods. Several researchers have suggested different methods to guarantee the closed-loop stability of the receding horizon scheme. In a recent paper by De Nicolao *et al.* [14], the receding horizon controller guarantees closed-loop stability by using a possibly non quadratic end point penalty which is the cost incurred if a locally stabilizing linear control law is applied at the end of the time horizon T. Jadbabaie *et al.* [15] have followed the method of De Nicolao *et al.* by using the control Lyapunov function as the end point penalty, and have shown that stability of the receding horizon scheme is guaranteed.

In this paper, we propose the nonlinear receding horizon control scheme for the visual feedback system. The stability of the visual feedback system is discussed with the manipulator dynamics. A control Lyapunov function and a corresponding control law which have been proposed by authors in [10] play a crucial role for the proposed scheme.

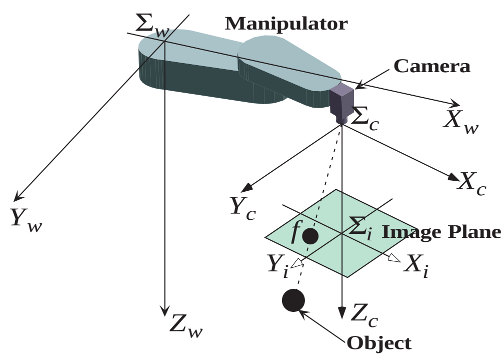

Figure 1: Planar Visual Feedback Configuration

The paper is organized as follows. Section 2 shows the model of the visual feedback system. Section 3 introduces the control Lyapunov function and the corresponding visual feedback control law which are important for the receding horizon control. In Section 4, we propose the stabilizing receding horizon control scheme for the visual feedback system. Finally the numerical example and the conclusion are shown in Section 5 and 6, respectively.

2 System Model

The manipulator model considered here is the well-known Euler-Lagrange system whose inputs are joint torques and whose measurement outputs are joint positions and velocities. A pinhole camera, mounted on the hand of the manipulator in Fig. 1, is modeled by an ideal perspective transformation.

2.1 Manipulator Model

The dynamics of a n-link rigid manipulator can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau, \qquad (1)$$

where

q	: $n \times 1$ vector of the joint angles
τ	: $n \times 1$ vector of control input torques
$M(q)$	: $n \times n$ inertia matrix of the manipulator
$C(q,\dot{q})\dot{q}$	: $n \times 1$ vector of the Coriolis and centrifugal torques
$g(q)$	: $n \times 1$ vector of the gravitational torques.

The following properties are well known [16].

Property 1 The inertia matrix $M(q)$ is positive definite.

Property 2 $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric.

Property 2 is concerned with the passivity property. These properties are important for the Lyapunov/passivity based control design.

By using the inertia parameters, the dynamic equation (1) can be transformed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})\theta = \tau, \qquad (2)$$

where $Y(q,\dot{q},\ddot{q})$ is an $n \times p$ matrix of known functions, called the regressor, and θ is a constant p-dimensional vector of the inertia parameters. The dimension of the parameter space is not unique.

2.2 Camera Model

We consider a planar manipulator with the world frame $\Sigma_w = \{X_w \; Y_w \; Z_w\}$. It is assumed that the manipulator end-effector evolves in the $X_w - Y_w$ plane of Σ_w. Suppose that a camera with the frame $\Sigma_c = \{X_c \; Y_c \; Z_c\}$ is mounted on the manipulator end-effector as depicted in Fig. 1. Hence, the manipulator kinematics gives the camera position ${}^w p_c(q) := [{}^w x_c(q) \; {}^w y_c(q)]'$ and the orientation ${}^w\theta_c(q)$ with respect to Σ_w. A frame $\Sigma_i = \{X_i \; Y_i\}$ is defined in the camera image plane and its origin is the intersection of the optical axis with the image plane. Here it is assumed that the axes X_i and Y_i parallel the axes X_c and Y_c respectively, and the planes $X_c - Y_c$ and $X_w - Y_w$ are separated by the focal length $l > 0$.

Next the object point ${}^w p_o$ is located at $[{}^w x_o \; {}^w y_o \; {}^w z_o]'$ with respect to the frame Σ_w. We assume that the object is static, i.e. ${}^w\dot{p}_o = 0$. ${}^i p_o = [{}^i x_o \; {}^i y_o]'$ is the image coordinate of ${}^w p_o$ through the perspective transformation with the frame Σ_i.

Taking the perspective transformation as the camera model (shown in Fig. 1) yields [6]–[10]

$${}^i p_o := f(q) = \frac{s\lambda}{{}^w z_o} R'({}^w\theta_c(q))({}^w p_o - {}^w p_c(q)), \qquad (3)$$

where $s > 0$ is the scaling factor in pixels/m, ${}^w p_o := [x_o \; y_o]'$ and $f : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$.

Then the differential kinematics of the manipulator gives the relationship between the manipulator joint velocities $\dot{q}$ and the velocities of the camera mounted on the end-effector. The relation can be represented using the manipulator Jacobian $J_p(q) \in \mathbb{R}^{n \times n}$:

$${}^w\dot{p}_c(q) = J_p(q)\dot{q}. \qquad (4)$$

The derivation of the equation (3) yields

$$\dot{f} = -\frac{s\lambda}{{}^w z_o} R' J_p \dot{q} - R'\dot{R} f. \qquad (5)$$

Now, we introduce Property 3 that is important for the Lyapunov/passivity based control design.

Property 3 [16] $R'\dot{R}$ is skew-symmetric.

The following assumptions will be made throughout the paper:

Assumption 1 There exists a manipulator joint configuration achieving $f = 0$.

Assumption 2 The Jacobian J_p is nonsingular.

Assumption 1 ensures that the control problem is solvable. Assumption 2 is required for technical reasons in the stability analysis.

3 Visual Feedback Control

In this section, we discuss the stability of the visual feedback control. Since the visual feedback system consists of the equations (1) and (5) which are obtained in

Section 2, we consider the visual feedback system model described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \quad (6)$$

$$\dot{f} = -\frac{s\lambda}{{}^w z_o} R' J_p \dot{q} - R'\dot{R} f. \quad (7)$$

By invoking the discussions in [10]

$$\begin{aligned} \tau &= u + \alpha M(q)\dot{\eta} + \alpha C(q,\dot{q})\eta + g(q) \\ &= u + Y(q,\dot{q},\alpha\eta,\alpha\dot{\eta})\theta, \end{aligned} \quad (8)$$

where $\eta := J_p' R f$ and $\alpha \in \mathbb{R}_+$. u is the control input that will be designed to achieve the visual feedback control objective.

Substituting the control law (8) into (6) gives us the following temporally closed loop system

$$\begin{aligned} \dot{\xi} &= -M^{-1}C\xi + M^{-1}u \\ \dot{f} &= -\frac{s\lambda\alpha}{{}^w z_o} R' J_p J_p' R f - \frac{s\lambda}{{}^w z_o} R' J_p \xi - R'\dot{R} f, \end{aligned} \quad (9)$$

where $\xi := \dot{q} - \alpha J_p' R f$.

The following lemma plays a key role in the stability analysis performed below.

Lemma 1 [10] Under the Assumption 1 and 2, consider the closed-loop system (9) with the following control law

$$u = -K_1\xi + J_p' R f, \quad (10)$$

where $K_1 \in \mathbb{R}^{n\times n}$ is positive definite. Then the equilibrium point $[\xi' \ f']' = 0$ of the system (9) is asymptotically stable. Furthermore the solutions of the closed loop system (9) asymptotically converge to zero, i.e., $[\dot{q}' \ f']' = 0$, as $t \to \infty$.

Proof: By using Property 1, i.e. $M(q) > 0$, we can consider the positive definite function (11) as a Lyapunov function candidate

$$V = \frac{1}{2}\xi' M(q)\xi + \frac{{}^w z_o}{2s\lambda}\|f\|^2. \quad (11)$$

The above Lyapunov function candidate is constructed by the full Lagrangian dynamics based on a potential function of the image feature parameter space, called image feature parameter potential which has been proposed by the authors in [6]. Evaluating the time derivative of V along the trajectories to the system (9) gives us

$$\begin{aligned} \dot{V} &= \xi' M\dot{\xi} + \frac{1}{2}\xi'\dot{M}\xi + \frac{{}^w z_o}{s\lambda} f'\dot{f} \\ &= \xi' M(-M^{-1}C\xi + M^{-1}u) + \frac{1}{2}\xi'\dot{M}\xi \\ &\quad + \frac{{}^w z_o}{s\lambda} f' \left(-\frac{s\lambda\alpha}{{}^w z_o} R' J_p J_p' R f - \frac{s\lambda}{{}^w z_o} R' J_p \xi - R'\dot{R} f \right). \end{aligned}$$

By using Property 2 and 3, i.e., $\dot{M} - 2C$ and $R'\dot{R}$ are skew-symmetric, $\dot{V}$ is transformed as

$$\begin{aligned} \dot{V} &= \frac{1}{2}\xi'(\dot{M} - 2C)\xi + \xi' u - \alpha f' R' J_p J_p' R f \\ &\quad - f' R' J_p \xi - \frac{{}^w z_o}{s\lambda} f' R' \dot{R} f \\ &= \xi' u - \alpha f' R' J_p J_p' R f - f' R' J_p \xi. \end{aligned} \quad (12)$$

Substituting the control law (10) into (12) yields

$$\begin{aligned} \dot{V} &= -\xi' K_1 \xi + \xi' J_p' R f - \alpha f' R' J_p J_p' R f - f' R' J_p \xi \\ &= -\xi' K_1 \xi - \alpha f' R' J_p J_p' R f, \end{aligned} \quad (13)$$

which is the negative definite function for all $[\xi' \ f']' \neq 0$, since R and J_p are nonsingular. Hence the asymptotic stability can be confirmed. Further, $[\xi' \ f']' \to 0$ is equivalent to $[\dot{q}' \ f']' \to 0$. □

Lemma 2 [10] The positive definite function (11) is a control Lyapunov function.

Proof: The equation (12) shows that

$$\begin{aligned} \inf_u\{\dot{V}\} &= \inf_u\{-\alpha f' R' J_p J_p' R f - f' R' J_p \xi + \xi' u\} \\ &= \begin{cases} -\alpha f' R' J_p J_p' R f & \text{if} \quad \xi = 0 \\ -\infty & \text{if} \quad \xi \neq 0 \end{cases}. \end{aligned} \quad (14)$$

Hence, the positive definite function (11) is a control Lyapunov function for the visual feedback system (9). □

The above discussions give us the control Lyapunov function and the corresponding feedback control law for the visual feedback system. These are important for the nonlinear receding horizon control.

4 Nonlinear Receding Horizon Control

4.1 Review of Nonlinear Receding Horizon Control

Here we would like to review the nonlinear receding horizon control scheme which has been proposed in [15]. Consider the following Finite Horizon Optimal Control Problem (FHOCP).

$$\begin{aligned} J(t,x,T,u) = \inf_{u_{[t,t+T]}} \int_t^{t+T} & h\big(x(\tau),u(\tau)\big)d\tau \\ & + M\big(x(t+T)\big), \end{aligned} \quad (15)$$
$$\text{subject to} \quad \dot{x} = f(x,u)$$

where h is a positive definite function of $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. Now let $J^*(t,x,T,u)$ be the optimal value of the cost function of the FHOCP. At time t, FHOCP is solved over $[t, t+T]$ and the corresponding optimal control $u^*(\tau)$, $t \le \tau < t+T$ is computed. Then, the current control is set equal to $u^*(t)$. Repeating these calculations yields a feedback control law. To ensure closed-loop stability, the following results was proved by Jadbabaie *et al.* [15].

Lemma 3 The receding horizon optimal control scheme (16) is asymptotically stabilizing if a stabilizing feedback control law $u_k(x)$ is available.

$$\begin{aligned} M(x(t+T)) &= \int_{t+T}^{\infty} h\Big(\phi_2\big(\tau; t+T, x_T^*, u_k\big), \\ &\qquad u_k\big(\phi_2(\tau; t+T, x_T^*, u_k)\big)\Big) d\tau \\ x_T^* &= \phi_1\big(t, x(t), T, u^*(t,x,T)\big) \qquad (16) \\ u^*(t,x,T) &= \arg\inf_u J\big(t,x,T,u\big), \end{aligned}$$

where ϕ_1 is the flow of the vector field along the open-loop receding horizon trajectory u^*, and ϕ_2 is the flow along the feedback control law u_k obtained in advance.

Lemma 4 Consider the FHOCP with the following terminal penalty:

$$M(x(t+T)) = \rho V(x(t+T)), \qquad (17)$$

where V is a control Lyapunov function obtained a priori and ρ is a design parameter. Denote the optimal cost associated with this problem by J_V^*. Then there exists a ρ_0 such that for all $\rho \geq \rho_0$, J_V^* is a Lyapunov function for the closed loop system with the receding horizon feedback.

4.2 Nonlinear Receding Horizon Control with Visual Feedback System

In this subsection, we consider the stability of the visual feedback system via the nonlinear receding horizon control approach.

Consider the Finite Horizon Optimal Control Problem (FHOCP) for the visual feedback system (9) which is based on the following optimization

$$\begin{aligned} J(t,x,T,u) = \inf_u \int_t^{t+T} & h(\xi(\tau), f(\tau), u(\tau)) d\tau \\ & + \rho V(x(t+T)). \quad (18) \end{aligned}$$

Let the cost $h(\xi, f, u)$ be as follows

$$h(\xi, f, u) = \xi' \mathcal{Q}_1 \xi + f' R' J_p \mathcal{Q}_2 J_p' R f + u' \mathcal{R} u, \quad (19)$$

where $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, and $\mathcal{R} > 0$. ρ is a design parameter and V is a terminal penalty.

To ensure closed-loop stability, we consider the following receding horizon optimal control scheme.

$$\begin{aligned} V(x(t+T)) &= \frac{1}{2} \xi'(t+T) M \xi(t+T) \\ &\quad + \frac{{}^w z_o}{2 s \lambda} f'(t+T) f(t+T) \quad (20) \\ x^*(t+T) &= \phi_1\big(t, x(t), T, u^*(t,x,T)\big) \quad (21) \\ u^*(t,x,T) &= \arg\inf_u J\big(t,x,T,u\big), \quad (22) \end{aligned}$$

where $x := [\xi' \ f']'$ and V is a control Lyapunov function which has been proved in Lemma 2. At time t, the finite horizon optimal control problem is solved over $[t, t+T]$ and the corresponding optimal control law $u^*(\tau)$, $t \leq \tau < t+T$ is computed (shown in Fig. 2). Then, the optimal control trajectory is set equal to $u^*(t,x,T)$ and the current optimal control law is defined as $u^*(t)$. At the next time instant, the whole procedure is repeated. ϕ_1 is the flow of the vector field along the open-loop receding horizon trajectory $u^*(t,x,T)$.

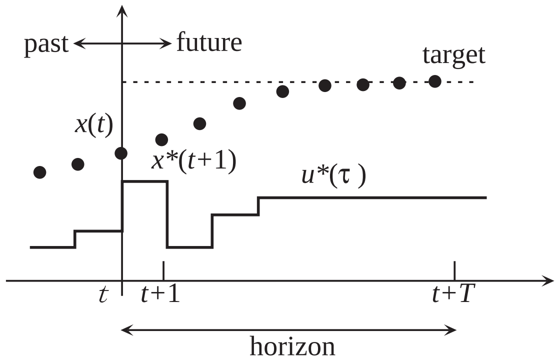

Figure 2: Receding horizon approach

Using the argument presented in Lemma 1–4 we have the following theorem.

Theorem 1 Consider the FHOCP (9) and (18) with the following control law

$$u_k = -K_1 \xi + J_p' R f, \qquad (23)$$

where $\xi := \dot{q} - \alpha \eta$ and $K_1 \in \mathbb{R}^{n \times n}$ is positive definite. Then the receding horizon optimal control scheme (20)–(22) is asymptotically stabilizing.

Remark 1 u_k is a stabilizing control law for the visual feedback system which has been proved in Lemma 1. An important note is that the stabilizing control law u_k is never actually applied, but it is just used to compute the end point penalty.

Proof: Our goal is to prove that $J^*(t,x,T,u)$ will qualify as a Lyapunov function for the closed loop system. We construct the following sub-optimal strategy for the time interval $[t, t+T+\delta]$

$$u = \begin{cases} u^*(\tau) & \tau \in [t, t+T] \\ u_k(\tau) & \tau \in [t+T, t+T+\delta] \end{cases}, \qquad (24)$$

where u_k is a stabilizing feedback control law for the closed-loop system (9). The stabilizing feedback control low (23) was proposed by the authors in [10].

$J^*(t,x,T,u)$ is positive definite, and the time derivative of $J^*(t,x,T,u)$ is as follows.

$$\begin{aligned}&\dot{J}^*(t,x,T,u)\\&=\rho\dot{V}(x_T^*)+\xi_T^{*\prime}\mathcal{Q}_1\xi_T^*+f_T^{*\prime}R_T^{*\prime}J_{pT}^*\mathcal{Q}_2J_{pT}^{*\prime}R_T^*f_T^*\\&\quad+u_k'\mathcal{R}u_k-\Big(\xi'\mathcal{Q}_1\xi+f'R'J_p\mathcal{Q}_2J_p'Rf+u^{*\prime}\mathcal{R}u^*\Big).\end{aligned} \tag{25}$$

Here, for the sake of simplicity, x_T^* is defined as $x^*(t+T)$, and ξ_T^*, f_T^*, R_T^*, J_{pT}^* are similarly defined. Evaluating the time derivative of the control Lyapunov function (20) along the trajectories to the system (9) gives us

$$\begin{aligned}\dot{V}(x_T^*)&=\xi_T^{*\prime}M\dot{\xi}_T^*+\frac{1}{2}\xi_T^{*\prime}\dot{M}\xi_T^*+\frac{{}^wz_o}{s\lambda}f_T^{*\prime}\dot{f}_T^*\\&=\xi_T^{*\prime}\left(-C\xi_T^*+u_k\right)+\frac{1}{2}\xi_T^{*\prime}\dot{M}\xi_T^*\\&\quad+\frac{{}^wz_o}{s\lambda}f_T^{*\prime}\left(-\frac{s\lambda\alpha}{{}^wz_o}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*\right.\\&\qquad\left.-\frac{s\lambda}{{}^wz_o}R_T^{*\prime}J_{pT}^*\xi_T^*-R_T^{*\prime}\dot{R}_T^*f_T^*\right).\end{aligned}$$

By using Property 2 and 3, $\dot{V}(x_T^*)$ is transformed as

$$\begin{aligned}\dot{V}(x_T^*)&=\frac{1}{2}\xi_T^{*\prime}(\dot{M}-2C)\xi_T^*+\xi_T^{*\prime}u_k\\&\quad-\alpha f_T^{*\prime}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*-f_T^{*\prime}R_T^{*\prime}J_{pT}^*\xi_T^*\\&=\xi_T^{*\prime}u_k-\alpha f_T^{*\prime}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*\\&\quad-f_T^{*\prime}R_T^{*\prime}J_{pT}^*\xi_T^*.\end{aligned} \tag{26}$$

Substituting the stabilizing feedback control law (23) into (26), we obtain

$$\begin{aligned}\dot{V}(x_T^*)&=\xi_T^{*\prime}\Big(-K_1\xi_T^*+J_{pT}^{*\prime}R_T^*f_T^*\Big)\\&\quad-\alpha f_T^{*\prime}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*-f_T^{*\prime}R_T^{*\prime}J_{pT}^*\xi_T^*\\&=-\xi_T^{*\prime}K_1\xi_T^*-\alpha f_T^{*\prime}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*.\end{aligned} \tag{27}$$

Hence, the equation (25) and the control law (23) give us

$$\begin{aligned}&\dot{J}^*(t,x,T,u)\\&=\rho\Big(-\xi_T^{*\prime}K_1\xi_T^*-\alpha f_T^{*\prime}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*\Big)\\&\quad+\Big(\xi_T^{*\prime}\mathcal{Q}_1\xi_T^*+f_T^{*\prime}R_T^{*\prime}J_{pT}^*\mathcal{Q}_2J_{pT}^{*\prime}R_T^*f_T^*+u_k'\mathcal{R}u_k\Big)\\&\quad-\Big(\xi'\mathcal{Q}_1\xi+f'R'J_p\mathcal{Q}_2J_p'Rf+u^{*\prime}\mathcal{R}u^*\Big)\\&=-\rho\xi_T^{*\prime}K_1\xi_T^*-\rho\alpha f_T^{*\prime}R_T^{*\prime}J_{pT}^*J_{pT}^{*\prime}R_T^*f_T^*\\&\quad+\xi_T^{*\prime}\mathcal{Q}_1\xi_T^*+f_T^{*\prime}R_T^{*\prime}J_{pT}^*\mathcal{Q}_2J_{pT}^{*\prime}Rf_T^*\\&\quad+\Big(-K_1\xi_T^*+J_{pT}^{*\prime}R_T^*f_T^*\Big)'\mathcal{R}\Big(-K_1\xi_T^*+J_{pT}^{*\prime}R_T^*f_T^*\Big)\\&\quad-\xi'\mathcal{Q}_1\xi-f'R'J_p\mathcal{Q}_2J_p'Rf-u^{*\prime}\mathcal{R}u^*\\&=-x_T^{*\prime}\begin{bmatrix}\rho K_1-K_1\mathcal{R}K_1-\mathcal{Q}_1 & K_1\mathcal{R}J_{pT}^{*\prime}R_T^*\\ R_T^{*\prime}J_{pT}^*\mathcal{R}K_1 & \rho\alpha I-\mathcal{R}-R_T^{*\prime}J_{pT}^*\mathcal{Q}_2J_{pT}^{*\prime}R_T^*\end{bmatrix}x_T^*\\&\quad-\xi'\mathcal{Q}_1\xi-f'R'J_p\mathcal{Q}_2J_p'Rf-u^{*\prime}\mathcal{R}u^*.\end{aligned}$$

Moreover, $\dot{J}^*(t,x,T,u)$ can be formulated as

$$\begin{aligned}\dot{J}^*(t,x,T,u)&=-x_T^{*\prime}\mathcal{P}x_T^*-\xi'\mathcal{Q}_1\xi\\&\quad-f'R'J_p\mathcal{Q}_2J_p'Rf-u^{*\prime}\mathcal{R}u^*,\end{aligned} \tag{28}$$

where

$$\mathcal{P}:=\begin{bmatrix}\rho K_1-K_1\mathcal{R}K_1-\mathcal{Q}_1 & K_1\mathcal{R}J_{pT}^{*\prime}R_T^*\\ R_T^{*\prime}J_{pT}^*\mathcal{R}K_1 & \rho\alpha I-\mathcal{R}-R_T^{*\prime}J_{pT}^*\mathcal{Q}_2J_{pT}^{*\prime}R_T^*\end{bmatrix}.$$

If $\rho>0$ is picked such that $\mathcal{P}$ is positive definite, then the total derivative of $J^*(t,x,T,u)$ is negative definite, which guarantees asymptotic stability. □

We discussed the stabilizing receding horizon control scheme for the visual feedback system. Our proposed scheme is based on the control Lyapunov function and the corresponding feedback control law.

5 Numerical Example

To illustrate the behavior of the visual feedback control, we apply the receding horizon control to a two-link planar manipulator with an eye-in-hand system. The entries of the inertia matrix $M(q)$ and the Coriolis and centrifugal matrix $C(q,\dot{q})$ are given by

$$\begin{aligned}M(q)&=\begin{bmatrix}M_1+M_2+2R_1\cos q_2 & M_2+R_1\cos q_2\\ M_2+R\cos q_2 & M_2\end{bmatrix}\\C(q,\dot{q})&=\begin{bmatrix}-R_1\dot{q}_2\sin q_2 & -R_1\left(\dot{q}_1+\dot{q}_2\right)\sin q_2\\ R_1\dot{q}_1\sin q_2 & 0\end{bmatrix}\\M_1&=I_1+m_1r_1^2+m_2l_1^2\\M_2&=I_2+m_2r_2^2\\R_1&=m_2r_2l_1.\end{aligned}$$

Since the robot moves in the horizontal plane, we have $g(q)=0\in\mathbb{R}^2$. The rotation matrix R and the Jacobian matrix J_p are described as

$$\begin{aligned}R&=\begin{bmatrix}\cos(q_1+q_2) & -\sin(q_1+q_2)\\ \sin(q_1+q_2) & \cos(q_1+q_2)\end{bmatrix}\\J_p&=\begin{bmatrix}-l_1\sin q_1-l_2\sin(q_1+q_2) & -l_2\sin(q_1+q_2)\\ l_1\cos q_1+l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2)\end{bmatrix}.\end{aligned}$$

The distance between the optical center and robot workspace plane is ${}^wz_o=1.86$ m. It is assumed that the focal length λ multiplied by the scale factor s, i.e. $s\lambda$, is equal to 2180 pixels. The target has been placed in the X_w-Y_w plane at ${}^wP_o=[\ 0.3414\quad -0.1414\ ]'$. The initial conditions of the robot positions and velocities are set as follows: $q_1(0)=0$ rad, $q_2(0)=\pi/2$ rad, and $\dot{q}_1(0)=\dot{q}_2(0)=0$ rad/s. Hence, the initial state is $x(0)=[\ \xi_1\quad \xi_2\quad f_1\quad f_2\ ]'=[\ 113\quad 33\quad -400\quad -165\ ]'$.

The gain matrices are chosen as $\mathcal{Q}_1=\mathrm{diag}\{4,2\}$, $\mathcal{Q}_2=\mathrm{diag}\{4,2\}$, and $\mathcal{R}_1=\mathrm{diag}\{4,2\}$. We pick $\rho=10$

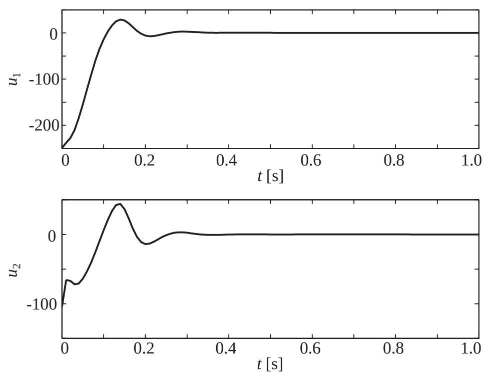

Figure 3: Control input

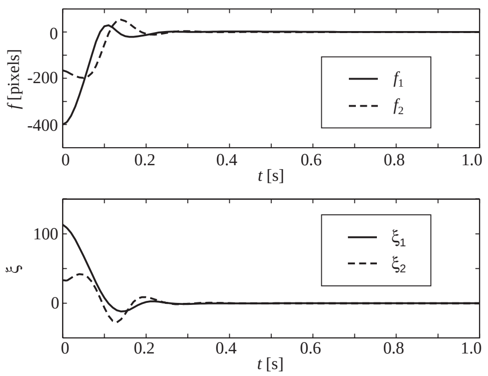

Figure 4: Image position errors and velocity errors

and the horizon length $T = 0.05$ with a sampling time 0.01 s.

In receding horizon control, the current control at state x and time t is obtained by determining on-line (the open-loop) optimal control $u^*(t, x, T)$ over the interval $[t, t+T]$ and setting the control equal to $u^*(t)$. Repeating this calculation continuously yields a feedback control (since $u^*(t)$ clearly depends on the current state x). The optimal control problem (9) and (18) is solved on-line. Fig. 3 shows the control input u, and Fig. 4 shows the image position error f and ξ tend asymptotically to zero. It can be seen that the equilibrium point of the visual feedback system is asymptotically stable.

6 Conclusion

In this paper, the nonlinear receding horizon control for the visual feedback system has been discussed. In particular, we proposed the stabilizing receding horizon control scheme which is based on the control Lyapunov function and the corresponding feedback control law. The proposed scheme has employed the cost function as a Lyapunov function for establishing stability of nonlinear receding horizon control. Moreover, the numerical example was reported to illustrate the effectiveness of the proposed scheme.

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