A Stabilizing Receding Horizon Control Approach to Visual Feedback Systems of Rigid Body Motion

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This paper investigates a robot motion control with visual information via the nonlinear receding horizon control approach. Firstly the model of the relative rigid body motion and the nonlinear observer are considered in order to derive the visual feedback system. Secondly the stabilizing feedback control law for the closed-loop system is discussed as a preparation for our main result. Finally we propose the stabilizing receding horizon control scheme for the 3-D visual feedback control problem by using an appropriate control Lyapunov function as the end point penalty. The proposed scheme employs the cost function as a Lyapunov function for establishing stability.

1 Introduction

Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision system to control the position of the robot end-effector in an efficient manner. The combination of mechanical control with visual information, so-called *visual feedback control* or *visual servo*, should become extremely important, when we consider a mechanical system working under *dynamical* environments.

This paper deals with the relative rigid motion control of the target with respect to the camera frame. This control problem is standard and important, and has gained much attention of researchers for many years [1]–[5]. The control objective is to move the end effector of the manipulators in a three-dimensional workspace by visual information. The typical example is shown in Fig. 1. While, there has recently

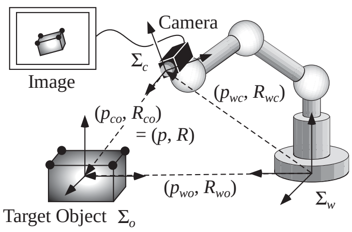

Fig. 1 Eye-in-hand visual feedback system.

been a rapidly growing interest in using receding horizon control, also known as model predictive control, for the nonlinear systems[6]. Especially, stability is an overriding requirement and much of literature has suggested different methods to guarantee the closed-loop stability of the receding horizon scheme[7][8].

In this paper, we discuss stability for the 3-D visual feedback control problem based on a nonlinear receding horizon control scheme. The proposed scheme employs the cost function as a Lyapunov function for establishing stability. In order to derive the 3-D visual feedback system, we will consider a relative rigid body motion dynamics and a nonlinear observer. A control Lyapunov function and a corresponding control law for the 3-D visual feedback system play a crucial role for the proposed scheme.

Let a rotation matrix $R_{ab} \in \mathbb{R}^{3\times3}$ represent the change of the principle axes of a frame b relative to a frame a. Then, R_{ab} is known to become orthogonal with unit determinant. Such a matrix belongs to a Lie group of dimension three, called $SO(3) = \{R_{ab} \in \mathbb{R}^{3\times3} | R_{ab}R_{ab}^T = R_{ab}^T R_{ab} = I, \det(R_{ab}) = +1\}$. The configuration space of the rigid body motion is the product space of $\mathbb{R}^3$ with $SO(3)$, which should be denoted as $SE(3)$ throughout this paper (see, e.g. [9]).

2 Relative Rigid Body Motion Model

Let us consider the eye-in-hand system[1] depicted in Fig. 1, where the coordinate frame Σ_w represents the world frame, Σ_c represents the camera (end-effector) frame, and Σ_o represents the object frame, respectively. Let $p_{co} \in \mathbb{R}^3$ and $R_{co} \in \mathbb{R}^{3\times3}$ denote the position vector and the rotation matrix from the camera frame Σ_c to the object frame Σ_o. Then, the relative rigid body motion from Σ_c to Σ_o can be represented by $(p_{co}, R_{co}) \in SE(3)$. Similarly, we will define the rigid body motion (p_{wc}, R_{wc}) from Σ_w to Σ_c, and (p_{wo}, R_{wo}) from Σ_w to Σ_o, respectively, as in Fig. 1.

In this section, let us derive a model of the relative rigid body motion. The rigid body motion (p_{wo}, R_{wo}) of the target object, relative to the world frame Σ_w, is given by

$$p_{wo} = p_{wc} + R_{wc}p_{co} \quad (1)$$

$$R_{wo} = R_{wc}R_{co} \quad (2)$$

which is a direct consequence of a transformation of the coordinates in Fig. 1. These coordinate transformations can be found in [9] (Chap.2, Eq.(2.3) and (2.22)). Using the property of a rotation matrix, i.e. $R^{-1} = R^T$, the rigid body motion (1) and (2) is now rewritten as

$$p_{co} = R_{wc}^T(p_{wo} - p_{wc}) \quad (3)$$

$$R_{co} = R_{wc}^T R_{wo}. \quad (4)$$

The dynamic model of the relative rigid body motion involves the velocity of each rigid body. Let $\hat{\omega}_{wc}$ and $\hat{\omega}_{wo}$ denote the instantaneous body angular velocities from Σ_w to Σ_c, and from Σ_w to Σ_o, respectively [9] (Chap.2, Eq.(2.49)). Here the operator '$\wedge$' (wedge), from $\mathbb{R}^3$ to the set of 3×3 skew-symmetric matrices $so(3)$, is defined as

$$\hat{a} = (a)^{\wedge} := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

While, the operator '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathbb{R}^3$. With these, it is possible to specify the velocities of each rigid body as follows [9] (Chap.2, Eq.(2.55)).

$$\dot{p}_{wc} = R_{wc} v_{wc}, \quad \dot{R}_{wc} = R_{wc} \hat{\omega}_{wc} \tag{5}$$

$$\dot{p}_{wo} = R_{wo} v_{wo}, \quad \dot{R}_{wo} = R_{wo} \hat{\omega}_{wo}. \tag{6}$$

Differentiating (3) and (4) with respect to time, we can obtain the model of the relative rigid body motion in a matrix form as follows [4][5].

$$\begin{bmatrix} \dot{p} \\ (\dot{R}R^T)^{\vee} \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} V_{wo} \tag{7}$$

where $u_c := [v_{wc}^T \ \omega_{wc}^T]^T$ represents the body velocity of the camera relative to the world frame Σ_w and $V_{wo} := [v_{wo}^T \ \omega_{wo}^T]^T$ represents the body velocity of the target object relative to Σ_w. Here (p, R) denotes (p_{co}, R_{co}) for short.

3 Estimation of Relative Rigid Body Motion

The visual feedback control task requires information of the relative rigid motion (p, R). However image information is only measured in the visual feedback systems.

Now, we consider the following dynamic model which just comes from the relative rigid body motion (7).

$$\begin{bmatrix} \dot{\bar{p}} \\ (\dot{\bar{R}}\bar{R}^T)^{\vee} \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} I & 0 \\ 0 & \bar{R} \end{bmatrix} u_e \tag{8}$$

where $(\bar{p}, \bar{R})$ is the estimated value of the relative rigid motion, and u_e is the estimated input which is constituted by image information in order to converge the estimated value to the actual relative rigid motion.

Next let us derive a pinhole camera model as shown in Fig. 2. Let λ be a focal length, p_{oi} and p_{ci} be coordinates of the object's i-th feature point relative to Σ_o and Σ_c, respectively. Then, from a transformation of the coordinates, we have

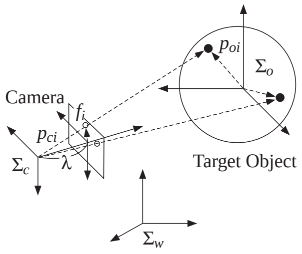

Fig. 2 Pinhole camera model.

$$p_{ci} = p + Rp_{oi} \tag{9}$$

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate f_i as follows.

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{10}$$

where $p_{ci} := [x_{ci} \ y_{ci} \ z_{ci}]^T$.

It is straightforward to extend this model to the n image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T \ \cdots \ f_n^T]^T \in \mathbb{R}^{2n}$.

Let us define the estimation error between the estimated value $(\bar{p}, \bar{R})$ and the actual relative rigid motion (p, R) as

$$(p_{ee}, R_{ee}) := (p - \bar{p}, \bar{R}^T R). \tag{11}$$

Let the matrix $\mathrm{sk}(R)$ denote $\frac{1}{2}(R - R^T)$ and let

$$e_R(R) := \mathrm{sk}(R)^{\vee} \tag{12}$$

represent the error vector of the matrix R. Using the notation $e_R(R)$, the vector of the estimation error is defined as

$$e_e := [p_{ee}^T \ e_R^T(R_{ee})]^T. \tag{13}$$

Then we consider the measurement equation from Eqs.(9) and (10). Suppose the estimation error is *small*, an approximation of image information f around the estimated value $(\bar{p}, \bar{R})$ can be given as follows [4][5].

$$f - \bar{f} = J(\bar{p}, \bar{R}) e_e \tag{14}$$

where the matrix $J(\bar{p}, \bar{R}) : SE(3) \to \mathbb{R}^{2n \times 6}$ is defined as

$$J(\bar{p}, \bar{R}) := \begin{bmatrix} L(\bar{p}, \bar{R}; p_{o1}) \\ L(\bar{p}, \bar{R}; p_{o2}) \\ \vdots \\ L(\bar{p}, \bar{R}; p_{on}) \end{bmatrix} \tag{15}$$

$$L(\bar{p}, \bar{R}; p_{oi}) := \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & -\bar{R}\hat{p}_{oi} \end{bmatrix}. \tag{16}$$

Note that the matrix $J(\bar{p}, \bar{R})$ can be considered as the image Jacobian[1]. The following assumption will be made.

Assumption 1 For all $(\bar{p}, \bar{R}) \in SE(3)$, the matrix $J(\bar{p}, \bar{R})$ is full column rank.

Under the Assumption 1, the relative rigid motion can be uniquely defined by the image feature vector. Moreover it is known that $n > 4$ is desirable for the visual feedback systems.

The above discussion shows that we can derive the vector of the estimation error e_e from image information f and the estimated value $(\bar{p}, \bar{R})$,

$$e_e = J^{\dagger}(\bar{p}, \bar{R})(f - \bar{f}) \tag{17}$$

where $\dagger$ denotes the pseudo-inverse.

In the next section, Eqs.(8) and (17) will be exploited in order to estimate the relative rigid body motion.

4 Visual Feedback Control

Let us consider the visual feedback control as a preparation for our main result. In order to derive the visual feedback system, let us define the control error as follows.

$$(p_{ec}, R_{ec}) := (\bar{p} - p_d, \bar{R}R_d^T). \tag{18}$$

Using the notation $e_R(R)$, the vector of the control error is defined as

$$e_c := [p_{ec}^T \ e_R^T(R_{ec})]^T. \tag{19}$$

Using Eqs.(7), (8), (11) and (18), the model of the visual feedback system can be given as follows [4][5].

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} & I & 0 \\ 0 & -I & 0 & \bar{R} \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} u + \begin{bmatrix} 0 \\ R_2 \end{bmatrix} V_{wo} \tag{20}$$

where $R_2 = \mathrm{diag}\{R, R_{ee}\}$ and $u := [u_c^T \ u_e^T]^T$ is defined as the input vector. Let us define the error vector of the visual feedback system as

$$e := \begin{bmatrix} c_c^T & e_e^T \end{bmatrix}^T. \tag{21}$$

It should be remarked that the actual relative rigid motion (p, R) tends to the reference (p_d, R_d) if $e \to 0$.

Now, let us consider the following control input.

$$u = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I \end{bmatrix} e \tag{22}$$

where $R_1 = \mathrm{diag}\{I, \bar{R}\}$. K_c and K_e are 6×6 positive definite matrices which are called the control gain and the estimation gain, respectively. B is defined as

$$B(a) = \begin{bmatrix} I & 0 \\ \hat{a} & I \end{bmatrix}$$

for any vector $a \in \mathbb{R}^3$.

The result with respect to stability of the closed-loop system (20) and (22) can be established as follows.

Lemma 1 [4][5] If $V_{wo} = 0$, then the equilibrium point $e = 0$ for the closed-loop system (20) and (22) is asymptotically stable.

Proof: Let us consider the following positive definite function

$$V := \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}) \tag{23}$$

where $\phi := \frac{1}{2}\mathrm{tr}(I - R) \ \ \forall R \in SO(3)$ is the error function of the rotation matrix and the following properties hold[10].

1. $\phi(R) = \phi(R^T) \geq 0$ and $\phi(R) = 0$ iff $R = I$
2. $\dot{\phi}(R) = e_R^T(R)(R^T\dot{R})^\vee = e_R^T(R)(\dot{R}R^T)^\vee$.

The positive definiteness of the function V is given by the property of the error function ϕ. Differentiating Eq.(23) yields

$$\dot{V} = e^T \begin{bmatrix} -B^T(p_d) & R_1 \\ 0 & -I \end{bmatrix} u = -e^T K e \tag{24}$$

where K is defined as

$$K := \begin{bmatrix} B^T(p_d) & -R_1 \\ 0 & I \end{bmatrix} \begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} B(p_d) & 0 \\ -R_1^T & I \end{bmatrix}. \tag{25}$$

Hence the asymptotic stability can be confirmed. ■

Remark 1 The control input u contains the error vector e which consists of the vector of the control error e_c and the vector of the estimation error e_e. e_c is derived from the proposed nonlinear observer. While, e_e can also be obtained from Eq.(17), Hence we can exploit the control input u.

5 Nonlinear Receding Horizon Control

In this section, we discuss stability of the visual feedback system via the nonlinear receding horizon control approach. Our approach is based on a control Lyapunov function and a corresponding feedback control law. The following lemma plays an important role in stability analysis performed below.

Lemma 2 If $V_{wo} = 0$, then the positive definite function (23) is a control Lyapunov function for the visual feedback system (20).

Proof: From Eq.(24), the time derivative of V along the trajectories to the system (20) can be derived as

$$\begin{aligned} \inf_u\{\dot{V}\} &= \inf_u \left\{ -e^T \begin{bmatrix} B^T(p_d) & -R_1 \\ 0 & I \end{bmatrix} u \right\} \\ &= -\infty \quad \text{if } e \neq 0. \end{aligned} \tag{26}$$

Hence, the positive definite function (23) is a control Lyapunov function for the visual feedback system (20). This completes the proof. ■

Let us consider the Finite Horizon Optimal Control Problem (FHOCP) for the visual feedback system (20) which is based on the following cost function

$$J(t, e, T, u) = \int_t^{t+T} h(e(\tau), u(\tau))d\tau + \rho V(e(t+T)) \tag{27}$$

where $\rho \in \mathbb{R}$ is positive and ρV is a terminal penalty. Let denote the optimal cost as

$$J^*(t, e, T) = \inf_u J(t, e, T, u). \tag{28}$$

Now, we propose the following receding horizon optimal control scheme in order to ensure closed-loop stability.

$$h(e, u) = e^T \mathcal{Q} e + u^T \mathcal{R} u \tag{29}$$

$$\begin{aligned} V(e(t+T)) &= \frac{1}{2}\|p_{ec}(t+T)\|^2 + \phi(R_{ec}(t+T)) \\ &\quad + \frac{1}{2}\|p_{ee}(t+T)\|^2 + \phi(R_{ee}(t+T)) \end{aligned} \tag{30}$$

$$e^*(t+T) = \phi_1\big(t, e, T, u^*(t, e, T)\big) \tag{31}$$

$$u^*(t, e, T) = \arg\inf_u J\big(t, e, T, u\big) \tag{32}$$

where $\mathcal{Q} > 0$ and $\mathcal{R} > 0$. V is a control Lyapunov function which has been proved in Lemma 2. At time t, the finite horizon optimal control problem is solved over $[t, t+T]$ and the corresponding optimal control law $u^*(\tau)$, $t \le \tau < t+T$ is computed.

Then, the optimal control trajectory is set equal to $u^*(t,e,T)$ and the current optimal control law is defined as $u^*(t)$. At the next time instant, the whole procedure will be repeated. ϕ_1 is the flow of the vector field along the open-loop receding horizon trajectory $u^*(t,e,T)$. If $V_{wo}=0$, then we have the following theorem.

Theorem 1 Consider the FHOCP (27) for the visual feedback system (20) with the following control law

$$u_k = -K_k e \tag{33}$$

where K_k is defined as

$$K_k := \begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ R_1^T & -I \end{bmatrix}. \tag{34}$$

Then the receding horizon optimal control scheme (29)–(32) is asymptotically stabilizing.

Remark 2 u_k is a stabilizing control law for the visual feedback system which has been proved in Lemma 1. Note that the stabilizing control law u_k is never actually applied, but it is just used to compute the end point penalty.

Proof: Our goal is to prove that $J^*(t,e,T)$ will qualify as a Lyapunov function for the closed-loop system. Let us consider the following sub-optimal strategy over the time interval $[t+\delta, t+T+\delta]$

$$\tilde{u} = \begin{cases} u^*(\tau) & \tau \in [t+\delta, t+T] \\ u_k(\tau) & \tau \in [t+T, t+T+\delta] \end{cases} \tag{35}$$

where u_k is a stabilizing feedback control law for the closed-loop system (20). Using Eq.(35), $J^*(t,e,T)$ can be transformed into

$$\begin{aligned} & J^*(t,e,T) \\ & = J(t+\delta, e^*(t+\delta), T, \tilde{u}) + \int_t^{t+\delta} h(e^*, u^*) d\tau \\ & +\rho[V(e^*(t+T) - V(\phi_2(t+T+\delta; e^*(t+T), u_k)] \\ & - \int_{t+T}^{t+T+\delta} h(\phi_2(t+T+\delta; e^*(t+T), u_k), u_k) d\tau \end{aligned} \tag{36}$$

where ϕ_2 is the flow along the feedback control law u_k. Since $\tilde{u}$ is sub-optimal strategy over the time interval $[t+\delta, t+T+\delta]$,

$$J^*(t+\delta, e^*(t+\delta), T) \le J(t+\delta, e^*(t+\delta), T, \tilde{u}) \tag{37}$$

holds. Substituting Eq.(36) into Eq.(37) yields

$$\begin{aligned} & J^*(t+\delta, e^*(t+\delta), T) - J^*(t,e,T) \\ & \le -\int_t^{t+\delta} h(e^*, u^*) d\tau \\ & +\rho[V(\phi_2(t+T+\delta; e^*(t+T), u_k) - V(e^*(t+T)] \\ & + \int_{t+T}^{t+T+\delta} h(\phi_2(t+T+\delta; e^*(t+T), u_k), u_k) d\tau. \end{aligned} \tag{38}$$

Dividing both sides of the above equation by δ and taking the limit as $\delta \to 0$, we have

$$\dot{J}^*(t,e,T) \le -e_T^{*T} \mathcal{P} e_T^* - e^T \mathcal{Q} e - u^{*T} \mathcal{R} u^* \tag{39}$$

where

$$\mathcal{P} := \rho K - \mathcal{Q} - K_k^T \mathcal{R} K_k.$$

Here, for the sake of simplicity, e_T^* is defined as $e^*(t+T)$. There exists $\rho > 0$ such that $\mathcal{P}$ is positive definite. Hence the total derivative of $J^*(t,e,T)$ is negative definite. This completes the proof. ■

The proposed scheme has employed the cost function as a Lyapunov function for establishing stability. Our proposed scheme is based on the control Lyapunov function and the corresponding feedback control law.

6 Conclusions

This paper has discussed stability for the full 3-D visual feedback control via receding horizon control approach. By using the representation of $SE(3)$, we have derived the model of the visual feedback system. Based on the control Lyapunov function and the corresponding feedback control law, we have proposed the stabilizing receding horizon control scheme for the visual feedback system. The proposed scheme has employed the cost function as a Lyapunov function for establishing stability.

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