Fatigue Life Prediction of Composites Using Micro-Mechanics of Failure

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Motivation – To Achieve Better Fatigue Life Prediction of Composite Structures

Most challenging task to predict the fatigue life!

Goodman diagram for fatigue life prediction:
Uniaxial only, Laminate dependent, Accuracy ??

The real fatigue loads and experimental data show…….

• Bi-axial loads
• Laminate dependent
• Nonlinear life diagram

Develop a generic methodology and tools!!
→ Minimize the number of tests
→ Multi-scale approach
→ Predict Life
Multi-Scale Modeling for Structural Analysis of Composites

- Strength Prediction
- Life prediction

**Structures**

- Laminates (+cohesive)
- Ply

**Micromechanics**

- Fiber, Matrix & Interface
- Molecular level

**Nano-Scale**

**Micromechanics: MMF, SIFT & MCT**

- μσ relations
- Yield
- Linear

Temperature [°C]
Static and Fatigue Strength Prediction Using MMF

- Micromechanics models to compute micro-stresses.
- Failure criteria and Fatigue life diagram for each constituent.

**Compute micro-stresses**

Homogeneous Strains calculated by FEM

\[ \bar{\sigma} = \bar{C}(\bar{\varepsilon} - \bar{\alpha}\Delta T) \]

**Macro \to micro**

\[ \bar{\sigma}_{\text{macro}} \Delta T \Rightarrow \sigma^{(i)}_{\text{micro}} \]

**Fatigue Life Diagram for each constituent**

**Mean Stress Effects**

\[ \bar{\sigma} = \bar{C}(\bar{\varepsilon} - \bar{\alpha}\Delta T) \]

**Predict static strengths**

\[ F(\sigma^{(i)}_{\text{micro}}) < 1 \]

**Failure Criteria for each constituent**

Matrix

Fiber

Interface

\[ k < 1 \quad \text{and} \quad k = 1 \]
Micromechanics: Regular & Random Fiber Arrays
Micromechanical Model – Regular Fiber Arrays

Cross-sectional view of continuous fiber reinforced composites

Square Array

Hexagonal Array

Diamond Array
Calculation of Micro Stresses – The Concept of Stress Amplification Factor (SAF)

\[ \sigma = M \bar{\sigma} + A \Delta T \]

- **M** Stress amplification factor for macro stresses
- **A** Stress amplification factor for temperature increment
- **\( \bar{\sigma} \)** Ply-level macro stresses
- **\( \sigma \)** Constituent-level micro stresses
- **\( \Delta T \)** Temperature increment
Calculation of Micro Stresses: Formulas in Extenso (1)

For a material point within the fiber

\[ \sigma_f^{(k)} = \mathbf{M}_f^{(k)} \bar{\sigma} + A_f^{(k)} \Delta T \]

\[ \begin{bmatrix} \sigma_1 = \sigma_{11} \\ \sigma_2 = \sigma_{22} \\ \sigma_3 = \sigma_{33} \\ \sigma_4 = \sigma_{23} \\ \sigma_5 = \sigma_{31} \\ \sigma_6 = \sigma_{12} \end{bmatrix}^{(k)} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & 0 & 0 \\ M_{21} & M_{22} & M_{23} & M_{24} & 0 & 0 \\ M_{31} & M_{32} & M_{33} & M_{34} & 0 & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & M_{56} \\ 0 & 0 & 0 & 0 & M_{65} & M_{66} \end{bmatrix}_f \]

For a material point within the matrix

\[ \sigma_m^{(k)} = \mathbf{M}_m^{(k)} \bar{\sigma} + A_m^{(k)} \Delta T \]

\[ \begin{bmatrix} \sigma_1 = \sigma_{11} \\ \sigma_2 = \sigma_{22} \\ \sigma_3 = \sigma_{33} \\ \sigma_4 = \sigma_{23} \\ \sigma_5 = \sigma_{31} \\ \sigma_6 = \sigma_{12} \end{bmatrix}^{(k)} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & 0 & 0 \\ M_{21} & M_{22} & M_{23} & M_{24} & 0 & 0 \\ M_{31} & M_{32} & M_{33} & M_{34} & 0 & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & M_{56} \\ 0 & 0 & 0 & 0 & M_{65} & M_{66} \end{bmatrix}_m \]
Calculation of Micro Stresses: Formulas in Extenso (2)

For a point at the fiber-matrix interface

\[ \sigma_i^{(k)} = M_i^{(k)} \bar{\sigma} + A_i^{(k)} \Delta T \]

\[
\begin{bmatrix}
    t_x^{(k)} \\
    t_n \\
    t_t_i
\end{bmatrix}
=
\begin{bmatrix}
    M_{11} & M_{12} & M_{13} & M_{14} & 0 & 0 \\
    M_{21} & M_{22} & M_{23} & M_{24} & 0 & 0 \\
    M_{31} & M_{32} & M_{33} & M_{34} & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    \bar{\sigma}_1 \\
    \bar{\sigma}_2 \\
    \bar{\sigma}_3 \\
    \bar{\sigma}_4 \\
    \bar{\sigma}_5 \\
    \bar{\sigma}_6
\end{bmatrix}
+
\begin{bmatrix}
    A_1 \\
    A_2 \\
    A_3
\end{bmatrix}
\Delta T
\]

- \( t_x \) Longitudinal interfacial traction
- \( t_t \) Tangential interfacial traction
- \( t_n \) Normal interfacial traction
Examples of Micro Stress Distribution in Unit Cells

- **Micro transverse stress due to unit macro transverse tensile stress**
  \[ \sigma_2 = M_{22} \]
  
  Contour of micro stress \( \sigma_2 \)
  
  \[ \overline{\sigma}_2 = 1 \]

- **Micro in-plane shear stress due to unit macro in-plane shear stress**
  \[ \sigma_6 = M_{66} \]
  
  Contour of micro stress \( \sigma_6 \)
  
  \[ \overline{\sigma}_6 = 1 \]
Micromechanical Model – Random Fiber Array

Cross-sectional view of continuous fiber reinforced composites
Finite Element Modeling of the Random Fiber Array

- $V_f = 0.6$
- Number of fibers: 120

Cross-sectional view

Mesh generation

3D view of fibers

3D view of matrix
Comparison between Random Array and Regular Arrays

- **Random fiber array** ($V_f = 0.6$)
- **Regular fiber arrays** ($V_f = 0.6$)

(a) random array ($V_f = 0.6$)

(b) regular arrays

- $\bar{\varepsilon}_2 = 0.1\%$
- $\bar{\varepsilon}_2 = 0.1\%$
Micromechanics of Failure (MMF): Constituent Failure Criteria
Constituent Failure Criteria

- **Fiber failure criterion**
  - Maximum longitudinal stress failure criterion

\[-C_{f1} < \sigma_{f1} < T_{f1}\]

\(T_{f1}\) Fiber longitudinal tensile strength
\(C_{f1}\) Fiber longitudinal compressive strength

- **Quadratic failure criterion**

\[\sum_{j=1}^{6} \sum_{i=1}^{6} F_{ij} \sigma_{fi} \sigma_{fj} + \sum_{i=1}^{6} F_i \sigma_{fi} = 1\]

\(F_{ij}, F_i\) Coefficients associated with fiber strengths

- **Matrix failure criterion**

\[\sigma_{VM}^2 + (C_m - T_m) I_1 = C_m T_m\]

\(T_m\) Matrix tensile strength
\(C_m\) Matrix compressive strength

- **Interface failure criterion**

\[\left(\frac{t_n}{Y_n}\right)^2 + \left(\frac{t_s}{Y_s}\right)^2 = 1\]

\(Y_n\) Interface normal strength
\(Y_s\) Interface shear strength
Incorporation of Kinking Model for Longitudinal Compressive Failure

3D kinking model

Fig. 10. (a) Kinking band; (b) fiber misalignment frame

\[
\begin{align*}
\sigma_{11}^m &= \frac{\sigma_{11} + \sigma_{22}}{2} + \frac{\sigma_{11} - \sigma_{22}}{2} \cos(2\varphi) + \tau_{12} \sin(2\varphi) \\
\sigma_{22}^m &= \frac{\sigma_{11} + \sigma_{22}}{2} - \frac{\sigma_{11} - \sigma_{22}}{2} \cos(2\varphi) - \tau_{12} \sin(2\varphi) \\
\tau_{12}^m &= -\frac{\sigma_{11} - \sigma_{22}}{2} \sin(2\varphi) + \tau_{12} \cos(2\varphi)
\end{align*}
\]

Kinking Macro Stresses

Micro stresses (Matrix)

SAF

MMF (Matrix FC)
Biaxial Tests Using Cruciform Specimens

Laminate specimen for biaxial test

Specimen held in grips of biaxial test machine

MMF failure envelope & biaxial test data

<table>
<thead>
<tr>
<th>Material Property</th>
<th>IM7/8551-7 (WWFE- I )</th>
<th>IM7/8552 (Hexcel data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E11 (GPa)</td>
<td>167</td>
<td>164</td>
</tr>
<tr>
<td>E22 (GPa)</td>
<td>8.4</td>
<td>12</td>
</tr>
<tr>
<td>nu12</td>
<td>0.27</td>
<td>0.27*</td>
</tr>
<tr>
<td>X (MPa)</td>
<td>2550</td>
<td>2724</td>
</tr>
<tr>
<td>X' (MPa)</td>
<td>1600</td>
<td>1690</td>
</tr>
<tr>
<td>Y (MPa)</td>
<td>72</td>
<td>111</td>
</tr>
<tr>
<td>Y' (MPa)</td>
<td>189</td>
<td>189*</td>
</tr>
<tr>
<td>S (MPa)</td>
<td>65</td>
<td>120</td>
</tr>
</tbody>
</table>

* Value not provided
Tri-Axial Strength Prediction: WWFE-II Test Case (2)

Test Case (2): Variation of the shear strength of UD carbon/epoxy with hydrostatic pressure

Stress Strain Curve: $\sigma_2 (=\sigma_3 = \sigma_1)$ vs. $\tau_{12}$ for T300/PR319 epoxy

**Triaxial Test Results For Fibre Reinforced Composites: Second World-Wide Failure Exercise Benchmark Data, M J Hinton and A S Kaddour, 2009**
Tri-Axial Strength Prediction: WWFE-II Test Case (6)

Test Case (6): Variation of the **longitudinal strength** with **through-thickness stress** for a UD S-glass/Epoxy2

**Failure Envelope:** $\sigma_1$ vs. $\sigma_3$ (= $\sigma_2$) for S-glass/Epoxy2

$\sigma_3 = \sigma_2$ (MPa)

**Final Matrix Failure due to Kinking**

**Final Fiber Failure**

**Initial Matrix Failure**

**Test Data**

**MMF prediction**
- Averaged SAF (Hex) for $M_{55}$ & $M_{66}$
- Quadratic fiber FC
- Kinking model ($\theta=4.5^\circ$)
- Progressive damage

Triaxial Test Results For Fibre Reinforced Composites: Second World-Wide Failure Exercise Benchmark Data, M J Hinton and A S Kaddour, 2009
Test Case (8): Variation of the axial compressive strength with through-thickness stress for $[\pm 35]s$ laminate

Failure Envelope: $\sigma_x (=\sigma_z)$ vs. $\sigma_y$ for $[\pm 35]s$; E-glass/MY750 epoxy

#### Triaxial Test Results

For Fibre Reinforced Composites: Second World-Wide Failure Exercise Benchmark Data, M J Hinton and A S Kaddour, 2009
Fatigue Life Prediction Using MMF
Key Issues to Address in Fatigue Life Prediction of Composites

- Why Micromechanics?

- Uni-axial fatigue loads: S-N curves

- Multi-axial fatigue loads

- Mean stress effects

- Random fatigue loads
  
  - Low cycle fatigue (N<100,000) or High cycle fatigue (N>100,000) ?
  - Strain or Stress based ?
  - Crack initiation, Crack propagation, or life prediction ?
Procedures of MMF-Based Fatigue Life Prediction

**Macro Stress Analysis**

3D beam or FEM

 Loads & Temperatures

\[ F \rightarrow \Delta T \rightarrow \text{FEM (3D beam)} \]

Laminate

\[ N_1 \quad N_6 \quad M_3 \quad M_6 \]

Multi-axial Random Fatigue loading

CLT

\[ \bar{\sigma} \]

**Micro Fatigue Analysis**

Multi-axial Stress

\[ \sigma_i, a \quad \sigma_i, m \]

Mean Stress Effects

\[ \sigma_{eq, a} \quad \sigma_{eq, m} \]

S-N cure

\[ D = \sum \frac{n_i}{N_i} \]

Predict Life

Damage accumulation and/or Rain-flow counting

\[ R = -0.7 \quad R = -0.5 \quad R = -0.3 \quad R = 0 \quad R = 0.5 \]

**Micro mechanical model**

Matrix

Fiber

\[ \sigma_{xx,f} \]

Micro stresses

Time

\[ \sigma_{eq} \quad \sigma_{eq,a} \quad \sigma_{eq,m} \]

Constant Life Diagram Of Each Constituent

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Laminates Subjected to Fatigue Loadings

• **Laminate**

  In-plane loads

  \[
  \begin{align*}
  N &= N(t) \\
  M &= M(t)
  \end{align*}
  \]

  \[
  \sigma_{xx}^{\text{on-axis}} = \sigma_{xx}^{\text{on-axis}}(t) \\
  \sigma_{yy}^{\text{on-axis}} = \sigma_{yy}^{\text{on-axis}}(t) \\
  \sigma_{ss}^{\text{on-axis}} = \sigma_{ss}^{\text{on-axis}}(t)
  \]

  \[
  \sigma_{ij}^{\text{in-plane loads}}
  \]

  FEM or CLT

• **Ply**

  on-axis macro stresses

  \[
  \begin{align*}
  \overline{\sigma}_x &= \overline{\sigma}_x(t) \\
  \overline{\sigma}_y &= \overline{\sigma}_y(t) \\
  \overline{\sigma}_s &= \overline{\sigma}_s(t)
  \end{align*}
  \]

  \[
  \Delta T
  \]

  In-plane loads
Calculation of Micro Stresses in Constituents

\[
\begin{align*}
\bar{\sigma}_1 &= \bar{\sigma}_1(t) \\
\bar{\sigma}_2 &= \bar{\sigma}_2(t) \\
\bar{\sigma}_6 &= \bar{\sigma}_6(t)
\end{align*}
\]

on-axis macro stresses

Stress Amplification Factors (SAF)

\[
\begin{bmatrix}
\sigma_1^{(k)} \\
\sigma_2^{(k)} \\
\sigma_3^{(k)} \\
\sigma_4^{(k)} \\
\sigma_5^{(k)} \\
\sigma_6^{(k)}
\end{bmatrix}
= 
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} & 0 & 0 \\
M_{21} & M_{22} & M_{23} & M_{24} & 0 & 0 \\
M_{31} & M_{32} & M_{33} & M_{34} & 0 & 0 \\
M_{41} & M_{42} & M_{43} & M_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & M_{55} & M_{56} \\
0 & 0 & 0 & 0 & M_{65} & M_{66}
\end{bmatrix}
\begin{bmatrix}
\bar{\sigma}_1 \\
\bar{\sigma}_2 \\
\bar{\sigma}_3 \\
\bar{\sigma}_4 \\
\bar{\sigma}_5 \\
\bar{\sigma}_6
\end{bmatrix}
\]

\[
\sigma_f^{(k)} = M_f^{(k)} \bar{\sigma} + A_f^{(k)} \Delta T
\]

\[
\sigma_m^{(k)} = M_m^{(k)} \bar{\sigma} + A_m^{(k)} \Delta T
\]

\[
t_i^{(k)} = M_i^{(k)} \bar{\sigma} + A_i^{(k)} \Delta T
\]

Ply level macro cyclic stress \(\rightarrow\) micro cyclic stresses (Micro-mechanics)

\(\rightarrow\) Applied to constituent fatigue models (multi-axial stress & mean stress effects)
**MMF-Based Fatigue Models**

Equivalent stress distribution in the **matrix**

\[
\sigma_{eq,m} = \frac{(\beta - 1)I_{1,m} \pm \sqrt{(\beta - 1)^2 I_1^2 + 4\beta I_{\mathcal{M},m}^2}}{2\beta}
\]

Critical plane model at the **interface**

\[
\sigma_{eq,i} = \text{sign}(\sigma_n, \tau) \sqrt{\sigma_n^2 + (k\tau)^2}
\]

On a plane with directions \(\theta, \phi\)

Maximum stress model for **fiber**

\[
\sigma_{eq,f} = \sigma_{f1}
\]
Uniaxial Fatigue Tests to Generate S-N Curves of Resin

• Uni-axial fatigue loads

\[ \sigma_{\text{max}} \rightarrow \sigma \rightarrow \sigma_{\text{min}} \]

• Life prediction using S-N curve

\[ \sigma_a = f(N_f) \]
Effects of Mean Stress on Fatigue Life

For isotropic material

\[
\begin{align*}
\sigma_{\text{max}} & \quad \sigma_a \\
\sigma_{\text{min}} & \quad \sigma_m
\end{align*}
\]

Stress ratio, \( R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \)

\( \sigma_m \) : mean stress, \( \sigma_{\text{max}} \) : stress amplitude

Life depends on both amplitude and mean stresses

\[\sigma_a = \text{const}\]

\( R = \infty \) \quad \( R = -2 \) \quad \( R = -1 \) \quad \( R = -0.5 \) \quad \( R = 0 \) \quad \( R = 0.5 \)

\( \sigma_{\text{eff}} = \frac{T + C}{2} - \frac{T - C}{2} \)

\( \sigma_{\text{eff}} = \left( 1 - \frac{\sigma_{\text{m}}}{T} \right) \left( 1 + \frac{\sigma_{\text{m}}}{C} \right)^n \)

\( R = 0\) \quad \( R = -0.3 \) \quad \( R = 0.3 \) \quad \( R = 0.5 \) \quad \( R = 0 \) \quad \( R = -0.7 \)

S-N curve

Nonlinear Mean Stress Effects

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Equivalent Stress Method for Multi-Axial Fatigue Loadings

For isotropic material

\[ \sigma_{eq,a} = \frac{(\beta - 1)I_{1,a} \pm \sqrt{(\beta - 1)^2 I_{1,a}^2 + 4\beta \sigma_{vm,a}^2}}{2\beta} \]

\[ \sigma_{eq,m} = \frac{(\beta - 1)I_{1,m} \pm \sqrt{(\beta - 1)^2 I_{1,m}^2 + 4\beta \sigma_{vm,m}^2}}{2\beta} \]

where, \( \beta = \frac{C}{T} \)

• Determination of Equivalent Stress

• Mean stress effects
Critical Plane Model for the Fiber-Matrix Interface

- Determination of normal and shear stresses at the interface

\[ \sigma_{eq} = \text{sign}(\sigma_n, \tau) \sqrt{\sigma_n^2 + (k\tau)^2} \]

- Effective stress

- Mean stress

\[ \sigma_a = R \sigma_m \]

On a plane with directions \( \theta, \phi \)

\[ \sigma_{eq} \]

Time

\[ \sigma_{eq,a} \]

Time

\[ \sigma_{eq,m} \]

Time

\[ \sigma_a \]

N

\[ \sigma_m \]

N1

N2

N3

R = -0.7

R = -0.5

R = -0.3

R = -0.3

R = 0

R = 0.3

R = 0.5

R = 1.0

R = 0

\[ n = 1, m = 1 \]

\[ n = 1.5, m = 1 \]

\[ n = 1, m = 2 \]

\[ n = 1.5, m = 2 \]

\[ n = 3, m = 3 \]
Cycle Counting for Random Fatigue Loadings

- Simple rainflow counting for only one signal

- Multi-axial rainflow counting for several signals
Cumulative Damage Law

Random Fatigue Load

Number of cycle

\[ n_1 \quad n_2 \quad n_3 \quad n_4 \quad n_5 \quad \cdots \]

Number of cycle to failure

\[ N_{f,1} \quad N_{f,2} \quad N_{f,3} \quad N_{f,4} \quad N_{f,5} \quad \cdots \]

Damage index

\[ D_1 \quad D_2 \quad D_3 \quad D_4 \quad D_5 \quad \cdots \]

- Linear cumulative damage law (load segments with same failure modes failure modes, Miner’s rule)

\[ D = \sum_{i=1}^{k} D_i \]

- Non-linear cumulative damage law

\[ D = \sum_{i=1}^{k} D_i \exp \left\{ F_i \left[ D_i - 1 \right] \right\} \quad \text{OR} \quad D = \sum_{i=1}^{k} D_i^m \]
Fatigue Life Prediction of Off-Axis UD

• Matrix Fatigue S-N curves from the UD *

\[
\sigma_x = 15^\circ \quad \omega = 15^\circ
\]

\[
R = \frac{\sigma_{x,\text{max}}}{\sigma_{x,\text{min}}} = 0.1
\]

Matrix

\[
f(N_f) = 100(N_f)^{-0.07} - 19
\]

MMF

• Well predict the fatigue life of UD with other fiber angle

\[
\sigma_x = 5^\circ \quad \omega = 5^\circ
\]

\[
\sigma_x = 10^\circ \quad \omega = 10^\circ
\]

\[
\sigma_x = 60^\circ \quad \omega = 60^\circ
\]

But off-axis UD with small fiber angles are slightly overestimated because of matrix fatigue models.

Material Properties: Off-Axis UD

Material properties and analysis options for fatigue life prediction of the off-axis UD

1. Matrix material properties
Matrix: Epoxy (Epon826)
Multi-axial stress: Equivalent stress
Mean stress effect: Modified Goodman
S-N curve: Basquin's eqn.

2. Fiber material properties
Fiber: E-glass
Multi-axial stress: Max. stress ($s_{xx,f}$)
Mean stress effect: Modified Goodman
S-N curve: Basquin's eqn.

3. Interface material properties
Fiber: E-glass/Epoxy
Multi-axial stress: Critical plane
Mean stress effect: Modified Goodman
S-N curve: Basquin's eqn.

4. Micromechanics model
SQR(FEM) HEX(FEM) MCT

log $\sigma_{eq} = -0.073 \log N_f + 1.955$

---

Fatigue life prediction of the off-axis UD : off-axis angle

Off-axis UD
• Material: E-glass/Epoxy (Epon826)
• Layup: [15°], [30°], [60°]

- Micro-mechanics models (No averaged SAF)

S-N curve @ θ = 15°

S-N curve @ θ = 30°

S-N curve @ θ = 60°
Fatigue Life Prediction of the **Angled Laminates**

**Angled Laminates**

- Material: E-glass/Epoxy (Epon826)
- Layup: $[\pm \theta^\circ]_S$
- Angle, $\theta = 15, 30, 45, 60, 90^\circ$

![schematic diagram of angled laminates]

- Micro-mechanics models (Averaged SAF)
  - SQR(FEM)
  - HEX(FEM)

**S-N curve using Square model**

![S-N curve graph using Square model]

**S-N curve using Hexagonal model**

![S-N curve graph using Hexagonal model]

- Static strengths prediction using Tsai-Wu criteria in E-glass/MY750 (WWFE)
Material Properties: UDT, BX and TX

Material properties and analysis options for fatigue life prediction for E-glass/Epoxy(Hexion)

1. Matrix material properties  a)  2. Fiber material properties  b)  3. Interface material properties

Matrix: Epoxy (Hexion)
- Multi-axial stress: Equivalent stress
- Mean stress effect: Modified Goodman
- S-N curve: Basquin’s eq.

Fiber: E-glass
- Multi-axial stress: Max. stress ($\sigma_f$)
- Mean stress effect: Modified Goodman
- S-N curve: Basquin’s eq.

Fiber: E-glass/Epoxy
- Multi-axial stress: Critical plane
- Mean stress effect: Modified Goodman
- S-N curve: Basquin’s eq.

$$\sigma_{eq,i} = \frac{(-1)I_{1,i} \pm \sqrt{(-1)^2 I_{1,i}^2 + 4\beta \sigma_{int,i}^2}}{2\beta}$$

$$T_m = 68 \text{ MPa}$$
$$C_m = 125 \text{ MPa}$$

$$T_f = 2150 \text{ MPa}$$
$$C_f = 1450 \text{ MPa}$$

$$\sigma_{eq,f} = \sigma_f$$

$$\sigma_{eq,d} = \text{sign}(\sigma_n, \tau) \sqrt{\sigma_n^2 + (k\tau)^2}$$

4. Micromechanics models

- SQR(FEM)
- HEX(FEM)
- MCT

$$V_f = 0.42$$

a) Tested by HSCL

Fatigue Life Prediction of the UDT, BX and TX

UDT, BX and TX
- Material: E-glass/Epoxy (Hexion)
- Layup: UDT[90°], BX[±45°]s and TX[0°₂/±45°]s
- Micromechanical models
  - SQR(FEM)
  - HEX(FEM)
  - MCT
- Fatigue loading
  
\[ R = 0.1 \]

\[ \sigma_x \]

\[ \sigma_x, \text{max} \]

\[ \log N_f \]

\[ \sigma_x \]

S-N curve @ UDT[90°]

[90°] ply, matrix fail

Static failure

S-N curve @ BX[±45°]s

[±45°] ply, matrix fail

S-N curve @ TX[0°₂/±45°]s

LPF [0°] ply, fiber fail

Material: E-glass/Epoxy (Hexion)
Layup: UDT[90°], BX[±45°]s and TX[0°₂/±45°]s
Micromechanical models
- SQR(FEM)
- HEX(FEM)
- MCT
Fatigue loading
\[ R = 0.1 \]
\[ \sigma_x \]
\[ \sigma_x, \text{max} \]
\[ \log N_f \]
\[ \sigma_x \]
Implementation of MMF-Based Fatigue Analysis into FEM

FEM Tools

- **SMM+** ➔ Preprocessor for SMM+
- **3D BEAM** ➔ Interface for 3D BEAM
- **NASTRAN** ➔ Interface for NASTRAN
- **ABAQUS** ➔ Interface for ABAQUS

**MMFatigue Program (Visual Basic)**

- Micromechanics based fatigue analysis
- Multi-axial fatigue stress
- Mean stress effect (modified Goodman, Harris)
- Random fatigue loads (rainflow counting method)
- Damage accumulation (Miner’s rule)

**GUI**

- Fatigue loading
- Fatigue parameters (constituents)
- Stress amplification factor
- Options

**Import** *.mmi file;
- Node & Element information

**Import** *.mms file;
- Ply macro stresses

**Export** *.out file;
- Damage index

**Import** *.mmf file

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Summary and Conclusion

• The MMF fatigue life prediction of laminates, applied to real composites structures.
• Fatigue loads with various mean stresses and multi-axial loads or spectrum, random fatigue loads.
• Environmental effects (temperature and moisture) can be readily investigated.

• A MMF based life prediction module for commercial SMM+ is developed.
• A linkage with the 3D-beam, ABAQUS and NASTRAN is under development.

• Need to generate constituent Failure and Fatigue D/B from Experiments.

Thank you for your attention.

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See you in ICCM18, 2011, Jeju island, Korea.