Unit cell simulation of fatigue damage in short fiber reinforced plastic composites

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Introduction

Discontinuous-fiber reinforced plastics

- Lightweight
- Superior formability
- High cycle molding
  (Thermoplastic matrix system)

Promising materials for automotive application

Radiator core support
(Injection molding)
http://www.calsonickansei.co.jp/

Door inner
(Press molding)
http://www.jgmt.net/
Lack of mechanical strength of discontinuous-fiber reinforced plastics is limiting its application.

Conventional short fiber composite couldn’t use up the strength of reinforcement fiber.
We simulated the random-oriented short fiber composites. The average fiber length is about 100mm and close to the real fiber length in polymer composites made by the injection molding.
Outline of numerical simulation

First, this approach divided the strain and displacement into macroscopic one and microscopic one. Then, the microscopic displacement field is calculated by controlling the macroscopic strain.

\[
\begin{align*}
\mathbf{u} &= \mathbf{u}_G + \mathbf{u}_L \\
\Delta \mathbf{\varepsilon} &= \Delta \mathbf{\varepsilon}_G + \Delta \mathbf{\varepsilon}_L \\
(K_f + K_m + K_{coh}) \Delta U_L &= F - \left( Q_f + Q_m + Q_{coh} \right) - \Delta Q_G \\
\Delta Q_G &= \sum_e \int_{V_e} B^T D_f \Delta \varepsilon_G dV + \sum_e \int_{V_e} (1 - D) B^T D_m \Delta \varepsilon_G dV
\end{align*}
\]

Therefore, the displacement field in a unit cell is calculated when the macroscopic average strain is given.
Modeling of micro damages in short fiber composites

Damage mechanics model is used to express the complicated damage in a matrix.

Constitutive law is given with internal damage $D$ by

$$\Delta \sigma = (1 - D)C^{ep} : \Delta \varepsilon - \frac{\Delta D}{1 - D} \sigma$$

$D = 0$ → Intact
$D = 1$ → Completely damaged

Evolution law of damage variable $D$ is given by

$$\Delta D = A(1 - D) \left\{ \Delta \left\{ D \left( \frac{\langle \sigma_m \rangle}{\sigma_y} \right)^2 \right\} \right\} + (B_0 + B_1 D) \Delta \varepsilon^p$$

Void expansion term referring to Gurson, Shizawa et al.

Shear failure term

$A, B_0, B_1$ are phenomenological parameters.
Simulated results

\[ \left[ \theta_{\text{lower}}, \theta_{\text{upper}} \right] = [0.4\pi, 0.6\pi] \]

* Mandell et al. (1982)
Fatigue damage progress of the matrix

**Kachanov damage mechanics**

\[
\frac{dD}{dt} = \left( \frac{1}{A} \right)^\gamma \frac{\sigma^\gamma}{(1 - D)^k}
\]

※ Kachanov LM. Introduction to Continuum Damage Mechanics

**Evolution law of fatigue damage**

\[
\Delta D = \Delta N \left( \frac{1}{A} \right)^\gamma \frac{\sigma_m^\gamma}{(1 - D)^k}
\]

\(\Delta N = 1000\) (cycles) is used in this study
Flow chart

1. Start
2. Static incremental analysis (Tension loading and unloading)
3. Calculate maximum hydrostatic pressure $\sigma_m$ during the incremental analysis (for all integration points)
4. Calculate fatigue damage increment and stress degradation for $\Delta N$ cycles
5. If $\Delta D > 0$, NO and go to FINAL FAILURE; if $\Delta D < 0$, YES and go to STOP.

Mathematical equations:

$$\Delta D = A(1 - D) \left\Delta \left\{ D \left( \frac{\sigma_m}{\sigma_y} \right)^2 \right\} \right\} + (B_0 + B_1 D) \Delta \varepsilon^p$$

$$\Delta D = \Delta N \left( \frac{1}{A} \right)^\gamma \frac{\sigma_m^\gamma}{(1 - D)^k}$$

$$\Delta \sigma = -\frac{\Delta D}{1 - D} \sigma$$
Previous experiments by Ha, Yokobori and Takeda (1999)

Tensile stress: 0 ↔ 32 MPa
Previous experiments by Ha, Yokobori and Takeda (1999)
Simulated results

Initial damage

DEG 0
$N = 20000$ (cycles)

90° 方向
$N = 1000$ (cycles)
Simulated results

DEG 45
$N = 4000$ (cycles)

DEG 90 $> \ DEG 45 > \ DEG 0$
→ Agree with the experiments
Effect of fiber length on damage mode

Fracture modes of short fiber reinforced composite differ from that of long fiber reinforced composite.

Continuous (long) fiber

Discontinuous (short) fiber

In this presentation, we discuss how the short fiber reinforced composites are broken.

* from Sato et al. (1991)
Thermoplastic press sheet with carbon fiber

Materials

Thermoplastic resin + Carbon fiber mat → Sheet

Features

- Long fiber length
- Good fiber dispersion
- In-plane random fiber orientation
- Thermoplastic matrix

Strength

Formability

Fig. Photos of the sheet

(a) Overview

(b) Cross sectional area
Strength and formability

Strength
- GMT: Glass mat thermoplastics
- SMC: Sheet molding compound

Formability
- Exhibits good flowability, complex shape is easily fabricated

Much superior to conventional short fiber composite
Unit cell model for the unidirectional short fiber composites

Fiber modeling

Cumulative failure probability

\[ P_f (\sigma) = 1 - \exp \left( \left( -\frac{\sigma}{\sigma_0} \right)^p \right) \]

Matrix modeling

Evolutionary equation of damage variable \( D \)

\[ \Delta D = A (1 - D) \left\{ D \left( \frac{\sigma_m}{\sigma_y} \right)^2 \right\} + (B_0 + B_1 D) \Delta \varepsilon^p \]

Void expansion term referring to Gurson, Shizawa et al.

Shear failure term

\( A, B_0, B_1 \) are phenomenological parameters.
Simulated results

$l_f = 0.02\ \text{mm}$  

$l_f = 0.2\ \text{mm}$  

$l_f = 0.3\ \text{mm}$  

$l_f = 1.0\ \text{mm}$
Initial stiffness of the composite increases as $l_f$ increases.

When $l_f \leq 0.1\text{mm}$, the initiation of matrix cracking causes the critical failure.

When $0.1\text{mm} < l_f < 0.5\text{mm}$, the crack is trapped. But, the failure is caused by the fiber-avoiding.

When $l_f \geq 0.5\text{mm}$, the failure is determined by the fiber-breaking of the neighboring.
Theoretical model for the fiber length effect on the composite’s strength


The length of discontinuous fibers is represented by the density of initial broken segments in the fiber.

\[ L \]

Averaged stress carried by the fibers is equal to the composite stress (Duva et al.):

\[ \sigma_\infty = V_j \sigma_f \frac{1}{\Phi} (1 - e^{-\Phi}) \]

\[ \Phi = 2d_b L_T = 2 \frac{N}{L} L_T = 2L_T \left\{ d_0 + \frac{1}{L_0} \left( \frac{\sigma_f}{\sigma_0} \right)^\rho \right\} \]

In calculating the stress-transfer length \( L_T \), we used the solution of elastic-plastic hardening shear-lag model (Okabe and Takeda, 2000).
Composite strength versus averaged fiber stress

Kelly & Tyson length is shorter to get fiber failure mode (high strength).

Duva & Curtin has a good agreement with experiments
Effect of fiber length on fatigue life

[Graphs showing the effect of fiber length on maximum stress and normalized maximum stress over cycles.]
Fatigue damage in UD composites (0.1mm)

$L_f = 0.1\text{mm} \quad \sigma_{\text{max}} = 29\text{MPa}$

Initial cycle

After 11 cycle

After 31 cycle
Fatigue damage in UD composites (2.1mm)

$L_f = 2.1\text{mm}$, $\sigma_{\text{max}} = 158\text{MPa}$

Initial cycle, After 3204 cycle, 3205 cycle, Applied strain=0.564%