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Passivity-Based Robust Visual Feedback Control of Robotic Systems with Parametric Uncertainties

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Abstract. In this paper, we investigate the vision-based robot motion control problem. The saturation-type switching controller which attempts to robustly control a robotic system is proposed. The control goal is to place the robot end-effector over a desired static target by using the vision system equipped with an eye-in-hand camera. Our design discussed is relying on the relative motion dynamic model, the error function on the rotational matrix group, $SO(3)$, the nonlinear observer, and the Euler-Lagrange equations.

We formulate the problem of the 3-D visual feedback control of the rigid motion in $SE(3)$ which is the product space of $\mathcal{R}^3$ with $SO(3)$. In order to measure the distance between the reference and the actual configuration in $SE(3)$, the error matrix and the error function of the rotation matrix are used. Since the visual feedback control task requires the information of the relative rigid motion, the nonlinear observer to estimate the relative motion is proposed. We prove that the combined system with the nonlinear observer satisfies the passivity property, which plays an important role in our discussion.

The robust saturation-type switching controller by invoking the passivity of the 3-D visual feedback system is proposed. We show the practical stability of the designed controller using the energy function derived from the passivity property. Specifically we derive a robust visual feedback control law that utilizes an auxiliary saturating controller to compensate for the uncertainty present in the robot dynamics. The novelty of our result lies in the fact that the proposed controller is able to guarantee the uniform ultimately boundedness for the 3-D visual feedback control system composed by the manipulator dynamics with parametric uncertainties.

1 Introduction

Vision based robust control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision system to control the position of the robot end-effector in an efficient manner. The combination of mechanical control with visual information should become extremely important, when we consider a mechanical system working with targets whose position is unknown, or with manipulators which may be uncertain. Non-contact sensing is useful in the achievement of many kinds of the robotics tasks. Recent research efforts toward this direction have been nicely collected in [1].

This paper deals with the relative rigid motion control of the desired static target with respect to the camera frame. This control problem is standard and important, and has gained much attention of researchers for many years [1]. The control objective is to place the robot end-effector over a desired static target in a three-dimensional workspace, called $SE(3)$, by the image information. The typical example is shown in Figure 1. Hence the dynamics of the relative rigid motion is described by the nonlinear systems on $SE(3)$ introduced in [6]. However few rigorous results have been obtained in terms of the nonlinear control aspects. For example, there exist no researches that explicitly show the *Lyapunov function* for the full 3-D visual feedback systems except the planar motion types [2]–[5]. Most of these works have been investigated under the assumption of the exact knowledge of the manipulator dynamics. However the exact manipulator dynamics is generally not available since the parametric uncertainties inevitably exist in the manipulator dynamics.

In this paper, we propose the robust saturation-type switching controller for the vision-based robotic systems with the parametric uncertainties. The main contribution of this paper is the answer to the above research challenge about nonlinear control aspects. The proposed controller is able to guarantee the uniform ultimately boundedness for the 3-D vision-based robotic system with the parametric uncertainties. Since the visual feedback control task requires the information of the relative rigid motion, the nonlinear observer to estimate the relative motion also is proposed. We prove that the combined system with the nonlinear observer satisfies the passivity property, which plays an important role in our discussion.

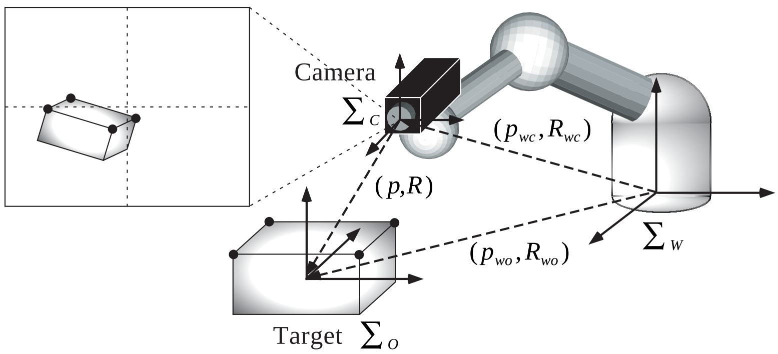

Figure 1: Eye-in-Hand Visual Feedback System

The paper is organized as follows. Section 2 reviews the rigid body motion on the manifold $SE(3)$, the error function on $SE(3)$. Section 3 introduces the model of the visual feedback systems. The state equation is the dynamics of the rigid body motion on $SE(3)$. In Section 4, we formulate the robust visual feedback control problem and propose the robust saturation-type switching controller. Some comments are finally discussed in Section 5.

2 Background and Notation

2.1 Representation of Rigid Motion

A transformation $g : \mathcal{R}^3 \rightarrow \mathcal{R}^3$ of the 3-D Euclidean space is a rigid body motion if the distance is preserved between points and the cross product is preserved between vectors. The action of a rigid body motion is written as

$$g(q) = R_{ab}q + p_{ab}, \tag{1}$$

where $q \in \mathcal{R}^3$ is the coordinates relative to the body frame B, p_{ab} is the position vector of the origin of the body frame B from the origin of the inertial frame A. $R_{ab} \in \mathcal{R}^{3\times3}$ which represents the changes of the principle axes of B relative to the inertial frame A is orthogonal with unit determinant, such a matrix belongs to a Lie group of dimension three called $SO(3)$. The configuration space of the rigid motion is the product space of $\mathcal{R}^3$ with $SO(3)$, which should be denoted as $SE(3)$.

$$SE(3) = \{(p, R)|p \in \mathcal{R}^3, R \in SO(3)\} = \mathcal{R}^3 \times SO(3)$$

A rigid motion can be composed to form a new rigid motion. Let $g_{ab} = (p_{ab}, R_{ab}) \in SE(3)$ be the rigid motion of frame B relative to frame A, and $g_{bc} = (p_{bc}, R_{bc}) \in SE(3)$ be the rigid motion of a frame C relative to frame B. Then the rigid motion of frame C relative to frame A is computed as

$$g_{ac} = g_{ab} \circ g_{bc} = (p_{ab} + R_{ab}p_{ac}, R_{ab}R_{ac}). \tag{2}$$

It is known that $SE(3)$ is a group with the operation $\circ$. The identity element in $SE(3)$ is $(0, I)$ and the inverse of $g \in SE(3)$ is given by $g^{-1} := (-R^T p, R^T)$.

If $g_{ab}(t) = (p_{ab}(t), R_{ab}(t)) \in SE(3)$ is a curve, then the velocity of that curve is the element of the tangent space to $SE(3)$ at $(p_{ab}(t), R_{ab}(t))$. The tangent space at $(0, I)$ of $SE(3)$ is a Lie algebra called $se(3)$. Hence, by left translation, the velocity of the rigid body can be written as

$$\dot{p}_{ab} = R_{ab}v_{ab}, \tag{3}$$
$$\dot{R}_{ab} = R_{ab}\hat{\omega}_{ab}, \tag{4}$$

where $v_{ab} \in \mathcal{R}^3$ and

$$\hat{\omega}_{ab} := \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix},$$

belong to the set of the skew-symmetric matrices $so(3)$. The vector space $so(3)$ is the Lie algebra of $SO(3)$ and isomorphic to $\mathcal{R}^3$ via the mapping $\hat{\omega}_{ab} \rightarrow \omega_{ab} = [\omega_1\ \omega_2\ \omega_3]^T$. The velocity $(v_{ab}, \hat{\omega}_{ab}) \in se(3)$ is called as the *body velocity*. The interpretation of the components of the body velocity is intuitive: v_{ab} is the velocity of the origin of the frame B relative to the frame A with respect to the current B frame. ω_{ab} is the angular velocity of the frame B, also as viewed in the current B frame. The details of the above review of the rigid motion is described by Murray et. al. [6].

2.2 Error Function on $SO(3)$

This paper discusses the problem of the visual feedback control of the rigid body motion in $SE(3)$. In order to measure the distance between reference and actual configuration in $SE(3)$, we introduce the notation of the error function. Let $g = (p, R) \in SE(3)$ be actual configuration and $g_d = (p_d, R_d)$ be reference. It is clearly that the error function of the position vector is given by $\frac{1}{2}\|p - p_d\|^2$. For example, we define the attitude error as $R_e = R_d^T R$. The error matrix R_e implies the relative rotation between the actual R and the reference R_d. Using the error matrix, the error function of the rotation matrix is then defined $\phi : SO(3) \to \mathcal{R}_+$ as

$$\phi(R_e) := \frac{1}{2}\mathrm{tr}(I - R_e), \tag{5}$$

and, for any 3×3 matrix A, $\mathrm{sk}(A) := \frac{1}{2}(A - A^T)$. The error function ϕ has the following properties.

Property 1 *Let $R \in SO(3)$. The following properties hold.*

1. $\phi(R) = \phi(R^T) \geq 0$ and $\phi(R) = 0$ if and only if $R = I$.
2. $\dot{\phi}(R) = e_R^T(R)(R^T\dot{R})^\vee = e_R^T(R)(\dot{R}R^T)^\vee$, where $e_R(R) := \mathrm{sk}(R)$ and the operator '$\vee$' extracts the 3-dimensional vector which parameterizes a 3×3 skew-symmetric matrix as follows.
$$\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}^\vee = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}.$$
3. There exist $b_1 \geq \frac{1}{2} \geq b_2 > 0$ such that
$$b_2\|e_R(R)\|^2 \leq \phi(R) \leq b_1\|e_R(R)\|^2, \tag{6}$$
for any $R \in SO(3)$ which satisfy $\phi(R) < 1$.

These properties are proved in [7].

3 Relative Motion Control Problem with Visual Information

3.1 Relative Motion Dynamic Model

In this section we begin with a model of the visual feedback system which is defined as a nonlinear dynamical model on $SE(3)$. Throughout this paper, we consider the visual feedback system as the eye-in-hand system depicted in Figure 1, where Σ_w represents the world frame, Σ_c represents the camera (end-effector) frame, Σ_o represents the object frame, and $g_{xy} = (p_{xy}, R_{xy}) \in SE(3)$ is the rigid body motion from Σ_x to Σ_y. Note that g_{wo} is not directly obtained, and g_{wc} can be calculated from the forward kinematics.

The control task of the visual feedback systems is the calculation of the changes in the camera's rigid motion, g_{wc}, required to bring the actually relative rigid motion, g_{co}, to the reference motion $g_d = (p_d, R_d) \in SE(3)$. First we derive the dynamic model of the relative motion g_{co}. Using the equation (2) gives the relative rigid motion

$$p_{co} = R_{wc}^T (p_{wo} - p_{wc}), \tag{7}$$

$$R_{co} = R_{wc}^T R_{wo}. \tag{8}$$

The equations (3) and (4) give us the following differential equations.

$$\dot{p}_{wc} = R_{wc} v_{wc}, \tag{9}$$

$$\dot{R}_{wc} = R_{wc} \hat{\omega}_{wc}, \tag{10}$$

where $u_c := [v_{wc}^T \ \omega_{wc}^T]^T \in \mathcal{R}^6$ is the body velocity of the camera relative to Σ_w, and we assume that the target object is static, in other words, the motion of the target object (p_{wo}, R_{wo}) is constant. Differentiating (7) and (8) with respect to time are

$$\begin{aligned} \dot{p}_{co} &= -R_{wc}^T \dot{R}_{wc} R_{wc}^T (p_{wo} - p_{wc}) - R_{wc}^T \dot{p}_{wc}, \\ \dot{R}_{co} &= -R_{wc}^T \dot{R}_{wc} R_{wc}^T R_{wo}, \end{aligned}$$

which lead to

$$\dot{p}_{co} = -\hat{\omega}_{wc} p_{co} - v_{wc}, \tag{11}$$

$$\dot{R}_{co} = -\hat{\omega}_{wc} R_{co}, \tag{12}$$

and then, the above equations are written in matrix form as

$$\begin{bmatrix} \dot{p} \\ (\dot{R} R^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \end{bmatrix} u_c. \tag{13}$$

Here (p, R) denotes (p_{co}, R_{co}) for short.

3.2 Camera Model

This subsection derives a pinhole camera model. Let λ be a focal length, p_{oi} and p_{ci} be coordinates of the object's i-th feature point relative to Σ_o and Σ_c, respectively. The equation (1) and the perspective projection of the i-th feature point onto the image plane give us the image plane coordinate f_i as follows.

$$p_{ci} = R p_{oi} + p, \tag{14}$$

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}, \tag{15}$$

where $p_{ci} := [x_{ci} \ y_{ci} \ z_{ci}]^T$. It is straightforward to extend this model to the n image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T \ \cdots \ f_n^T]^T \in \mathcal{R}^{2n}$. Then the camera model is expressed by the mapping $\pi : SE(3) \to \mathcal{R}^{2n}$

$$f = \pi(p, R), \tag{16}$$

where π is defined by the equations (14) and (15).

3.3 Nonlinear Observer of Rigid Body Motion

The visual feedback control task often requires the information of the relative rigid motion (p, R). However the image information f is only measured in the visual feedback systems. This paper propose the nonlinear observer which estimates the relative motion. The nonlinear observer is given by the following differential equation which is similar to the differential equation of the actually relative rigid motion.

$$\begin{bmatrix} \dot{\bar{p}} \\ (\dot{\bar{R}}\bar{R}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{\bar{p}} \\ 0 & -I \end{bmatrix} u_c + \begin{bmatrix} I & 0 \\ 0 & \bar{R} \end{bmatrix} u_e, \tag{17}$$

where $(\bar{p}, \bar{R})$ is the estimated value of the relative motion, and u_e is the estimated input which is computed from the image position f in order to converge the estimated value to the actual rigid motion.

We rigorously discuss the stability of the visual feedback control system with the nonlinear observer (17) based on the dissipation theoretical approach. First, we consider the state of the visual feedback system as,

$$(p_{ec}, R_{ec}) := (p - p_d, RR_d^T), \tag{18}$$
$$(p_{ee}, R_{ee}) := (p - \bar{p}, \bar{R}^T R), \tag{19}$$

where $(p_d, R_d) \in SE(3)$ is the time invariant reference of the relative rigid body motion, $(p_{ec}, R_{ec}) \in SE(3)$ represents the control error and $(p_{ee}, R_{ee}) \in SE(3)$ represents the estimation error. It should be remarked that the actually relative motion (p, R) tends to the reference (p_d, R_d) if $(p_{ec}, R_{ec}, p_{ee}, R_{ee}) \to (0, I, 0, I)$. From the equations (13) and (17), the state equation is given by

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} u, \tag{20}$$

where $u := \begin{bmatrix} u_c^T & u_e^T \end{bmatrix}^T$. Next we consider the measurement equation from the camera model (16). If the estimation error is assumed to be *small* enough that we can let $R_{ee} \simeq I + \mathrm{sk}(R_{ee})$, the equation (14) then becomes

$$p_{ci} = \bar{p}_{ci} - \bar{R}\hat{p}_{oi} e_R(R_{ee}) + p_{ee},$$

where $\bar{p}_{ci} := \bar{R}p_{oi} + \bar{p}$. Further, using the Taylor expansion, the equation (15) can be written as

$$f_i = \bar{f}_i + \begin{bmatrix} \frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci}) \tag{21}$$

where $\bar{p}_{ci} := [\bar{x}_{ci}\ \bar{y}_{ci}\ \bar{z}_{ci}]^T$ and $\bar{f}_i := \frac{\lambda}{\bar{z}_{ci}}[\bar{x}_{ci}\ \bar{y}_{ci}]^T$. Then an approximation of the nonlinear function π around the estimated value $(\bar{p}, \bar{R})$ is given by

$$f = \bar{f} + J(\bar{p}, \bar{R})e_e \tag{22}$$

where $e_e := [p_{ee}^T \ e_R^T(R_{ee})]^T$ and the matrix $J : SE(3) \to \mathcal{R}^{2n\times 6}$ is defined as

$$J(\bar{p},\bar{R}) := \begin{bmatrix} L(\bar{p},\bar{R};p_{o1}) \\ L(\bar{p},\bar{R};p_{o2}) \\ \vdots \\ L(\bar{p},\bar{R};p_{on}) \end{bmatrix}, \quad L(\bar{p},\bar{R};p_{oi}) := \begin{bmatrix} \frac{\lambda\bar{x}_{ci}}{\bar{z}_{ci}} & 0 & -\frac{\lambda\bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda\bar{y}_{ci}}{\bar{z}_{ci}} & -\frac{\lambda\bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & (-\bar{R}\hat{p}_{oi}) \end{bmatrix}.$$

Note that the matrix J is same as the image Jacobian which plays an important role in many researches of the visual feedback control [1]. We make the following assumption on the image Jacobian.

Assumption 1 *For all $(p,R) \in SE(3)$, the image Jacobian is full row rank.*

Under Assumption 1, the relative rigid motion can be uniquely defined by the image feature vector[1]. Moreover it is known that $n > 4$ is desirable for the visual feedback control systems. The measurement vector, which is calculated by

$$y = J^\dagger(\bar{p},\bar{R})(f - \bar{f}), \tag{23}$$

where $\dagger$ denotes the pseudo-inverse, which leads to the following measurement equation:

$$y = e_e, \tag{24}$$

We assume that the quantization and the lens distortion effects are negligible.

Before deriving the visual feedback control algorithm, we show an important lemma.

Lemma 1 *The system (20) from the control input u to the output ν*

$$\nu := \begin{bmatrix} -I & 0 & 0 & 0 \\ -\hat{p}_d & -I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} e, \tag{25}$$

where $e := [e_c^T \ e_e^T]^T$ and $e_c := [p_{ec}^T \ e_R^T(R_{ec})]^T$, is passive relative to the following storage function.

$$V(p_{ec},R_{ec},p_{ee},R_{ee}) := \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}). \tag{26}$$

Proof : The positive definiteness of the function V is given by Property 1. Differentiating (26) with respect to time gives

$$\dot{V} = e^T \begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix}. \tag{27}$$

[1]It should be noted that the relative motion can not be explicitly calculated because of the high nonlinearity of the function π.

Observing that the skew-symmetry of the matrices $\hat{p}_{ec}$ and $\hat{p}_{ee}$, the above equation along the trajectories of the system (20) becomes

$$\dot{V} = u^T \nu. \tag{28}$$

We then have

$$V(t) - V(0) = \int_0^t \dot{V}(\tau) d\tau = \int_0^t u^T \nu d\tau. \tag{29}$$

□

It is well known that there is the direct link between passivity and (internal) Lyapunov stability. Therefore we derive the following visual feedback control by invoking the passivity of the system (20).

$$u = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \nu = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} e_c \\ y \end{bmatrix}, \tag{30}$$

where K_c and K_e are 6×6 positive definite matrices called by *control* and *estimation* gains, respectively, and $B : \mathcal{R}^3 \rightarrow \mathcal{R}^{6\times6}$ is defined by

$$B(a) = \begin{bmatrix} I & 0 \\ \hat{a} & I \end{bmatrix},$$

for any vectors $a \in \mathcal{R}^3$.

4 Robust Visual Feedback Control

4.1 Manipulator Dynamics

In this subsection, we will introduce the manipulator dynamics with parametric uncertainties. Since the camera is mounted on the end effector of the manipulator, the control input u_c is given by

$$J_b(q)\dot{q} = u_c, \tag{31}$$

where J_b is the manipulator body Jacobian [6]. The velocity of the joints $\dot{q}$ is not directly controlled because there exists the manipulator dynamics based Euler–Lagrange equations. We propose the visual feedback controller with the manipulator dynamics and construct its Lyapunov function.

Before formulating the visual feedback control problem with the manipulator dynamics, we make the following assumption on the Jacobian J_b.

Assumption 2 *The manipulator Jacobian J_b is the nonsingular matrix.*

From Assumption 2, we can consider the visual feedback control problem without the kinematics problems.

Under the assumption, we obtain

$$\dot{q} = J_b^{-1} u_c, \tag{32}$$

where $\dot{q}$ is the velocity of the joints. However there exists the manipulator dynamics that relates the input torques to the joints angle velocities

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})\theta = \tau, \tag{33}$$

where q, $\dot{q}$, and $\ddot{q}$ are the joints angle, velocity, and acceleration, respectively, τ is the vector of the input torques, $\theta \in \mathcal{R}^m$ is a constant vector of inertia parameters and $Y(q,\dot{q},\ddot{q}) \in \mathcal{R}^{n\times m}$ is a matrix of known time functions. We express a nominal model as

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + g_0(q) = Y(q,\dot{q},\ddot{q})\theta_0, \tag{34}$$

where $M_0(q)$, $C_0(q,\dot{q})$, $g_0(q)$ represent nominal values vis-a-vis to parameter uncertainty of $M(q)$, $C(q,\dot{q})$, $g(q)$, respectively. We suppose only that the parameter vector θ is uncertain by which we mean that there exists θ_0 and ρ, both known, such that

$$\|\tilde{\theta}\| := \|\theta_0 - \theta\| \leq \rho. \tag{35}$$

The joints velocity (32) is regarded as the reference, and the error vector is defined as

$$\xi := \dot{q} - J_b^{-1} u_c. \tag{36}$$

Differentiating (36) with respect to time gives

$$\dot{\xi} = \ddot{q} + J_b^{-1}\dot{J}_b J_b^{-1} u_c - J_b^{-1}\dot{u}_c. \tag{37}$$

Now we propose the visual feedback controller as follows

$$\begin{aligned}\tau &= u_\xi + M_0(q)(J_b^{-1}\dot{u}_c - J_b^{-1}\dot{J}_b J_b^{-1}u_c) + C_0(q,\dot{q})J_b^{-1}u_c + g_0(q) \\ &= u_\xi + Y(q,\dot{q},\dot{q}-\xi,\ddot{q}-\dot{\xi})\theta_0. \end{aligned} \tag{38}$$

Substituting the control law (38) into (33) we obtain after some algebra

$$M(q)\dot{\xi} = -C(q,\dot{q})\xi + u_\xi + Y(q,\dot{q},\dot{q}-\xi,\ddot{q}-\dot{\xi})\tilde{\theta}, \tag{39}$$

where u_ξ is the control input that achieves the control objectives.

We associate the visual feedback control algorithm (20) and (30) with the control input u_ξ.

$$M(q)\dot{\xi} = -C(q,\dot{q})\xi + u_\xi + Y(q,\dot{q},\dot{q}-\xi,\ddot{q}-\dot{\xi})\tilde{\theta} \tag{40}$$

$$\begin{bmatrix} \dot{p}_{ec} \\ (\dot{R}_{ec}R_{ec}^T)^\vee \\ \dot{p}_{ee} \\ (\dot{R}_{ee}R_{ee}^T)^\vee \end{bmatrix} = \begin{bmatrix} -I & \hat{p} \\ 0 & -I \\ 0 & \hat{p}_{ee} \\ 0 & 0 \end{bmatrix} J_b(q)\xi + \begin{bmatrix} -I & \hat{p} & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & \hat{p}_{ee} & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} u_c \\ u_e \end{bmatrix} \tag{41}$$

$$\begin{bmatrix} u_c \\ u_e \end{bmatrix} = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -B(p_d) & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} e_c \\ y \end{bmatrix} \tag{42}$$

$$y = e_e \tag{43}$$

Then we derive the following robust 3-D visual feedback control problem with the parametric uncertainties for the system (40)–(43).

Robust 3-D Visual Feedback Control Problem : Given $\varepsilon > 0$ and an a priori bound on the parametric uncertainties, find a control input u_ξ such that the closed loop system (40)–(43) satisfies the uniform ultimate boundedness with respect to a small neighborhood of an equilibrium point.

4.2 Robust Visual Feedback Control Algorithm

In this subsection, we propose the following robust visual feedback control law

$$u_\xi = -J_b^T B(p_d)e_c - K_\xi \xi + Y(q, \dot{q}, \dot{q} - \xi, \ddot{q} - \dot{\xi})u_s,$$
$$u_{si} := \begin{cases} -\rho_i \dfrac{(Y^T\xi)_i}{\left|(Y^T\xi)_i\right|}, & \left|(Y^T\xi)_i\right| > \varepsilon_i \\ -\dfrac{\rho_i}{\varepsilon_i}(Y^T\xi)_i, & \left|(Y^T\xi)_i\right| \leq \varepsilon_i \end{cases} \quad i = 1, \cdots, m \tag{44}$$

where $u_s \in \mathcal{R}^m$ is an auxiliary input vector to compensate for the parametric uncertainty [8], [9]. Here Y denotes $Y(q, \dot{q}, \dot{q}-\xi, \ddot{q}-\dot{\xi})$ for short. Let $(Y^T\xi)_i$ denote the i-th component of the vector $Y^T(q, \dot{q}, \dot{q} - \xi, \ddot{q} - \dot{\xi})\xi$ and choose positive constants ε_i, $i = 1, \cdots, n$. We define a Lyapunov function candidate as

$$W = \frac{1}{2}\xi^T M(q)\xi + \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee}). \tag{45}$$

A simple calculation shows along the trajectories of the system (40)–(43)

$$\begin{aligned} \dot{W} &= -\xi^T(J_b^T B(p_d)e_c + K_\xi \xi) + \xi^T Y(\tilde{\theta} + u_s) + \frac{1}{2}\xi^T(\dot{M}(q) - 2C(q,\dot{q}))\xi \\ &\qquad + e_c^T B^T(p_d)J_b\xi + \left[\begin{array}{cc} u_c^T & u_e^T \end{array}\right] \left[\begin{array}{cc} -B(p_d) & 0 \\ 0 & -I \end{array}\right] \left[\begin{array}{c} e_c \\ e_e \end{array}\right] \\ &= -\xi^T K_\xi \xi + (Y^T\xi)^T(\tilde{\theta} + u_s) - e^T K e. \end{aligned} \tag{46}$$

where

$$K = \left[\begin{array}{cc} B^T(p_d) & 0 \\ 0 & I \end{array}\right] \left[\begin{array}{cc} K_c & 0 \\ 0 & K_e \end{array}\right] \left[\begin{array}{cc} B(p_d) & 0 \\ 0 & I \end{array}\right].$$

The differentiating V with respect to time along the trajectories of the system (40)–(43) gives

$$\dot{W} = -x^T Q x + (Y^T\xi)^T(\tilde{\theta} + u_s) \tag{47}$$

where $x = [\xi^T \ e^T]^T$ and $Q = \mathrm{diag}\{K_\xi, K\}$. The above discussion leads to the following.

Theorem 1 Given $\varepsilon > 0$ and a bound on the parametric uncertainty ρ for all K_ξ, K_c and K_e which are 6×6 positive definite matrices, the closed loop system (40)–(43) and (44) is uniformly ultimately bounded (u.u.b.) with respect to a small neighborhood B of the equilibrium point $(\xi, p_{ec}, e_R(R_{ec}), p_{ee}, e_R(R_{ee})) = 0$.

Proof : By using Cauchy-Schwartz inequality and an a priori bound on the parametric uncertainty ρ, differentiating W with respect to time along the trajectories gives

$$\begin{aligned}\dot{W} &\leq -x^TQx + \sum_{i=1}^{m} \big|(Y^T\xi)_i\big||\tilde{\theta}_i| + \sum_{i=1}^{m}(Y^T\xi)_i u_{si} \\ &\leq -x^TQx + \sum_{i=1}^{m} \big|(Y^T\xi)_i\big|\rho_i + \sum_{i=1}^{m}(Y^T\xi)_i u_{si}. \end{aligned} \tag{48}$$

If $(Y^T\xi)$ satisfies $\big|(Y^T\xi)_i\big| > \varepsilon_i$ for all $i \in \{1, \cdots, m\}$, the equation (44) gives $u_{si} = -\rho_i \frac{(Y^T\xi)_i}{|(Y^T\xi)_i|}$. Therefore, differentiating W with respect to time along the trajectories yields

$$\dot{W} \leq -x^TQx. \tag{49}$$

The error vector x is composed of ξ and e, Q is the positive matrix. Moreover, both ξ and e are not equal to zero, Lyapunov function candidate (45) satisfies $\dot{W} < 0$.

While, let $r(r < m)$ denote the number of the component of $(Y^T\xi)$ which satisfies $\big|(Y^T\xi)_i\big| \leq \varepsilon_i$, an auxiliary input u_{si} will be

$$u_{si} = -\frac{\rho_i}{\varepsilon_i}(Y^T\xi)_i \quad (i \leq r), \qquad u_{si} = -\rho_i \frac{(Y^T\xi)_i}{\big|(Y^T\xi)_i\big|} \quad (i > r).$$

Therefore we obtain the following inequality.

$$\begin{aligned}\dot{W} &\leq -x^TQx + \sum_{i=1}^{r}\rho_i\big|(Y^T\xi)_i\big| - \sum_{i=1}^{r}\frac{\rho_i}{\varepsilon_i}(Y^T\xi)_i^2 + \sum_{j=r+1}^{m}\rho_j\big|(Y^T\xi)_j\big| - \sum_{j=r+1}^{m}\rho_j\frac{(Y^T\xi)_j^2}{\big|(Y^T\xi)_j\big|} \\ &= -x^TQx - \sum_{i=1}^{r}\frac{\rho_i}{\varepsilon_i}\left(\big|(Y^T\xi)_i\big| - \frac{\varepsilon_i}{2}\right)^2 + \sum_{i=1}^{r}\frac{\varepsilon_i\rho_i}{4} \leq -x^TQx + \sum_{i=1}^{r}\frac{\varepsilon_i\rho_i}{4} \end{aligned} \tag{50}$$

Let $\lambda_{\min}(Q)$ denote the minimum eigenvalue of the matrix Q, the equation (50) gives

$$\dot{W} \leq -\lambda_{\min}(Q)\|x\|^2 + \sum_{i=1}^{r}\frac{\varepsilon_i\rho_i}{4}. \tag{51}$$

Lyapunov function candidate (45) satisfies $\dot{W} < 0$ for

$$\|x\| > \left(\frac{\sum_{i=1}^{r}\varepsilon_i\rho_i}{4\lambda_{\min}(Q)}\right)^{\frac{1}{2}}. \tag{52}$$

From the above discussion, the closed loop system (40)–(43) and (44) is uniformly ultimately bounded (u.u.b.) with respect to the following small neighborhood B

$$B(\rho, \varepsilon, K, K_\xi) := \Big\{(\xi, e) \;\big|\; \|x\| \leq \omega\Big\} \tag{53}$$

$$\text{where} \quad x := \left[\xi^T \; e^T\right]^T, \quad \omega = \left(\frac{\sum_{i=1}^{r}\varepsilon_i\rho_i}{4\lambda_{\min}(Q)}\right)^{\frac{1}{2}}.$$

□

5 Conclusion

This paper has derived the design of the robust 3-D visual feedback control law for the vision-based robotic systems with parametric uncertainties. Specifically, we designed the saturation-type switching controller that produced uniform ultimately bounded within a neighborhood of an equilibrium point. Based on the energy function derived from the passivity property, the practical stability has been performed. By using the nonlinear observer which satisfies the passivity property, we estimated the relative motion between the target and the camera.

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