Damage Evaluation with Multi-scale Failure Analysis using Material Degradation Model based on SIFT
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1. Introduction
Over the years, the macro-mechanical approaches have been applied for the failure of composite materials.

Practical failure mechanism of composite materials cannot be predicted in the macroscopic level.

Micro-level of the composite failure criteria were proposed by Hashin and Sun.

In recent years, SIFT is proposed by Gosse that a more efficient method to predict the failure of micro-level.
Damage initiation is caused by dilatational and distortional deformations in fiber or matrix phase. Micro-mechanical approach is required for the failure analysis of composite materials. In this study, Strain Invariant Failure Theory was applied as the microscopic criterion.

Failure modes are separated into the fiber failure, matrix failure and fiber/matrix interaction.
Multi-scale failure approach based on SIFT

Macro-scale

- Static analysis for composite laminate with macroscopic loading

- Determination of mesoscopic behavior ($\sigma/\varepsilon$ in each ply)

Micro-scale

- Micro-mechanical modification by strain amplification factors (Calculating effective strain invariants)
- Microscopic failure analysis by strain invariant failure criterion

Strain component in each ply

Micro-mechanical failure mode
2. Strain-based Failure Theory of Composite
Strain Invariant Failure Theory (SIFT)

- Strain Invariant Failure Theory was proposed by Gosse in 2001.
- SIFT defines the failure characteristic of each fiber and matrix by using the volumetric strain and the equivalent strain.

- Volumetric strain (sum of strain invariants) → Dilatational (volumetric) deformation
- Equivalent strain (von-Mises strain) → Distortional (deviatoric) deformation

Dilatational deformation

Distortional deformation
The strain invariants can be defined as functions of three principal strains from cubic characteristic equation of strain tensor.

\[ \varepsilon^3 - J_1 \varepsilon^2 + J_2 \varepsilon - J_3 = 0 \]

Cubic characteristic equation

- Actually, the volumetric strain is defined by the sum of each strain invariant.
- In practice, \( J_1 \) is considered as significant component of the volumetric strain.

\[ \varepsilon_v = J_1 + J_2 + J_3 \approx J_1 \]

Most significant component of the volumetric strain
The equivalent strain (von-Mises strain) is defined as a function of the second invariant of the deviatoric strain tensor.

\[ \varepsilon_{vm} = \sqrt{3}J'_2 = \sqrt{0.5[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2]} \]

The deviatoric strain invariants are obtained from cubic characteristic equation of the deviatoric strains tensor.

\[ \varepsilon'_3 - J'_2 \varepsilon' - J'_3 = 0 \]  

: Cubic characteristic equation of deviatoric strain  
\( \varepsilon' = \varepsilon - \overline{\varepsilon} \) 

Deviatoric strain invariants

\[ J'_2 = \frac{1}{6}\left[ (\varepsilon_{xx} - \varepsilon_{yy})^2 + (\varepsilon_{yy} - \varepsilon_{zz})^2 + (\varepsilon_{zz} - \varepsilon_{xx})^2 \right] - \frac{1}{4} \left( \varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{zx}^2 \right) \]

\[ J'_3 = \varepsilon'_{xx} \varepsilon'_{yy} \varepsilon'_{zz} + \frac{1}{4} \left( \varepsilon'_{xy} \varepsilon'_{yz} \varepsilon'_{zx} + \varepsilon'_{xx} \varepsilon'_{yy} \varepsilon'_{zz} - \varepsilon'_{xy} \varepsilon'_{yy} \varepsilon'_{zx} - \varepsilon'_{xx} \varepsilon'_{yz} \varepsilon'_{zx} \right) \]
Strain Invariant Failure Criterion

- Effective first invariant of strain
- Effective equivalent strain (von-Mises strain)

The effective strain invariants are calculated through the micro-mechanical modification.

Failure will occur at either the fiber or the matrix phases if any of the effective strain invariants ($J_1$, $\varepsilon_{vm}$) exceed the critical value.

Failure criterion in SIFT for matrix and fiber:

- **For matrix**
  - $J_1 \geq 0$, $J_1 \geq J_1^{critical}$
  - $J_1 < 0$, $\varepsilon_{vm} \geq \varepsilon_{vm}^{critical}$

- **For fiber**
  - $\varepsilon_{vm} \geq \varepsilon_{vm}^{critical}$
3. Micromechanical Modification
Homogenized lamina solution provides an average state of strain representing both the fiber and matrix phase at the same point in space.

In order to perform the microscopic analysis, a method of the micromechanical modification is required to amplified the average state of strain.

In micro-scale analysis, the mechanical strain amplification factor is very important parameter for connecting the micro-level and macro-level strain tensor.

\[
M_{ij} = \frac{\varepsilon_{ij}}{\left(\Delta L_{ij} / L_o\right)} \quad \Leftrightarrow \quad \{\varepsilon_{ij}\}_{total} = [M_{ij}] \{\varepsilon\}_{mech}
\]

- \( \varepsilon_{ij} \): local strain
- \( \Delta L_{ij} \): prescribed unit displacement
- \( L_o \): initial length of RVE which is parallel with loading direction
Strain amplification factor is calculated by finite element analysis for micro-mechanical block model.

The micromechanical block is called the RVE (Representative Volume Element) which is explicit model of fiber and matrix.

The type of RVE is divided by fiber packing arrays, namely square, hexagonal and diamond.

(a) Square  (b) Hexagonal  (c) Diamond
Micromechanical Block Modeling

- In order to obtain the strain amplification factor of IM7/K3B composite materials, a finite element model of the RVE was generated by MSC.Patran.
- The single cell model of square array type was used to perform the microscopic analysis.

<table>
<thead>
<tr>
<th>Mechanical Property</th>
<th>Fiber (IM7)</th>
<th>Matrix (K3B)</th>
<th>Composite Lamina</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ [GPa]</td>
<td>303</td>
<td>3.31</td>
<td>177.075</td>
</tr>
<tr>
<td>$E_{22}$ [GPa]</td>
<td>15.2</td>
<td>3.31</td>
<td>10.403</td>
</tr>
<tr>
<td>$G_{12}$ [GPa]</td>
<td>9.65</td>
<td>1.23</td>
<td>6.105</td>
</tr>
<tr>
<td>$G_{23}$ [GPa]</td>
<td>6.32</td>
<td>1.23</td>
<td>6.105</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.2</td>
<td>0.35</td>
<td>0.2947</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.2</td>
<td>0.35</td>
<td>0.2947</td>
</tr>
</tbody>
</table>

RVE (Representative Volume Element)
Boundary Condition of Micromechanical block

- RVE is given prescribed unit displacement in three case of normal deformation for one of the faces.
- Also, the other five faces are constrained by symmetric condition.

Definition of boundary conditions

<table>
<thead>
<tr>
<th>Mechanical strain amplification factor</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{11}$ (longitudinal direction)</td>
<td>$\varepsilon_{11} = 1, \varepsilon_{22} = \varepsilon_{33} = \gamma_{12} = \gamma_{13} = \gamma_{23} = 0$</td>
</tr>
<tr>
<td>$M_{22}$ (transverse direction)</td>
<td>$\varepsilon_{22} = 1, \varepsilon_{11} = \varepsilon_{33} = \gamma_{12} = \gamma_{13} = \gamma_{23} = 0$</td>
</tr>
<tr>
<td>$M_{33}$ (transverse direction)</td>
<td>$\varepsilon_{33} = 1, \varepsilon_{11} = \varepsilon_{22} = \gamma_{12} = \gamma_{13} = \gamma_{23} = 0$</td>
</tr>
</tbody>
</table>
10 points were selected in the single cell for the extraction of local strain values.

- Matrix region: M00 (Interstitial location), M01-M02 (Inter-fiber location), M1-M3 (Fiber/matrix interface)
- Fiber region: F1-F3 (Fiber/matrix interface), F4 (Center of fiber)
Matrix Phase Fiber Phase

- $M_{11}$ for longitudinal direction are all 1.0 at any selected points in fiber and matrix.
- $M_{22}$ and $M_{33}$ are equal due to rotational symmetry for square array.

- The result of the strain amplification factor, $M_{22}$

- Maximum strain amplification factors for each direction

<table>
<thead>
<tr>
<th>Region</th>
<th>Strain invariant</th>
<th>$M_{11}$</th>
<th>$M_{22}$</th>
<th>$M_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix phase</td>
<td>Case of $J_1$</td>
<td>1</td>
<td>2.61523</td>
<td>2.61523</td>
</tr>
<tr>
<td></td>
<td>Case of $\varepsilon^m_{vm}$</td>
<td>1</td>
<td>2.72389</td>
<td>2.72389</td>
</tr>
<tr>
<td>Fiber phase</td>
<td>Case of $\varepsilon^f_{vm}$</td>
<td>1</td>
<td>0.77291</td>
<td>0.77291</td>
</tr>
</tbody>
</table>
4. Multi-scale Failure Prediction of Composite Plate
In order to perform the failure analysis, the strain tensor was extracted for composite open-hole plate model through the finite element analysis.

- The materials of composite plate is IM7(graphite) / K3B(epoxy).
- The stacking sequence is \([45^{\circ}/90^{\circ}/-45^{\circ}/0^{\circ}]_s\)
Multi-scale Failure Analysis based on SIFT

Flow chart for multi-scale failure analysis

Macro- / Meso-scale Analysis

- Macroscopic loading for (Laminate)
- Determination of Stress/Strain in each ply

Micro-scale Analysis

- Micro-mechanical Modification by Strain Amplification Factors
- Calculating Effective Strain Invariants $J_1^m, \varepsilon_{vm}^m, \varepsilon_{vm}^f$

- Micro-mechanical Failure Prediction for each ply
- Calculating Failure Indices for each Failure Criterion
Strain distribution of composite plate

Layer 1 (45°)
X Component (Longitudinal direction)

Layer 2 (90°)
X Component (Longitudinal direction)
Strain distribution of composite plate

Layer 3 (-45°)
X Component (Longitudinal direction)

Layer 4 (0°)
X Component (Longitudinal direction)

Y Component (Transverse direction)
The strain components were amplified by the strain amplification factors. And then, the effective strain invariants \((J_1, \varepsilon_{vm})\) were calculated.

\[
J_1 = M_{11} \varepsilon_1 + M_{22} \varepsilon_2 + M_{33} \varepsilon_3
\]

\[
\varepsilon_{vm} = \sqrt{0.5[(M_{11} \varepsilon_1 - M_{22} \varepsilon_2)^2 + (M_{11} \varepsilon_1 - M_{33} \varepsilon_3)^2 + (M_{22} \varepsilon_2 - M_{33} \varepsilon_3)^2]}
\]

Critical strain invariants for graphite/epoxy laminated composite (IM7/K3B) were used experimental data from reference [2].

<table>
<thead>
<tr>
<th>Critical invariant</th>
<th>Value</th>
<th>Laminate stacking sequence</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J_{1-cr}^m)</td>
<td>0.0274</td>
<td>([90]_{10})</td>
<td>Matrix / Dilatational</td>
</tr>
<tr>
<td>(\varepsilon_{vm-cr}^m)</td>
<td>0.103</td>
<td>([10]_{10})</td>
<td>Matrix / Distortional</td>
</tr>
<tr>
<td>(\varepsilon_{vm-cr}^f)</td>
<td>0.0182</td>
<td>([0]_{10})</td>
<td>Fiber / Distortional</td>
</tr>
</tbody>
</table>
Failure indices for each layer $[45^\circ/90^\circ/-45^\circ/0^\circ]_s$

Layer 1 ($45^\circ$)

Layer 2 ($90^\circ$)

Layer 3 ($-45^\circ$)

Layer 4 ($0^\circ$)

*J_1* failure is dominant

$\varepsilon_{vm}$ failure is dominant
Comparing with existing theories (Maximum strain, Tsai-Wu, Hashin, Sun failure criterion)

Similar tendency was confirmed from the distribution of failure index.
5. Damage Evaluation of Composite Plate using Property Degradation Model
Micro-mechanical failure modes can be determined with Strain invariant failure criteria.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Loading Condition</th>
<th>Failure Mode</th>
<th>Failure Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>Tensile (Longitudinal)</td>
<td>Fiber fracture</td>
<td>$\varepsilon_{vm}^f$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fiber pullout</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Compression (Longitudinal)</td>
<td>Fiber kinking</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fiber buckling</td>
<td></td>
</tr>
<tr>
<td>Matrix</td>
<td>Tensile</td>
<td>Matrix cracking ($J_1 \geq 0$)</td>
<td>$J_1$</td>
</tr>
<tr>
<td></td>
<td>Compression</td>
<td>Matrix compression failure ($J_1 &lt; 0$)</td>
<td>$\varepsilon_{vm}^m$</td>
</tr>
<tr>
<td>Fiber/Matrix</td>
<td>Tensile / Compression</td>
<td>Fiber/Matrix shearing (de-bonding) failure</td>
<td>$\varepsilon_{vm}^m$</td>
</tr>
</tbody>
</table>

![Matrix cracking (Tens. / Comp.)](image1)
![Fiber fracture](image2)
![Fiber pullout](image3)
![Fiber/Matrix shearing (de-bonding) failure](image4)
Material degradation models were defined for each failure mode.

- Matrix failure: \( J_1 \geq 0 \), \( J_1^m \geq J_{1-cr}^m \)
  \( J_1 < 0 \), \( \varepsilon_{vm}^m \geq \varepsilon_{vm-cr}^m \)

  \[ E_y = \nu_{xy} = 0 \]

- Fiber failure: \( \varepsilon_{vm}^f \geq \varepsilon_{vm-cr}^f \)

  \[ E_x = \nu_{xy} = 0 \]

- Fiber/matrix shearing: \( \varepsilon_{vm}^m \geq \varepsilon_{vm-cr}^m \)

  \[ \nu_{xy} = G_{xy} = 0 \]

User-subroutine (USDFLD) in ABAQUS/CAE was used to degrade the material properties.

<table>
<thead>
<tr>
<th>Material State (Failure Mode)</th>
<th>Elastic Properties</th>
<th>Field variable #1</th>
<th>Field variable #2</th>
<th>Field variable #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No failure</td>
<td>( E_x ) ( E_y ) ( \nu_{xy} ) ( G_{xy} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Matrix failure</td>
<td>( E_x ) 0 0 0 ( G_{xy} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fiber failure</td>
<td>0 ( E_y ) 0 ( G_{xy} )</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Fiber/matrix shearing failure</td>
<td>( E_x ) ( E_y ) 0 ( G_{xy} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Matrix &amp; Fiber failure</td>
<td>0 0 0 ( G_{xy} )</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Matrix failure &amp; Fib/mtx shear</td>
<td>( E_x ) 0 0 0 ( G_{xy} )</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Fiber failure &amp; Fib/mtx shear</td>
<td>0 ( E_y ) 0 0 ( G_{xy} )</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>All failure modes</td>
<td>0 0 0 0 ( G_{xy} )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Damage Evaluation using Property Degradation Model

- Flow chart for damage evaluation using property degradation models

Start of Increment

Macro- / Meso-scale Analysis
- Macroscopic loading for (Laminate)
- Determination of Stress/Strain in each ply

Micro-scale Analysis
- Micro-mechanical Modification by Strain Amplification Factors
- Micro-scale Stress/Strain (Fiber, Matrix)

Re-define Mechanical Properties by Material Degradation Models

Calculating Damage for each failure modes ($D_m, D_i, D_p$)

Calculating Effective Strain Invariants $J_1^m, \varepsilon_{vm}^m, \varepsilon_{vm}'$

Failure Occur? ($D \geq 1$)

YES
- USDFLD

NO
- Start of Next Increment
- The materials of composite plate is IM7/K3B.
- The stacking sequence is \([(-45^\circ/+45^\circ)]_6\).

Composite open-hole plate model was generated by ABAQUS/CAE.
Static analysis was performed using property degradation models.
Damage distribution of composite plate

Damage for Matrix failure mode

>> Strain Invariant : $J_1$ (Tensile) / $\varepsilon_{vm}^m$ (Compression)

Increment 1
Increment 6
Critical location

Increment 2
Increment 7

Increment 3
Increment 8

Increment 4
Increment 9
Failure region is extended

Increment 5
Increment 10
Local failure occurs
Damage distribution of composite plate

Damage for Fiber failure mode

>> Strain Invariant: $\varepsilon_{vm}^f$ (Tensile / Compression)
Damage Evaluation of Composite Plate using Property Degradation Model

- Damage distribution of composite plate

- Damage for Fiber/matrix shearing failure mode

\[ \varepsilon_{vm}^m \text{ (Tensile / Compression)} \]

Increment 1
Increment 2
Increment 3
Increment 4
Increment 5
Increment 6
Increment 7
Increment 8
Increment 9
Increment 10
6. Conclusions
Conclusions

Damage Evaluation with Multi-scale Failure Analysis using Material Degradation Model based on SIFT

- The strain amplification factors were obtained from the result of micro-mechanical analysis for RVE model.

- Micro-mechanical modification was performed for macroscopic strains of composite plate model.

- The strain-based failure analysis was performed by using the SIFT.

- The result of failure analysis was verified by comparing with existing theories.

- Micro-mechanical progressive damage was evaluated for composite plate using the material property degradation models.
Thank you very much!